Chapter 10
Bond Prices and Yields

Concept Questions

1. Premium (par, discount) bonds are bonds that sell for more than (the same as, less than) their face or par value.

2. The face value is normally $1,000 per bond. The coupon is expressed as a percentage of face value (the coupon rate), so the annual dollar coupon is calculated by multiplying the coupon rate by $1,000. Coupons are normally paid semi-annually; the semi-annual coupon is equal to the annual coupon divided by two.

3. The coupon rate is the annual dollar coupon expressed as a percentage of face value. The current yield is the annual dollar coupon divided by the current price. If a bond’s price rises, the coupon rate won’t change, but the current yield will fall.

4. Interest rate risk refers to the fact that bond prices fluctuate as interest rates change. Lower coupon and longer maturity bonds have greater interest rate risk.

5. For a premium bond, the coupon rate is higher than the yield. The reason is simply that the bonds sells at a premium because it offers a coupon rate that is high relative to current market required yields. The reverse is true for a discount bond: it sells at a discount because its coupon rate is too low.

6. A bond’s promised yield is an indicator of what an investor can expect to earn if (1) all of the bond’s promised payments are made and (2) market conditions do not change. The realized yield is the actual, after-the-fact return the investor receives. The realized yield is more relevant, of course, but it is not knowable ahead of time. A bond’s calculated yield to maturity is the promised yield.

7. The yield to maturity is the required rate of return on a bond expressed as a nominal annual interest rate. For noncallable bonds, the yield to maturity and required rate of return are interchangeable terms. Unlike YTM and required return, the coupon rate is not used as the interest rate in bond cash flow valuation, but is a fixed percentage of par over the life of the bond used to set the coupon payment amount. For the example given, the coupon rate on the bond is still 10 percent, and the YTM is 8 percent.

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9. a. Bond price is the present value term when valuing the cash flows from a bond; YTM is the interest rate used in valuing the cash flows from a bond. They have an inverse relationship.

b. If the coupon rate is higher than the required return on a bond, the bond will sell at a premium, since it provides periodic income in the form of coupon payments in excess of that required by investors on other similar bonds. If the coupon rate is lower than the required return on a bond, the bond will sell at a discount, since it provides insufficient coupon payments compared to that required by investors on other similar bonds. For premium bonds, the coupon rate exceeds the YTM; and for discount bonds, the YTM exceeds the coupon rate. For bonds selling at par, the YTM is equal to the coupon rate.

c. Current yield is defined as the annual coupon payment divided by the current bond price. For premium bonds, the current yield exceeds the YTM; for discount bonds the current yield is less than the YTM; and for bonds selling at par value, the current yield is equal to the YTM. In all cases, the current yield plus the expected one-period capital gains yield of the bond must be equal to the required return.

10. A premium bond is one with a relatively high coupon, and, in particular, a coupon that is higher than current market yields. These are precisely the bonds that the issuer would like to call, so a yield to call is probably a better indicator of what is likely to happen than the yield to maturity (the opposite is true for discount bonds). It is also the case that the yield to call is likely to be lower than the yield to maturity for a premium bond, but this can depend on the call price. A better convention would be to report the yield to maturity or yield to call, whichever is smaller.

**Core Questions**

1. \[ P = 40 \times (PVIFA_{3.5\%, 20}) + 1000 \times (PVIF_{3.5\%, 20}) = 1,071.06 \]

2. \[ P = 1,289 = 47.5 \times (PVIFA_{R\%, 42}) + 1000 \times (PVIF_{R\%, 42}) ; R = 3.439\% , \ YTM = 6.88\% \\
\text{current yield} = 95.00/1,289 = 7.37\% \]

3. \[ P = 42.5 \times (PVIFA_{3.35\%, 36}) + 1000 \times (PVIF_{3.35\%, 36}) = 1,186.62 \]

4. \[ P = 35 \times (PVIFA_{4.5\%, 50}) + 1000 \times (PVIF_{4.5\%, 20}) = 802.38 \]

5. \[ P = 864.50 = 30 \times (PVIFA_{R\%, 32}) + 1000 \times (PVIF_{R\%, 32}) ; R = 3.732\% , \ YTM = 7.46\% \]

6. \[ P = 1,132 = 30 \times (PVIFA_{R\%, 46}) + 1000 \times (PVIF_{R\%, 46}) ; R = 2.513\% , \ YTM = 5.03\% \]

7. \[ P = 1,080 = 37.5 \times (PVIFA_{R\%, 12}) + 1000 \times (PVIF_{R\%, 12}) ; R = 2.949\% , \ YTM = 5.90\% \]

