Chapter 14 - Stock Options

American Options - An option that can be exercised any time up to its maturity date. Common for individual stock options traded in the US.

[Options are rarely exercised early]

European Options - An option that can only be exercised the day before expiration.

An Example: for Microsoft, June 2000

\[ S = \text{Stock Price} \quad C = \text{Call Option Price} \]
\[ K = 80 \quad \text{(Strike Price)} \]

<table>
<thead>
<tr>
<th>DATE</th>
<th>S</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/20/00</td>
<td>74.94</td>
<td>2.88</td>
</tr>
<tr>
<td>6/21/00</td>
<td>80.69</td>
<td>5.75</td>
</tr>
</tbody>
</table>

\[
\text{Stock hpr} = \frac{EV - BV}{BV} = \frac{80.69 - 74.94}{74.94} = 7.67\% \\
\text{for 1 day}
\]

\[
\text{Call hpr} = \frac{5.75 - 2.88}{2.88} = 99.65\% 
\]

Call options typically have a higher hpr than the underlying stock in a rising market (and vice versa). Therefore buying calls is "bullish"
Option Writing - Taking the selling side on an options transaction.

Call [put] Writer - Has the obligation to sell [purchase] the stock at the strike price if the option is exercised.

Payoff to Call Buyer = Max [S_T - K, 0]

Payoff to Call Writer = -Max [S_T - K, 0]

Payoff to a Put Buyer = Max [K - S_T, 0]

Payoff to a Put Writer = -Max [K - S_T, 0]
Option Profit to Buyer
= Payoff - Purchase Price

Option Profit to Write
= Purchase Price - Payoff

Back to Example
- Consider the following portfolio. On June 20 hold one stock and write 2 calls.

<table>
<thead>
<tr>
<th>Date</th>
<th>Value of Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/20</td>
<td>74.94 - 2 x 2.88 = 69.18</td>
</tr>
<tr>
<td>6/21</td>
<td>80.69 - 2 x 5.75 = 69.19</td>
</tr>
</tbody>
</table>

Value of PfH = S - 2C

Note: You have created a risk free portfolio as the gain on your stock is offset by the loss on the options.

To create a risk free portfolio you had to choose the appropriate number of calls to write for each share held. In this example writers 2 calls balanced holding one share.

An option's delta (S, Δ) describe the price change of the option as the price of the underlying stock changes. In our example
The option price changed $0.50 for every $1 change in the price of the stock, so the option delta = $0.50 / $1.00 = 0.5.

Creating a risk-free portfolio is called hedging. To hedge a stock portfolio, you can either write calls, or buy puts. The number of option you write (or buy) needed to hedge your portfolio depends on the option’s delta.

Option Strategies

1. Protective Put – Buying a put on a stock that you own. This puts a lower limit on the stock price as you can exercise at K.

2. Covered Call – Write a call on a stock you own. One reason to do this is to increase your “yield” from the stock by receiving the option premium, when you believe the stock is not going up prior to the maturity date.

3. Straddle – Buying or selling a call and a put on the same stock with the same strike price and some maturity date. Buying is a long straddle; selling is a short straddle.
Upper Bound on a Call Option
- The option will never sell for more than the price of the stock.
- If the option sold for more than the stock you could sell the options and buy the stock and thus make arbitrage profits.

Arbitrage Opportunity

1. Requires no net investment on your part.
2. Has no possibility of loss.
3. Has the potential for gain.

Upper Bound for a Put Option
- To prevent arbitrage a put option must sell for less than its strike price.

Intrinsic Value of An Option - This is the payoff (if positive) that one would receive from immediate exercise.

For a Call = Max [S - K, 0]
For a Put = Max [K - S, 0]
Lower Bounds for Options
- Options are never worse less than their intrinsic value.
  (They are usually worth more than their intrinsic value)

Time Value of an Option
- The difference between the market price of an option and its intrinsic value.

In the Money - An option is said to be "in the money" if its intrinsic value is positive.
An option is said to be "out of the money" if its intrinsic value is zero.
(If $S = K$, the option is "At the Money")

Example (Continued) - What is the intrinsic value and time value of the MSFT call option on June 20th?

Intrinsic Value = $\max [S-K, 0]$
= $\max [74.94-80, 0]$
= 0
Time Value - Market Price - Intrinsic Value

= 2.88 - 0 = 2.88

This option is "out of the money".

