Chapter 15
Option Valuation

Concept Questions

1. The six factors are the stock price, the strike price, the time to expiration, the risk-free interest rate, the stock price volatility, and the dividend yield.

2. Increasing the time to expiration increases the value of an option. The reason is that the option gives the holder the right to buy or sell. The longer the holder has that right, the more time there is for the option to increase in value. For example, imagine an out-of-the-money option that is about to expire. Because the option is essentially worthless, increasing the time to expiration obviously would increase its value.

3. An increase in volatility acts to increase both put and call values because greater volatility increases the possibility of favorable in-the-money payoffs.

4. An increase in dividend yields reduces call values and increases put values. The reason is that, all else the same, dividend payments decrease stock prices. To give an extreme example, consider a company that sells all its assets, pays off its debts, and then pays out the remaining cash in a final, liquidating dividend. The stock price would fall to zero, which is great for put holders, but not so great for call holders.

5. Interest rate increases are good for calls and bad for puts. The reason is that if a call is exercised in the future, we have to pay a fixed amount at that time. The higher the interest rate, the lower is the present value of that fixed amount. The reverse is true for puts in that we receive a fixed amount.

6. The time value of both a call option and a put option is the difference between the price of the option and the intrinsic value. For both types of options, as maturity increases, the time value increases since you have a longer time to realize a price increase (decrease). A call option is more sensitive to the maturity of the contract.

7. An option’s delta tells us the (approximate) dollar change in the option’s value that will result from a change in the stock price. If a call sells for $5.00 with a delta of .60, a $1 stock price increase will add $.60 to the option price, increasing it to $5.60.

8. The delta relates dollar changes in the stock to dollar changes in the option. The eta relates percentage changes. So, if the stock price rises by 4 percent ($100 to $104), an eta of 10 implies that the option price will rise by 40 percent.

9. Vega relates the change in volatility in percentage points to the dollar change in the option’s price. If volatility rises from 40 to 41 percent, a 1 point rise, and vega is .80, then the option’s price will rise by 80 cents.

10. Rho measures option price sensitivity to a change in the interest rate, where a 1 percent change in the interest rate causes the option price to change by approximately the amount rho. If the interest rate rises from 4 percent to 5 percent (a 1 percent increase), the call price will increase by $.14 from $10.00 to $10.14.
Core Questions

1. \[ d_1 = \frac{\ln(98/105) + (.04 + .62^2/2) \times 270/365}{.62 \times \sqrt{270/365}} = .1927 \]
\[ d_2 = .1927 - .62 \sqrt{270/365} = -.3405 \]

The standard normal probabilities are:
\[ N(d_1) = .5764 \quad N(d_2) = .3667 \]

Calculating the price of the call option yields:
\[ C = (98 \times .5764) - (100 \times e^{-0.04 \times 270/365} \times .3667) = 19.10 \]

2. \[ d_1 = \frac{\ln(23/25) + (.04 - .02 + .50^2/2) \times 60/365}{.50 \times \sqrt{60/365}} = -.2937 \]
\[ d_2 = -.2937 - .50 \sqrt{60/365} = -.4965 \]

The standard normal probabilities are:
\[ N(d_1) = .3845 \quad N(d_2) = .3098 \]

Calculating the price of the call option yields:
\[ C = (23 \times e^{-0.02 \times 60/365} \times .3845) - (25 \times e^{-0.04 \times 60/365} \times .3098) = 1.12 \]

3. \[ d_1 = \frac{\ln(81/75) + (.05 + .60^2/2) \times 180/365}{.60 \times \sqrt{180/365}} = .4518 \]
\[ d_2 = .4518 - .60 \sqrt{180/365} = .0305 \]

The standard normal probabilities are:
\[ N(d_1) = .6743 \quad N(d_2) = .5122 \]

Calculating the price of the call option yields:
\[ C = (81 \times .6743) - (75 \times e^{-0.05 \times 180/365} \times .5122) = 17.14 \]
4. \[ d_1 = \frac{\ln(87/95) + (.06 - .02 + .55^2/2) \times 45/365}{.55 \times \sqrt{45/365}} = -.3334 \]
\[ d_2 = -.3334 - .55 \sqrt{45/365} = -.5265 \]

The standard normal probabilities are:
\[ N(d_1) = .3694 \quad N(d_2) = .2993 \]

Calculating the price of the call option yields:
\[ C = ($87 \times e^{-0.02 \times 45/365} \times .3694) - ($95 \times e^{-0.06 \times 45/365} \times .2993) = $3.84 \]