8. \[ YTM = [(1,000/150)^{1/40} - 1] \times 2 = 9.71\% \]

9. \[ YTC = [(500/150)^{1/20} - 1] \times 2 = 12.41\% \]

10. \[ YTC = [(475/150)^{1/10} - 1] \times 2 = 11.87\% \]
11. \[ P = $1,084 = $C(PVIFA_{3.75\%,23}) + $1000(PVIFA_{3.75\%,23}) \; ; \; C = $43.01, \]
coupon rate = 2(4.301) = 8.60%

12. \[ P = $51.25(PVIFA_{4.63\%,18}) + $1000(PVIF_{4.63\%,18}) = $1,059.57 \]

13. \[ P = $650 = $17.50(PVIFA_{R\%,20}) + $1000(PVIF_{R\%,20}) \; ; \; R = 4.423\%; \; YTM = 8.85\% \]

14. Assuming a $1,000 face value, the current price of the bond is \( P = \frac{1,000}{(1 + .045)^{10}} = $267.00 \). Two years later the bond has 13 years to maturity and the same price, so the new yield to maturity must be 
\[ \left[\frac{1,000}{267.00}\right]^{1/12} - 1 \] 
\times 2 = 10.42%.

15. If held to maturity, a zero-coupon bond will always have a realized yield equal to its original yield to maturity, which in this case is 9 percent.

16. X: \[ P_0 = $40(PVIFA_{3\%,30}) + $1000(PVIF_{3\%,30}) = $1,196.00 \]
\[ P_1 = $40(PVIFA_{3\%,28}) + $1000(PVIF_{3\%,28}) = $1,187.64 \]
\[ P_5 = $40(PVIFA_{3\%,20}) + $1000(PVIF_{3\%,20}) = $1,148.77 \]
\[ P_{10} = $40(PVIFA_{3\%,10}) + $1000(PVIF_{3\%,10}) = $1,085.30 \]
\[ P_{14} = $40(PVIFA_{3\%,2}) + $1000(PVIF_{3\%,2}) = $1,019.13 \]
\[ P_{15} = $1,000 \]

Y: \[ P_0 = $40(PVIFA_{5\%,30}) + $1000(PVIF_{5\%,30}) = $846.28 \]
\[ P_1 = $40(PVIFA_{5\%,28}) + $1000(PVIF_{5\%,28}) = $851.02 \]
\[ P_5 = $40(PVIFA_{5\%,20}) + $1000(PVIF_{5\%,20}) = $875.38 \]
\[ P_{10} = $40(PVIFA_{5\%,10}) + $1000(PVIF_{5\%,10}) = $922.78 \]
\[ P_{14} = $40(PVIFA_{5\%,2}) + $1000(PVIF_{5\%,2}) = $981.41 \]
\[ P_{15} = $1,000 \]

All else held equal, the premium over par value for a premium bond declines as maturity is approached, and the discount from par value for a discount bond declines as maturity is approached. This is sometimes called the “pull to par.”
17. If both bonds sell at par, the initial YTM on both bonds is the coupon rate, 7 percent. If the YTM suddenly rises to 9 percent, then:

\[
P_A = 35(PVIFA_{4.5\%,4}) + 1000(PVIF_{4.5\%,4}) = $964.12
\]

\[
P_B = 35(PVIFA_{4.5\%,30}) + 1000(PVIF_{4.5\%,30}) = $837.11
\]

\[
\frac{P_A - 1000}{1000} = -3.59\%
\]

\[
\frac{P_B - 1000}{1000} = -16.29\%
\]

If the YTM suddenly falls to 7 percent, then:

\[
P_A = 35(PVIFA_{2.5\%,4}) + 1000(PVIF_{2.5\%,4}) = $1,037.62
\]

\[
P_B = 35(PVIFA_{2.5\%,30}) + 1000(PVIF_{2.5\%,30}) = $1,209.30
\]

\[
\frac{P_A - 1000}{1000} = +3.76\%
\]

\[
\frac{P_B - 1000}{1000} = +20.93\%
\]

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

18. Initially, at a YTM of 7 percent, the prices of the two bonds are:

\[
P_J = 20(PVIFA_{3.5\%,20}) + 1000(PVIF_{3.5\%,20}) = $786.81
\]

\[
P_K = 50(PVIFA_{3.5\%,20}) + 1000(PVIF_{3.5\%,20}) = $1,213.19
\]

