Chapter 13
Performance Evaluation and Risk Management

Concept Questions

1. The Sharpe ratio is calculated as a portfolio’s risk premium divided by the standard deviation of the portfolio’s return. The Treynor ratio is the portfolio risk premium divided by the portfolio’s beta coefficient.

2. A common weakness of both the Jensen alpha and the Treynor ratio is that both require an estimate of beta, which can differ a lot depending on the source, which in turn can lead to a mismeasurement of risk adjusted return.

3. Jensen’s alpha is the difference between a stock’s or a portfolio’s actual return and that which is predicted by the CAPM. A positive alpha implies returns above the CML line (as drawn using the CAPM).

4. An advantage of the Sharpe ratio is that a beta estimate is not required; however, the Sharpe ratio is not appropriate when evaluating individual stocks because it uses total risk rather than systematic.

5. The mean and standard deviation completely specify the normal distribution.

6. A Sharpe optimal portfolio is the portfolio with the highest possible Sharpe ratio given the available investments. This portfolio has the characteristic of having the highest possible return for the least amount of risk.

7. The Markowitz efficient frontier is closely related to the Sharpe ratio. The Markowitz efficient frontier tells us which portfolios are efficient (highest return for a given level of risk), but the Sharpe model helps to identify which of these efficient portfolios is actually the best.

8. After establishing the desired probability (x), the VaR statistic provides the minimum loss you would receive x% of the time. As an example, given:
   \[
   \text{Prob}(R \leq -0.20) = 5\%
   \]
   we would expect at least a 20% loss in one out of twenty periods (5% of the time). This is equivalent to saying that 5% of the time the minimum loss is 20%, similar to the previous answer.

9. This is equivalent to saying that 5% of the time the minimum loss is 20%, similar to the previous answer.

10. On a standard normal distribution, the mean is zero and standard deviation is one. One-half (50%) of the observations lie below the mean and one-half above. Thus, if Pr(X < x) is 50%, then x = 0.

Core Questions

1. \[48\% \sqrt{3/12} = 24\%\]
2. Annual standard deviation = \(\sqrt{0.0420} = 20.49\%\)
   
   2-month standard deviation = 20.49\% \sqrt{2/12} = 8.37\%

3. \(14.10\% / \sqrt{1/12} = 48.84\%\)

4. Weekly: \(8.16\% / \sqrt{1/4} = 16.32\%\)
   
   Annual: \(8.16\% / \sqrt{1/52} = 58.84\%\)

5.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Sharpe ratio</th>
<th>Treynor ratio</th>
<th>Jensen's alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0.26000</td>
<td>0.1000</td>
<td>2.60%</td>
</tr>
<tr>
<td>Y</td>
<td>0.19643</td>
<td>0.0917</td>
<td>1.40%</td>
</tr>
<tr>
<td>Z</td>
<td>0.15152</td>
<td>0.0625</td>
<td>-1.40%</td>
</tr>
<tr>
<td>Market</td>
<td>0.36364</td>
<td>0.0800</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

6. There is a 67\% probability of being within one standard deviation of the mean; therefore, there is a 1/3 probability of being outside of one standard deviation. Since we are only concerned with below the mean, this cuts the 1/3 in half, giving a final probability of 1/6, or 15.87\%.

7. On a standard normal distribution, the mean is zero and standard deviation is one. These standard deviations just represent \(z\) values on this standard curve. Thus, we have the following percentages for the values:
   
   \(5\% = -1.645\)
   
   \(2.5\% = -1.96\)
   
   \(1\% = -2.326\)

8. This is simply the previous question in reverse, so we have:
   
   \(5\% = -1.645\)
   
   \(2.5\% = -1.96\)
   
   \(1\% = -2.326\)

9. \(\text{Prob}(R \leq 0.14 - 1.645(0.28)) = 5\%\)
    \(\text{Prob}(R \leq -3.206) = 5\%\)

10. \(\text{Prob}(R \leq 0.18/12 - 1.96(0.45)(1/12)^{1/2}) = 2.5\%\)
    \(\text{Prob}(R \leq -2.396) = 2.5\%\)

11. \(E(R) = (0.14 + 0.18)/2 = 0.16\)
    \(\sigma = (0.5^2 \times 0.28^2 + 0.5^2 \times 0.45^2)^{1/2} = 0.2650\)
    \(\text{Prob}(R \leq 0.16/12 - 1.96(0.2650)(1/12)^{1/2}) = 2.5\%\)
    \(\text{Prob}(R \leq -1.366) = 2.5\%\)

12. \(\text{Prob}(R \leq 0.13 - 2.326(0.32)) = 1\%\)
    \(\text{Prob}(R \leq -0.6144) = 1\%\)
13. \[
\text{Prob}(R \leq -0.13) - 1.96(0.22)(1/52)^{1/2} = 2.5\%
\]
\[
\text{Prob}(R \leq -1.111) = 2.5\%
\]

14. \[
E(R) = (0.13 + 0.16)/2 = 0.145
\]
\[
\sigma = (0.5^2 \times 0.3^2 + 0.5^2 \times 0.4^2)^{1/2} = 0.2640
\]
\[
\text{Prob}(R \leq -0.145/12) - 1.645(0.2640)(1/12)^{1/2} = 5\%
\]
\[
\text{Prob}(R \leq -1.133) = 5\%
\]