If the call is out of the money, put options with the same strike price are in the money (and vice versa).

Note: On June 21, the call option was in the money as the stock price rose above $80.

Index Options

- An option on a specific index such as the S&P 500, DJIA, and others.
- Are settled in cash (rather than through delivery of stock).
- As with other options, there are 100 units to a contract.

Options Clearing Corporation

- A private agency that guarantees that terms on the contract will be fulfilled.
- Issues and clears all the option contracts trading on the US exchanges.
Put Call Parity - A mathematical relationship between the price of a put and the price of a call for European Options with the same strike price and expiration. (is required to prevent arbitrage)

Consider the following portfolio

1. One share of a non dividend paying stock
2. Short (i.e. Write) one Call Option
3. One put option

Payoff to this Portfolio

<table>
<thead>
<tr>
<th>Payoff at $T$</th>
<th>$S_T &lt; K$</th>
<th>$S_T &gt; K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_T$</td>
<td>$S_T$</td>
<td>$S_T$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$(S_T - K)$</td>
</tr>
<tr>
<td>$K - S_T$</td>
<td>$(K - S_T)$</td>
<td>0</td>
</tr>
</tbody>
</table>

This is the payoff = $K$ of a risk free portfolio. Its present value = $K \frac{1}{(1+r)^T}$

where: $r =$ risk free discount rate
\( \frac{K}{(1+r)^T} \approx Ke^{-rT} \)

\[ \text{discrete discounting} \quad \text{continuos time discounting} \]

\[ \frac{K}{(1+r)^T} \]

is the value of the portfolio today

\[ S - C + P = \frac{K}{(1+r)^T} \]

this is Put-Call Parity

In words, this says if you know the call price you can compute the put price (and vice versa). If put and call prices don't follow this relationship there is an arbitrage opportunity.
\[ C = 7 \quad K = 60 \quad T = 6 \text{mo} = 0.5 \text{ yr} \]
\[ S = 61 \quad r = 0.05 \]
\[ S - C + P = \frac{K}{(1+r)^T} \]
\[ 61 - 7 + P = \frac{60}{(1.05)^{0.5}} \]
\[ 54 + P = 58.5540 \]
\[ P = 58.5540 - 54 = 4.55 \]

Is this put "in the money"? No

What is the time value of this put? 4.55 - 0 = 4.55

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\[ P = 484 \]
\[ C = 9 \quad K = 75 \quad T = 0.5 \]
\[ P = 3 \quad r = 0.04 \quad S = ? \]

\[ S - 9 + 3 = \frac{75}{(1.04)^{0.5}} \]
\[ S = \frac{75}{(1.04)^{0.5}} + 6 \]
\[ = 73.54 + 6 = 79.54 \]

Is the call in the money? Yes

Intrinsic Value = 4.54

What is the time value of this call? 9 - 4.54 = 4.46
Buy 2 Calls & Put

Cost: 2(5) + 4 = $14

Payoff:
Call = \text{Max}[S_T - K, 0] = \text{Max}[55 - 45, 0] = 10
Put = \text{Max}[K - S_T, 0] = \text{Max}[45 - 55, 0] = 0
Payoff = 2 \times 10 + 0 = 20
Profit = Payoff - Cost
= 20 - 14 = 6

Break-even memo
BV = EV

What does it cost to create this position?
Buy Stock = 75 Sell Put is $5 income
BV = 75 - 5 = $70

EV = S_T - \text{Max}[S_T - K, 0]
If the call finishes out of the money
BV = EV
70 = S_T - 0 \implies S_T = 70

What if the call is in the money? When the call is in the money, you pay out $1 for each $1 gain in stock price which is neutral for you.
Payoff for covered call

\[ \text{Payoff} = \begin{cases} 75 & \text{if } S_t < K \\ S_t - K & \text{if } S_t \geq K \end{cases} \]

\[ \text{For } S_t = 75, \quad K = 75 \]

\[ \text{For } S_t = 114, \quad K = 90 \]

\[ 100 + p = S_t + \max [K - S, 0] \]

\[ S_t = 114, \quad K = 90 \]

\[ 100 + p = 114 + \max [90 - 114, 0] \]

\[ p = 14 \]