Chapter 9
Interest Rates

Concept Questions

1. Short-term rates have ranged between zero and 14 percent. Long-term rates have fluctuated between about two and 13 percent. Long-term rates, which are less volatile, have historically been in the four-to-five percent range (the 1960 - 1980 experience is the exception). Short-term rates have about the same typical values, but more volatility (and lower rates in the unusual 1930 - 1960 period).

2. A pure discount security is a financial instrument that promises a single fixed payment (the face value) in the future with no other payments in between. Such a security sells at a discount relative to its face value, hence the name. Treasury bills and commercial paper are two examples.

3. The Fed funds rate is set in a very active market by banks borrowing and lending from each other. The discount rate is set by the Fed at whatever level the Fed feels is appropriate. The Fed funds rate changes all the time; the discount rate only changes when the Fed decides; the Fed funds rate is therefore much more volatile. The Fed funds market is much more active. Banks usually borrow from the Fed only as a last resort, which is the primary reason for the Fed’s discount rate-based lending.

4. Both are pure discount money market instruments. T-bills, of course, are issued by the government; while commercial paper is issued by corporations. The primary difference is that commercial paper has default risk, so it offers a higher interest rate.

5. LIBOR is the London Interbank Offered Rate. It is the interest rate offered by major London banks for dollar-denominated deposits. Interest rates on loans are often quoted on a LIBOR–plus basis, so the LIBOR is an important, fundamental rate in business lending, among other things.

6. Such rates are much easier to compute by hand; they predate (by hundreds of years or more) computing machinery.

7. They are coupon interest, note principal, and bond principal, respectively. Recalling that each STRIPS represents a particular piece of a Treasury note or bond, these designations tell us which piece is which. A “ci” is one of the many coupon payments on a note or bond; an “np” is the final principal payment on a Treasury note; and a “bp” is the final principal payment on a Treasury bond.

8. We observe nominal rates almost exclusively. Which one is more relevant actually depends on the investor and, more particularly, what the proceeds from the investment will be used for. If the proceeds are needed to make payments that are fixed in nominal terms (like a loan repayment, perhaps), then nominal rates are more important. If the proceeds are needed to purchase real goods (like groceries) and services, then real rates are more important.

9. Trick question! It depends. Municipals have a significant tax advantage, but they also have default risk. Low risk municipals usually have lower rates; higher risk municipals can (and often do) have higher rates.
AIMR suggested answer:

a. The pure expectations theory states the term structure of interest rates is explained entirely by interest rate expectations. The theory assumes that forward rates of interest embodied in the term structure are unbiased estimates of expected future spot rates of interest. Thus, the pure expectations theory would account for a declining yield curve by arguing that interest rates are expected to fall in the future rather than rise. Investors are indifferent to holding (1) a short-term bond at a higher rate to be rolled over at a lower expected future short-term rate, and (2) a longer-term bond at a rate between the higher short-term rate and the lower expected future short-term rate.

b. Liquidity preference theory (Maturity preference) states that the term structure is a combination of future interest rate expectations and an uncertainty “risk” or uncertainty yield “premium.” The longer the maturity of a bond, the greater the perceived risk (in terms of fluctuations of value) to the investor, who accordingly prefers to lend short term and thus requires a premium to lend longer term. This yield “premium” is added to the longer-term interest rates to compensate investors for their additional risk. Theoretically, liquidity preference could account for a downward slope if future expected rates were lower than current rates by an amount greater than their respective term risk premium. Liquidity preference theory is consistent with any shape of the term structure but suggests and upward bias or “tilt” to any term structure shape given by unbiased expectations.

c. Market segmentation theory states that the term structure results from different market participants establishing different yield equilibriums between buyers and seller of funds at different maturity preferences. Market segmentation theory can account for any term structure shape because of the different supply/demand conditions posted at maturity ranges. Borrowers and lenders have preferred maturity ranges, based largely on institutional characteristics, and the yield curve is the average of these different suppliers’ and demanders’ maturity preferences. These maturity preferences are essentially fixed; that is, the participants do not tend to move between or among maturity ranges, so different supply and demand conditions exist across the maturity spectrum. In each maturity range, a higher demand for funds (supply of bonds) relative to the supply of funds will drive bond prices down, and rates up, in that maturity range. A downward sloping yield curve, in the context of market segmentation, indicates that a larger supply of short-term debt relative to demand has led to lower short-term bond prices and/or a small supply of long-term debt relative to demand has led to higher long-term bond prices. Either set of supply/demand conditions works to drive long-term rates lower and short-term rates higher.