5. \[ d_1 = \frac{\ln(53/50) + (.07 - .03 + .45^2/2) \times 65/365}{.45 \times \sqrt{65/365}} = .4534 \]
\[ d_2 = .4534 - .45 \sqrt{65/365} = .2635 \]

The standard normal probabilities are:
\[ N(d_1) = .6749 \quad N(d_2) = .6039 \]

Calculating the price of the call option yields:
\[ C = ($53 \times e^{-0.03 \times 65/365} \times .6749) - ($50 \times e^{-0.07 \times 65/365} \times .6039) = $5.85 \]

6. \[ d_1 = \frac{\ln(86/85) + (.06 + .67^2/2) \times 48/365}{.67 \times \sqrt{48/365}} = .2021 \]
\[ d_2 = .2021 - .67 \sqrt{48/365} = -.0409 \]

The standard normal probabilities are:
\[ N(d_1) = .5801 \quad N(d_2) = .4837 \]
\[ N(-d_1) = .4199 \quad N(-d_2) = .5163 \]

Calculating the price of the put option yields:
\[ P = ($85 \times e^{-0.06 \times 48/365} \times .5163) - ($86 \times .4199) = $7.43 \]
7. \[ d_1 = \frac{\ln(75/80) + (0.05 - 0.02 + 0.62^2/2) \times 120/365}{0.62 \times \sqrt{120/365}} = 0.0239 \]
\[ d_2 = 0.0239 - 0.62 \times \sqrt{120/365} = -0.3315 \]

The standard normal probabilities are:

\[ N(d_1) = 0.5096 \quad N(d_2) = 0.3701 \]
\[ N(-d_1) = 0.4904 \quad N(-d_2) = 0.6299 \]

Calculating the price of the put option yields:

\[ P = \left(80 \times e^{-0.05 \times 120/365} \times 0.6299\right) - \left(75 \times e^{-0.02 \times 120/365} \times 0.4904\right) = 13.03 \]

8. \[ d_1 = \frac{\ln(104/115) + (0.06 - 0.03 + 0.75^2/2) \times 150/365}{0.75 \times \sqrt{150/365}} = 0.0569 \]
\[ d_2 = 0.0569 - 0.75 \times \sqrt{150/365} = -0.4239 \]

The standard normal probabilities are:

\[ N(d_1) = 0.5227 \quad N(d_2) = 0.3358 \]
\[ N(-d_1) = 0.4773 \quad N(-d_2) = 0.6642 \]

Calculating the price of the put option yields:

\[ P = \left(115 \times e^{-0.06 \times 150/365} \times 0.6642\right) - \left(104 \times e^{-0.03 \times 150/365} \times 0.4773\right) = 25.49 \]

9. Number of option contracts = \[ \frac{\text{Portfolio beta} \times \text{Portfolio value}}{\text{Option delta} \times \text{Option contract value}} \]

Number of option contracts = \[ \frac{1.2 \times 400,000,000}{0.60 \times 1050 \times 100} = 7,619 \text{ contracts to write} \]

10. You can either buy put options or sell call options. In either case, gains or losses on your stock portfolio will be offset by gains or losses on your option contracts. To calculate the number of contracts needed to hedge a $200 million portfolio with a beta of 1.5 using an option contract value of $150,000 (100 times the index) and a delta of .50, we use the formula from the chapter:

\[ \text{Number of option contracts} = \frac{\text{Portfolio beta} \times \text{Portfolio value}}{\text{Option delta} \times \text{Option contract value}} \]

Filling in the numbers, we need to write \( (1.4 \times 200M)/(0.5 \times 110,000) = 5,091 \) call contracts.
Intermediate Questions

11.  \( E = 0 \), so \( C = S = $85 \)

12.  \( \sigma = 0 \), so \( d_1 \) and \( d_2 \) go to +8, so \( N(d_1) \) and \( N(d_2) \) go to 1.
    \[
    C = ($80 \times 1) - ($75 \times e^{-0.05 \times 6/12 \times 1}) = $6.85
    \]

13.  for \( \sigma = 8 \), \( d_1 \) goes to +8 so \( N(d_1) \) goes to 1, and \( d_2 \) goes to −8 so \( N(d_2) \) goes to 0; \( C = S = $35 \)