15. For a portfolio with two investments having zero correlation, the Sharpe ratio would be calculated as follows:

\[
\text{Sharpe ratio} = \frac{x_SE(R_S) + x_BE(R_B) - R_f}{(x_SE^2 \sigma_S^2 + x_BE^2 \sigma_B^2)^{1/2}}
\]

16. \[
\text{Sharpe ratio} = \frac{0.5E(R_S) + 0.5E(R_B) - R_f}{[0.5^2 \sigma_S^2 + 0.5^2 \sigma_B^2 + 2(0.5)(0.5)(\sigma_S)(\sigma_B)(\text{Corr}(R_S, R_B))]}^{1/2}
\]

17. Any portfolio of the two securities will also have the same expected return.

\[
\text{Sharpe ratio} = \frac{E(R_S) - R_f}{(x_SE^2 \sigma_S^2 + x_BE^2 \sigma_B^2)^{1/2}} = \frac{E(R_B) - R_f}{(x_BE^2 \sigma_B^2 + x_BE^2 \sigma_B^2)^{1/2}}
\]

18. \[
\text{Prob}(R \leq -0.13 - 2.326(0.65)) = 1\%
\]
\[
\text{Prob}(R \leq -1.3821) = 1\%
\]
This number does not make sense since it is impossible to lose more than 100\% in a stock.

19. \[
\text{Prob}(R \geq 0.13 + 2.326(0.65)) = 1\%
\]
\[
\text{Prob}(R \geq 1.6421) = 1\%
\]
While this is a large return, it is plausible, and even possible. Since it is not possible for a stock to lose more than 100\% but it is possible for a stock to gain more than 100\%, stock returns are not truly normal.

20. \[
E(R) = (0.14 + 0.22)/2 = 0.18
\]
\[
\sigma = [(0.5^2)(0.31^2) + (0.5^2)(0.56^2) + 2(0.5)(0.5)(0.31)(0.56)(0.5)]^{1/2} = 0.3819
\]
\[
\text{Prob}(R \leq -0.18/12) - 1.645(0.3819)(1/12)^{1/2} = 5\%
\]
\[
\text{Prob}(R \leq -1.1663) = 5\%
\]

21. \[
E(R) = (0.14 + 0.22)/2 = 0.18
\]
\[
\sigma = [(0.5^2)(0.31^2) + (0.5^2)(0.56^2) + 2(0.5)(0.5)(0.31)(0.56)(-0.5)]^{1/2} = 0.2430
\]
\[
\text{Prob}(R \leq -0.18/12) - 1.645(0.2430)(1/12)^{1/2} = 5\%
\]
\[
\text{Prob}(R \leq -1.1004) = 5\%
\]

22. \[
E(R) = 0.18
\]
\[
\sigma = [(0.333^2)(0.35^2) + (0.333^2)(0.45^2) + (0.333^2)(0.55^2) + 2(0.333)(0.333)(0.35)(0.45)(0) + 2(0.333)(0.333)(0.35)(0.55)(0) + 2(0.333)(0.333)(0.45)(0.55)(0)]^{1/2} = 0.2640
\]
\[
\text{Prob}(R \leq 0.18 - 2.326(0.2640)) = 1\%
\]
\[
\text{Prob}(R \leq -0.4343) = 1\%
\]
23. \( E(R) = 0.18 \)
\[ \sigma = \left[ (0.333^2)(0.35^2) + (0.333^2)(0.45^2) + (0.333^2)(0.55^2) + 2(0.333)(0.333)(0.35)(0.45)(0.20) + 2(0.333)(0.333)(0.35)(0.55)(0.20) + 2(0.333)(0.333)(0.45)(0.55)(0.20) \right]^{1/2} = 0.3103 \]
\[ \text{Prob}(R \leq 0.18 - 2.326(0.3103)) = 1\% \]
\[ \text{Prob}(R \leq -0.5418) = 1\% \]

24. \( E(R) = 0.16 \)
\[ \sigma = 0.24 \]
\[ \text{Prob}(R \leq (0.16/4) - 1.645(0.24)(1/4)^{1/2}) = 5\% \]
\[ \text{Prob}(R \leq -0.1574) = 5\% \]
Spreadsheet Problem

25. The Solver inputs are:

![Solver Parameters](image)

based on the following spreadsheet.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Chapter 13</td>
<td>Question 25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Input Area</td>
<td></td>
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<td></td>
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<tr>
<td>7</td>
<td>Expected return A</td>
<td>14.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Standard deviation A</td>
<td>65.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Expected return B</td>
<td>11.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Standard deviation B</td>
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<td></td>
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<tr>
<td>11</td>
<td>Correlation</td>
<td>0.25</td>
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<tr>
<td>12</td>
<td>Risk-free rate</td>
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<tr>
<td>13</td>
<td>Starting weight of Asset A</td>
<td>37.58%</td>
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<tr>
<td>18</td>
<td>Output Area</td>
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</tr>
<tr>
<td>19</td>
<td>Weight of Asset A</td>
<td>37.58% =D13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Weight of Asset B</td>
<td>62.42% =1-D19</td>
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<td></td>
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<tr>
<td>22</td>
<td>Portfolio expected return</td>
<td>12.13% =D19<em>D7+(D20</em>D9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Portfolio standard deviation</td>
<td>49.56% =SQRT((D19^2)<em>(D8^2)+(D20^2)</em>(D10^2)+2<em>D19</em>D20<em>D8</em>D10*D11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Sharpe ratio</td>
<td>0.2004 =D22-D12/D23</td>
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<td></td>
</tr>
</tbody>
</table>

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