Core Question

1. Price = $100 / (1 + .052/2)^2(6) = $73.4905 = $73.4905/$100 = 73.4905% or 73:16

2. Price = $100,000 / (1 + .068/2)^2(9.5) = $52,979.73 = $52,979.73/$100,000 = 52.97973% or 52:31

3. YTM = 2 × [(100 / 75.875)^(1/(2 × 7)) – 1] = .0398

4. YTM = 2 × [(100 / 50.375)^(1/(2 × 10)) – 1] = .0698

5. 13.4% – 3.9% = 9.5%

6. 10.1% – 6.2% = 3.9%
7. \[ d = .048 = \frac{($1M - P)}{P} \frac{360}{110}; \quad P = $985,333.33 \]

8. \[ y = \frac{[365(.048)]}{[360 - (110)(.048)]} = 4.939\% \]

9. \[ d = .0205 = \frac{($1M - P)}{P} \frac{360}{43}; \quad P = $997,551.39 \]

10. \[ y = \frac{[365(.0205)]}{[360 - (43)(.0205)]} = 2.084\% \]

**Intermediate Questions**

11. \[ 99.07 = 100 \times [1 - (80/360) \times \text{DY}); \quad \text{discount yield} = .04185 \]
    bond equivalent yield = \[ \frac{[365(.04185)]}{[360 - (80)(.04185)]} = .04283 \]
    EAR = \[ \left[1 + \frac{.04283}{(365/80)}\right]^{365/80} - 1 = .04355 \]

12. \[ d = .0424 = \frac{($100 - P)}{P} \frac{360}{34}; \quad P = 99.600\% \text{ of par or 99:19} \]
    \[ y = \frac{[366(.0424)]}{[360 - (34)(.0424)]} = .04328\% \]

13. \[ 1.062 = \left[1 + \frac{\text{APR}}{2}\right]^{\frac{365}{140}}; \quad \text{APR} = \text{bond equivalent yield} = 6.085\% \]
    discount yield = \[ \frac{[360(.06085)]}{[365 + (140)(.06085)]} = 5.865\% \]

14. Recall that the prices are given as a percentage of par value, and the units after the colon are 32nds of 1 percent by convention.

    Feb05 STRIP: \[ 96.15625 = \frac{100}{[1 + \left(\frac{y}{2}\right)]^2}; \quad y = 3.958\% \]
    Feb06 STRIP: \[ 91.96875 = \frac{100}{[1 + \left(\frac{y}{2}\right)]^4}; \quad y = 4.230\% \]
    Feb07 STRIP: \[ 86.56250 = \frac{100}{[1 + \left(\frac{y}{2}\right)]^6}; \quad y = 4.868\% \]
    Feb08 STRIP: \[ 82.03125 = \frac{100}{[1 + \left(\frac{y}{2}\right)]^8}; \quad y = 5.014\% \]
    Feb09 STRIP: \[ 75.09375 = \frac{100}{[1 + \left(\frac{y}{2}\right)]^{10}}; \quad y = 5.811\% \]
    Feb10 STRIP: \[ 70.28125 = \frac{100}{[1 + \left(\frac{y}{2}\right)]^{12}}; \quad y = 5.965\% \]

    Note that the term structure is upward sloping; the expectations hypothesis then implies that this reflects market expectations of rising interest rates in the future.

15. \[ \text{EAR} = \left[1 + \left(\frac{.04230}{2}\right)\right]^2 - 1 = 4.275\% \]
16. \( [1 + (.04230/2)]^4 = [1 + (.03958/2)]^2 \ (1 + f_{1,1}); f_{1,1} = 4.553\% = \text{EAR} \)

\[ f_{1,1} = 100/[1.04553] = 95.6451\% \ \text{of par} = 94.21 \ \text{rounded to the nearest 32nd.} \]

Note that this price can be found directly from the relationship \( f_{t,k} = 100P_{t+k}/P_t \); where the first subscript refers to the time when the forward rate/price begins, the second subscript refers to the length of the forward rate/price, and \( P \) represents current or spot prices of various maturities. Similarly, \( f_{1,1} = \left[(P_t/P_{t+k})\right]^{1/n} - 1 \) Thus

\[ f_{1,1} = 100(91.96875/96.15625) = 95.6451 \ ; f_{1,1} = (96.15625/91.96875) - 1 = 4.553\% \]

The implied 1-year forward rate is larger than the current 1-year spot rate, reflecting the expectation that interest rates will go up in the future. Hence, for upward-sloping term structures, the implied forward rate curve lies above the spot rate curve.

17. \( f_{1,5} = 100(70.28125/96.15625) = 73.0906\% \ \text{of par} = 73:03 \ \text{rounded to the nearest 32nd.} \)

\[ 73.0906 = 100/(1 + f_{1,5})^5 ; f_{1,5} = 6.470\% \]

\[ f_{3,2} = 100(75.09375/86.56250) = 86.7509\% \ \text{of par} = 87:24 \ \text{rounded to the nearest 32nd.} \]

\[ 86.7509 = 100/(1 + f_{3,2})^2 ; f_{3,2} = 7.365\% \]

18. \( [1 + (.04230/2)]^4 = [1 + (.03958/2)]^2 \ (1 + f_{1,1} + .0030) ; f_{1,1} = 4.253\% \)

\[ f_{1,1} = 100/(1.04253) = 95.920\% \ \text{of par} = 95:29 \ \text{rounded to the nearest 32nd.} \]

Intuitively, the maturity premium on 2-year investments makes the future 1-year STRIP more valuable; hence, the forward price is greater and the forward rate lower. Alternatively, verify that if the forward rate and 1-year spot rate stayed the same as before, the spot 2-year price would become 91.7240% of par and the corresponding yield would be 4.414%; i.e., the longer maturity investment would be less valuable.