If the YTM rises from 7 percent to 9 percent:

\[
P_J = 20(PVIFA_{4.5\%,20}) + 1000(PVIF_{4.5\%,20}) = $674.80
\]

\[
P_K = 50(PVIFA_{4.5\%,20}) + 1000(PVIF_{4.5\%,20}) = $1,065.04
\]

\[
\frac{P_J - 786.81}{786.81} = -14.24\%
\]

\[
\frac{P_K - 1213.19}{1213.19} = -12.21\%
\]

If the YTM declines from 7 percent to 5 percent:

\[
P_J = 20(PVIFA_{2.5\%,20}) + 1000(PVIF_{2.5\%,20}) = $922.05
\]

\[
P_K = 50(PVIFA_{2.5\%,20}) + 1000(PVIF_{2.5\%,20}) = $1,389.73
\]

\[
\frac{P_J - 786.81}{786.81} = +17.19\%
\]

\[
\frac{P_K - 1213.19}{1213.19} = +14.55\%
\]

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.
19. Current yield = \( \frac{.0721}{P_0} \); \( P_0 = \frac{80}{.0721} = $1,109.57 \)

\[
P_0 = $1,109.57 = \frac{40(1 - (1/1.0345)^N)}{.0345} + \frac{1,000}{1.0345^N}
\]

\[
1,109.57(1.0345)^N = 1,159.42(1.0345)^N - 1,159.42 + 1,000
\]

\[
159.42 = 49.85(1.0345)^N \quad ; \quad 3.198 = 1.0345^N \quad ; \quad N = \log 3.198 / \log 1.0345 = 34.27 = 17.14 \text{ yrs.}
\]

20. The maturity is indeterminate; a bond selling at par can have any maturity length.

21. \( a. P_0 = $1,100 = 55(PVIFA_{R\%,30}) + 1000(PVIF_{R\%,30}) \quad ; \quad R = 4.860\%, \text{ YTM} = 9.72\% \)

This is the rate of return you expect to earn on your investment when you purchase the bond.

\( b. \) Price when sold = $55(PVIFA_{5.86\%,26}) + 1000(PVIF_{5.86\%,26}) = $952.56

Future value of reinvested interest payments = $55(FVIFA_{4.86\%,4}) = $236.56

Realized return = \( (952.56 - 1,100 + 236.56) / 1,100 = 8.10\% \)

The realized yield is less than the expected yield when the bond was purchased because interest rates have increased by 2 percent; bond prices fall when yields rise.

22. The yield to call can be computed as:

\[
P = $1,180 = 50(PVIFA_{R\%,10}) + 1,080(PVIF_{R\%,10}) \quad ; \quad R = 3.515\%, \text{ YTC} = 7.03\%
\]

Since the bond sells at a premium to par value, you know the coupon rate must be greater than the yield. Thus, if interest rates remain at current levels, the bond issuer will likely call the bonds to refinance (at a lower coupon rate) at the earliest possible time, which is the date when call protection ends. The yield computed to this date is the YTC, and it will always be less than the YTM for premium bonds with a zero call premium. In the present example,

\[
P = $1,180 = 50(PVIFA_{3.964\%,10}) + 1,000(PVIF_{3.964\%,10}) \quad ; \quad R = 3.964\%, \text{ YTM} = 7.93\%
\]

where if the bond is held until maturity, no call premium must be paid. Note that using the same analysis, a break-even call premium can also be computed:

\[
P = $1,180 = 50(PVIFA_{3.964\%,10}) + (1,000 + X)(PVIF_{3.964\%,10}) \quad ; \quad X = $141.30
\]

Thus, if interest rates remain unchanged, the bond will not be called if the call premium is greater than $141.30.

23. \( P = $1,084.50 = 30(PVIFA_{R\%,16}) + 1,000(PVIF_{R\%,16}) \quad ; \quad R = 2.360\%, \text{ YTM} = 4.72\% \)

Duration = \( (1.0236/.0472) - [(1.0236 + 8(.06 -.0472)) / (.0472 + .06(1.0236^{16} - 1)) = 6.541 \text{ years} \)

Modified duration = \( 6.541/(1.0236) = 6.390 \text{ years} \)
24. Estimated percent change in price = –6.390(.02) = –.1278 = \( \frac{P_1}{P_0} - 1 \)

so \( P_1 = (1 - .1278)(\$1,084.50) = \$945.91 \)

Actual \( P_1 = \$30(PVIFA_{3.36\%,16}) + \$1,000(PVIF_{3.36\%,16}) = \$956.00 \)

25. Duration = \( \frac{1.04}{.08} - \frac{[(1.04 + 13(.07 - .08))/(.08 + .07(1.04^{26} - 1))]} = 8.541 \) years

Modified duration = \( 8.541/(1.04) = 8.212 \) years

26. Duration = \( \frac{1.03}{.06} - \frac{[(1.03 + 13(.07 - .06))/(.06 + .07(1.03^{26} - 1))]} = 8.937 \) years

Modified duration = \( 8.937/(1.03) = 8.677 \) years

For an option free bond, at a lower YTM, the duration is higher.