14.  To calculate the implied volatility, we use the following inputs in the formula supplied in the text.

\[
S = \text{current stock price} = $105
\]
\[
K = \text{option strike price} = $120,
\]
\[
r = \text{risk-free rate} = 6%,
\]
\[
T = \text{time to expiration} = 100 \text{ days},
\]
\[
y = \text{dividend yield} = 3%,
\]
\[
C = \text{call option price} = $8.25
\]

\[
\sigma \approx \frac{\sqrt{2\pi} / T}{Y + X} \left( \frac{C - Y + X}{2} + \sqrt{\left( \frac{C - Y - X}{2} \right)^2 - \left( \frac{Y - X}{\pi} \right)^2} \right)
\]

\[
Y = S e^{-yT}, \quad X = K e^{-rT}
\]

The result yields this implied standard deviation value:

\[
Y = 104.1405 \quad X = 118.0435
\]

\[
\sigma \approx \frac{\sqrt{2\pi} / (100/365)}{104.1 + 118.0} \left( 8.25 - \frac{104.1 + 118.0}{2} + \sqrt{\left( 8.25 - \frac{104.1 - 118.0}{2} \right)^2 - \left( \frac{104.1 - 118.0}{\pi} \right)^2} \right)
\]

\[
\sigma \approx 0.6083 = 60.83%
\]

15.  Notice that the put price is given, not the call price. First, we must use put-call parity to find the put price:

\[
C = P + Se^{-yT} - Ke^{-rT}
\]

\[
= $19.15 + $110 \times e^{-0.03 \times 100/365} - $120 \times e^{-0.06 \times 100/365}
\]

\[
= $10.21
\]

\[
Y = 109.0996 \quad X = 118.0435
\]

\[
\sigma \approx \frac{\sqrt{2\pi} / (100/365)}{109.1 + 118.0} \left( 8.25 - \frac{109.1 + 118.0}{2} + \sqrt{\left( 8.25 - \frac{109.1 - 118.0}{2} \right)^2 - \left( \frac{109.1 - 118.0}{\pi} \right)^2} \right)
\]

\[
\sigma \approx 0.6001 = 60.01%
\]
16. Using the following inputs:

\[ S = \text{current stock price} = 93 \]
\[ K = \text{option strike price} = 90 \]
\[ r = \text{risk-free rate} = .05 \]
\[ s = \text{stock volatility} = .50 \]
\[ T = \text{time to expiration} = 180 \text{ days} \]
\[ y = \text{dividend yield} = 3 \text{ percent} \]

we compute these values for \( d_1 \) and \( d_2 \).

\[
d_1 = \frac{\ln(93/90) + (.05 \cdot .03 + .50^2/2) \times (180/365)}{.50 \times \sqrt{180/365}} = .2970
\]

\[
d_2 = .2970 - .50 \sqrt{180/365} = -.0541
\]

The standard normal probabilities are:

\[
\begin{align*}
N(d_1) & = .6168 & N(d_2) & = .4784 \\
N(-d_1) & = .3832 & N(-d_2) & = .5216
\end{align*}
\]

The option prices are:

\[
\text{C} = (93 \times e^{-.03 \times 180/365} \times .6168) - (90 \times e^{-.05 \times 180/365} \times .4784) = 14.51
\]

\[
\text{P} = (90 \times e^{-.05 \times 180/365} \times .5216) - (93 \times e^{-.03 \times 180/365} \times .3832) = 10.68
\]

Option deltas are then calculated as:

\[
\begin{align*}
\text{Call option Delta} & = e^{yt} N(d_1) = .9856 \times .6168 = .6077 \\
\text{Put option Delta} & = -e^{yt} N(-d_1) = -.9856 \times .3832 = -.3776
\end{align*}
\]

Option etas are calculated as:

\[
\begin{align*}
\text{Call option Eta} & = e^{yt} N(d_1) S/C = .6077 \times 93/14.51 = 3.8957 \\
\text{Put option Eta} & = -e^{yt} N(-d_1) S/P = -.3776 \times 93/10.68 = -3.2875
\end{align*}
\]

For the call and put option vega, we first compute the standard normal density value for \( d_1 \) as:

\[
n(d_1) = \frac{e^{d_1/2}}{\sqrt{2\pi}} = \frac{e^{-0.088209/2}}{\sqrt{2\pi}} = .38173
\]

and then compute vega as

\[
\text{Vega} = S e^{yt} n(d_1) (\sqrt{T}) = 93 \times e^{-0.03 \times 180/365} \times .38173 \times \sqrt{180/365} = 24.5640
\]
17. Using the following inputs:

\[ S = \text{current stock price} = \$47.61 \]
\[ K = \text{option strike price} = \$36.05 \]
\[ r = \text{risk-free rate} = .0503 \]
\[ s = \text{stock volatility} = .3066 \]
\[ T = \text{time to expiration} = 7.8 \text{ years} \]
\[ y = \text{dividend yield} = .0109 \]

we compute these values for \( d_1 \) and \( d_2 \).