19. Feb05 STRIPS: \( P* = 100/[1 + (.03958 + .0025) / 2 ]^2 = 95.9210\% \ \text{of par} \)

\[ ?\%P = (95.9210 - 96.15625)/96.15625 = - 0.245\% \]

Feb07 STRIPS: \( P* = 100/[1 + (.04868 + .0025) / 2 ]^6 = 85.9314\% \ \text{of par} \)

\[ ?\%P = (85.9314 - 86.56250)/86.56250 = - 0.729\% \]

Feb10 STRIPS: \( P* = 100/[1 + (.05965 + .0025) / 2 ]^{12} = 69.2656\% \ \text{of par} \)

\[ ?\%P = (69.2656 - 70.28125)/70.28125 = - 1.445\% \]

For equal changes in yield, the longer the maturity, the greater the percentage price change. Hence, for parallel yield curve shifts, the price volatility is greater for longer-term instruments.
Feb05 STRIPS: \[96.15625 - .50 = \frac{100}{1 + (y*/2)^2}; \quad y* = 4.491\%\]
\[?y = 4.491 - 3.958 = + 0.533\%; \quad ?\% = .533/3.958 = 13.45\%\]

Feb07 STRIPS: \[86.56250 - .50 = \frac{100}{1 + (y*/2)^6}; \quad y* = 5.066\%\]
\[?y = 5.066 - 4.868 = + 0.198\%; \quad ?\% = .198/4.868 = 4.06\%\]

Feb10 STRIPS: \[70.28125 - .50 = \frac{100}{1 + (y*/2)^{12}}; \quad y* = 6.088\%\]
\[?y = 6.088 - 5.965 = + 0.123\%; \quad ?\% = .123/5.965 = 2.06\%\]

For equal changes in price, the absolute yield volatility is greater the shorter the maturity; the effect is magnified for percentage yield volatility when the yield curve is upward sloping, because yields (the divisor) are smaller for short maturities. Because of this, note that for sharply downward sloping yield curves, it’s possible for shorter maturity instruments to have less percentage yield volatility, but greater absolute yield volatility, than slightly longer maturity instruments.

20. Approximate real rate = 8.15% – 3.50% = 4.65%
Real interest rates are not observable because they do not correspond to any traded asset (at least not until very recently in the U.S.); hence, they must be inferred from nominal interest rates (which do correspond to traded assets), and from estimated inflation data. Real interest rate estimates are therefore only as good as (1) the inflation estimates used in the Fisher relation and (2) the degree to which the Fisher relation itself actually describes the behavior of economic agents.

21. \[f_{1,1} = (1.054^2/1.045)^{1/1} - 1 = 6.31\%\]
   \[f_{1,2} = (1.048^3/1.045)^{1/2} - 1 = 4.95\%\]
   \[f_{1,3} = (1.059^4/1.045)^{1/3} - 1 = 6.37\%\]

22. \[f_{2,1} = 1.048^3/1.054^2 - 1 = 3.61\%\]
   \[f_{3,1} = 1.059^4/1.048^3 - 1 = 9.27\%\]

23. \[I_1 = r_1 - 2\% = 4.50\% - 2\% = 2.50\%\]
   \[I_2 = f_{1,1} - 2\% = 6.31\% - 2\% = 4.31\%\]
   \[I_3 = f_{2,1} - 2\% = 3.61\% - 2\% = 1.61\%\]
   \[I_4 = f_{3,1} - 2\% = 9.27\% - 2\% = 7.27\%\]
## Spreadsheet Problems

**Chapter 9**

**Question 24**

### Input Area:

<table>
<thead>
<tr>
<th>Settlement date</th>
<th>3/14/04</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maturity date</td>
<td>7/1/04</td>
</tr>
<tr>
<td>Par value</td>
<td>$100,000</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.053</td>
</tr>
</tbody>
</table>

### Output Area:

| Price | $98,395.28 =TBILLPRICE(D7,D8,D10)*(D9/100) |
| Bond equivalent yield | 5.461% =TBILLEG(D7,D8,D10) |

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**Chapter 9**

**Question 25**

### Input Area:

<table>
<thead>
<tr>
<th>APR</th>
<th>7.80%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compounding periods per year</td>
<td>12</td>
</tr>
</tbody>
</table>

### Output Area:

| EAR             | 8.08% =EFFECT(D7,D8) |