27. Duration = \( \frac{1.03}{.06} - \frac{[(1.03 + 20(.08 - .06))/(.06 + .08(1.03^{40} - 1))]} = 11.232 \) years

Modified duration = \( 11.232/(1.03) = 10.905 \) years

28. Initial price = \( \$40(PVIFA_{3\%,40}) + \$1000(PVIF_{3\%,40}) = \$1,231.15 \)

If interest rates rise .25%:

Estimated percent change in price = –10.905(.0025) = –.02726 = \( \frac{P_1}{P_0} - 1 \)

so \( P_1 = (1 - .02726)(\$1,231.15) = \$1,197.59 \)

Actual \( P_1 = \$40(PVIFA_{3.125\%,40}) + \$1,000(PVIF_{3.125\%,40}) = \$1,198.23 \)

If interest rates rise 1%:

Estimated percent change in price = –10.905(.01) = –.1095 = \( \frac{P_1}{P_0} - 1 \)

so \( P_1 = (1 - .1095)(\$1,231.15) = \$1,096.89 \)

Actual \( P_1 = \$40(PVIFA_{3.5\%,40}) + \$1,000(PVIF_{3.5\%,40}) = \$1,106.78 \)

If interest rates rise 2%:

Estimated percent change in price = –10.905(.02) = –.2181 = \( \frac{P_1}{P_0} - 1 \)

so \( P_1 = (1 - .2181)(\$1,231.15) = \$962.63 \)

Actual \( P_1 = \$40(PVIFA_{4\%,40}) + \$1,000(PVIF_{4\%,40}) = \$1,000.000 \)

If interest rates rise 5%:

Estimated percent change in price = –10.905(.05) = –.54525 = \( \frac{P_1}{P_0} - 1 \)

so \( P_1 = (1 - .54525)(\$1,231.15) = \$559.86 \)

Actual \( P_1 = \$40(PVIFA_{5.5\%,40}) + \$1,000(PVIF_{5.5\%,40}) = \$759.31 \)

While duration gives an effective estimate for small interest rate changes, duration does not produce a good estimate of the price change for large interest rate changes.
29. Strategy I:

\[ \% \text{ MV}_{5 \text{ Year}} = -4.83(-0.0075) = 0.036225; \text{ New MV} = (1 + 0.036225)(5,000,000) = 5,181,125 \]

\[ \% \text{ MV}_{25 \text{ Year}} = -23.81(0.0050) = -0.11905; \text{ New MV} = (1 - 0.11905)(5,000,000) = 4,404,750 \]

New market value of portfolio = $5,181,125 + 4,404,750 = $9,585,875

Strategy II:

\[ \% \text{ MV}_{15 \text{ Year}} = -14.35(0.0025) = -0.03575; \text{ New MV} = (1 - 0.03575)(10,000,000) = 9,641,250 \]

For this interest rate change, Strategy II is a better alternative.

30. Zero coupon YTM = $945 = $1,000 / (1 + r); \ r = 5.82\%

Two year spot rate: $1,032 = $80/(1 + 0.0582) + $1,080/(1 + r)^2; \ r_2 = 6.27\%

Three year spot rate: $1,081 = $90/(1 + 0.0582) + $90/(1 + 0.0627)^2 + $1,090/(1 + r_3)^3; \ r_3 = 5.96\%

31. \[ P = \frac{80}{1 + 0.055} + \frac{80}{(1 + 0.0625)^2} + \frac{80}{(1 + 0.0680)^3} + \frac{1,080}{(1 + 0.0730)^4} = 1,027.12 \]

\[ P = 1,027.12 = 80(PVIFA_{r\%,4}) + 1,000(PVIF_{r\%,4}) ; \ YTM = 7.20\% \]

### Spreadsheet Problems

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10-7
### Chapter 10

#### Question 33

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**Output Area:**

- **Yield to maturity**: 8.30% =YIELD(D7,D8,D9,D10,100,2)
- **Yield to call**: 9.40% =YIELD(D7,D11,D9,D10,D12,2)

### Chapter 10

#### Question 34

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**Output Area:**

- **Macaulay duration**: 7.7425 =DURATION(D7,D8,D9,D10,2)
- **Modified duration**: 7.5500 =MDURATION(D7,D8,D9,D10,2)