\[
d_1 = \frac{\ln(\frac{47.61}{36.05}) + (0.0503 - 0.0109 + 0.3066^2 / 2) \times 7.8}{0.3066 \times \sqrt{7.8}} = 1.1119
\]

\[
d_2 = 1.1119 - 0.3066 \times \sqrt{7.8} = 0.2556
\]

These standard normal probabilities are given:

\[ N(d_1) = N(1.1119) = .8669 \]
\[ N(d_2) = N(.2556) = .6009 \]

Calculating the price of the employee stock options yields:

\[
\text{ESO} = (47.61 \times e^{-0.0109 \times 7.8 \times .8669}) - (36.05 \times e^{-0.0503 \times 7.8 \times .60086})
\]
\[
= (47.61 \times .91849 \times .8669) - (36.05 \times .67547 \times .60086)
\]
\[
= $23.28
\]

18. This is a hedging problem in which you wish to hedge one option position with another. Your employee stock option (ESO) position represents 10,000 shares, and you need to know how many put option contracts are required to establish the hedge. First, we need to calculate deltas for both options.

Using values from the previous answer, the ESO delta is

\[
ESO \ (Call) \ Delta = e^{-yT}N(d_1) = e^{-0.0109(7.8)} \times .8669 = .7962
\]

For the put option, we use the following inputs:

\[ S = \text{current stock price} = \$47.61 \]
\[ K = \text{option strike price} = \$50 \]
\[ r = \text{risk free interest rate} = .0503 \]
\[ y = \text{dividend yield} = .0109 \]
\[ s = \text{stock volatility} = .3066 \]
\[ T = \text{time to expiration} = .25 \text{ years} \]

We obtain this value for \( d_1 \)

\[
d_1 = \frac{\ln(\frac{47.61}{50}) + (0.0503 - 0.0109 + 0.3066^2 / 2) \times 0.25}{0.3066 \times \sqrt{0.25}} = -.1786
\]
and this put option delta

\[ \text{Put option Delta} = -e^{-yT}N(-d_1) = -.99728 \times .57087 = -.5693 \]

The number of put option contracts is then calculated as

\[
\text{Number of option contracts} = \frac{\text{ESO delta} \times 10,000}{\text{Put option delta} \times 100} = \frac{.7962 \times 10,000}{.5693 \times 100}
\]

Performing the calculation and ignoring the minus sign yields 139.856, or 140, put option contracts.

19. After the volatility shift, we need to recalculate deltas for both options. The new value of \( d_1 \) for the ESO is:

\[
d_1 = \frac{\ln(47.61/3.605) + (.0503 - .0109 + .40^2/2) \times 7.8}{.40 \times \sqrt{7.8}} = 1.08264
\]

In turn, the new ESO delta is

\[
\text{ESO (Call) Delta} = e^{-yT}N(d_1) = .91849 \times .8605 = .7904
\]

For the put option, we obtain this value for \( d_1 \)

\[
d_1 = \frac{\ln(47.61/5.0) + (.0503 - .0109 + .40^2/2) \times .25}{.40 \times \sqrt{.25}} = -.09565
\]

and this put option delta

\[ \text{Put option Delta} = -e^{-yT}N(-d_1) = -.99728 \times .5381 = -.5366 \]

The new number of contracts required is:

\[
\text{Number of option contracts} = \frac{.7904 \times 10,000}{-.5366 \times 100}
\]

Which yields (ignoring the minus sign) 147.298, or 147, put option contracts.
To calculate the implied volatility for the put option, we use the following inputs.

\[
\begin{align*}
S &= \text{current stock price} = \$47.61 \\
K &= \text{option strike price} = \$50 \\
r &= \text{risk free interest rate} = .0503 \\
y &= \text{dividend yield} = .0109 \\
T &= \text{time to expiration} = .25 \text{ years}
\end{align*}
\]

First, using put-call parity we calculate the corresponding call option price.

\[
C = P + Se^{-yT} - Ke^{-rT}
\]

\[
C = \$6.00 + \$47.61e^{-0.0109(0.25)} - \$50e^{-0.0503(0.25)}
\]

\[
C = \$4.105
\]

This result yields this implied standard deviation value.

\[
\sigma \approx \left(\frac{6.28318 / .25}{96.856}\right)^{1/2} \times \left[4.105 - (-1.895/2) + [(4.105-(-1.895/2))^2 - ((-1.895)^2/3.1416)]^{1/2}\right]
\]

\[
\sigma \approx .5171 = 51.71%
\]

AIMR suggested answers:

20. Donie should choose the long strangle. A long strangle consists of buying a put and a call with the same expiration date and the same underlying asset. In a strangle strategy, the call has an exercise price above the stock price and the put has an exercise price below the stock price. An investor who buys (goes long) a strangle expects that the price of the underlying asset (TRT in this case) will either move substantially below the exercise price on the put or above the exercise price on the call. With respect to TRT, the long strangle investor buys both the put and call options for a total cost of $9.00, and will experience large profits of the stock moves more than $9.00 above the call exercise price of $9.00 below the put exercise price. This strategy would enable Donie’s client to profit from a large move in stock price, either up or down, in reaction to the court decision.

21. The maximum possible loss per share is $9.00, which is the total cost of the two options = $5.00 + $4.00. The maximum possible gain is unlimited if the stock price moves above the call strike price. The breakeven prices are $46.00 and $69.00. The put will just cover costs if the stock price finishes $9.00 below the put exercise price ($55.00 – $9.00 = $46.00), and the call will just cover costs of the stock price finishes $9.00 above the call exercise price ($60.00 + $9.00 = $69.00).

22. The delta for a call option is always positive, so the value of the call option will increase if the stock price increases. Specifically, if the stock price increases by $1.00, the price of the call will increase by approximately $0.63:

\[
\text{Change in call price} = (0.6250 \times 1.00) = \$0.625 \text{ increase.}
\]
### Spreadsheet Answers

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<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<td><strong>Chapter 15</strong></td>
<td><strong>Question 24</strong></td>
<td></td>
<td><strong>Input Area</strong></td>
<td></td>
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<td>Exercise price</td>
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<td>Risk-free rate</td>
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<td>Standard deviation</td>
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<td>Dividend yield</td>
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<td>Call premium</td>
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<tr>
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<td>Put premium</td>
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</tbody>
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### Chapter 15

#### Question 25

**Input Area**

| Stock price | $83 |
| Exercise price | $80 |
| Expiration (days) | 180 |
| Risk-free rate | 4% |
| Standard deviation | 40% |
| Dividend yield | 2% |

**Output Area**

\[
\begin{align*}
&d_1 = \frac{\ln(83/80) + (0.0256 + 0.5103) \times (0.4/365) \times 180}{0.4 \times \sqrt{0.4/365}} \\
&d_2 = d_1 - 0.5 \times 0.4/365 \\
&N(d_1) = \text{NORMDIST}(d_1) \\
&N(d_2) = \text{NORMDIST}(d_2) \\
&N(-d_1) = 1 - N(d_1) \\
&N(-d_2) = 1 - N(d_2) \\
&\text{Call premium} = \exp(-0.0256 \times 180) \times \left( \text{N}(d_2) - \text{N}(-d_1) \right) - 80 \\
&\text{Put premium} = \exp(-0.0256 \times 180) \times \left( \text{N}(-d_2) - \text{N}(-d_1) \right) + 83 \\
&\text{Call option delta} = \exp(0.0256 \times 180) \times \left( \text{N}(d_2) - \text{N}(-d_1) \right) \\
&\text{Put option delta} = \exp(0.0256 \times 180) \times \left( \text{N}(-d_2) - \text{N}(-d_1) \right) \\
&\text{Call option eta} = \frac{83 - \text{Call premium}}{83 - 80} \times \exp(-0.0256 \times 180) \times \left( \text{N}(d_2) - \text{N}(-d_1) \right) \\
&\text{Put option eta} = \frac{80 - \text{Put premium}}{83 - 80} \times \exp(-0.0256 \times 180) \times \left( \text{N}(-d_2) - \text{N}(-d_1) \right) \\
&\text{Vega} = \frac{\text{Call premium} - \text{Put premium}}{83 - 80} \times \exp(-0.0256 \times 180) \times \left( \text{N}(d_2) - \text{N}(-d_1) \right) \\
&\text{For the call and put option vega, we first compute the standard normal density value for } N(d_1) \text{ as:} \\
&\text{value for } N(d_1) = 0.38812 = \exp(-0.5 \times (d_1 - 0.5) \times \sqrt{0.4/365}) \\
&\text{Vega} = \frac{0.38812 \times \exp(-0.0256 \times 180) \times \left( \text{N}(d_2) - \text{N}(-d_1) \right)}{\sqrt{0.4/365}} \\
\end{align*}
\]