Chapter 10: Bond Prices & Yields

Coupon Rate: The bond's annual coupon divided by the face value

\[
\text{Coupon Rate} = \frac{\text{Annual Coupon}}{\text{Face Value}}
\]

Current Yield = \frac{\text{Annual Coupon}}{\text{Current Bond Price}}

**Bond Computations**

- A bond can be seen as a series of CF's.
- The price of a bond is the PV of the CF's.
- The CF's have 2 components:
  1. Every 6 mo (typical bond) you get a stated coupon payment which is ½ of the annual coupon.
  2. The Face Value is paid to you at maturity.

\[
P = \sum_{t=1}^{T} \frac{CF}{(1+i)^t}
\]

Using this formula is tedious. Using a financial calculator is quick.
BOND COMPUTATIONS USING A FINANCIAL CALCULATOR: Assuming Semi Annual Coupon Payment and Face Value = 1000

[for Annual Coupon Bond]

\[ P/YR = 2 \quad \text{[P/YR = 1]} \]

\[ N = \text{Number of Periods, which is } 2 \times \text{the number of years until maturity} \]

\[ \text{[Numbers of years to maturity]} \]

\[ I/YR = \text{The YTM of the Bond (Yield to Maturity) or the required return on the Bond. It is an APR (i.e. BEY) which is } 2\times \text{ the 6 mo rate.} \]

\[ PV = -\text{PV of the Bond} \]

\[ \text{[The coupon payment from the Bond]} \]

\[ \text{[10 \times \text{Coupon Rate \%}]} \]

More generally:

\[ PMT = \frac{\text{FACE VALUE} \times \text{COUPON RATE}}{\# \text{ of PMTS per year}} \]

\[ FV = \text{The Face Value of the Bond, which is typically 1000.} \]

Usually we are solving for either

\[ \text{Price or } \frac{\text{YTM}}{I/YR} \]
For the Bond Example 10.1 Page 326

\( N = 2 \times 20 = 40 \) (20 yr bond)

I/YR = \underline{V}e\underline{r}y

P/YR = \underline{S}olve\underline{F}or

Pmt = 5 \times 8 = 40 \) (8\% Coupon-Semi Annual)

FV = 1000

\( P/YR = 2 \)

<table>
<thead>
<tr>
<th>DISC RATES YTM</th>
<th>PRICE</th>
<th>CURR YLD</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1376.54</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1231.15</td>
<td>80/1231.15 = 6.5%</td>
</tr>
<tr>
<td>7</td>
<td>1106.78</td>
<td>80/1000 = 8.0%</td>
</tr>
<tr>
<td>8</td>
<td>1000.00</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>907.99</td>
<td>80/907.99 = 8.8%</td>
</tr>
<tr>
<td>10</td>
<td>828.41</td>
<td>80/759.31 = 10.54%</td>
</tr>
<tr>
<td>11</td>
<td>759.31</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>699.07</td>
<td></td>
</tr>
</tbody>
</table>

Note: For Premium Bonds

Coupon Rate > CURR YLD > YTM
8\% > 6.5\% > 6.0\%

For Discount Bond

Coupon Rate < CURR < YTM
8\% < 10.54\% < 11\%
Callable Bonds - A bond that can be repurchased by the issuer prior to maturity (i.e. at the Call Date or after the call Date)

Call Price = The price at which the bond will be bought back (maybe the face value but is often above the face value)

YTC = Yield to Call - Analogous to YTM but substitute Call Date for maturity and substitute Call Price for face value.

Interest Rate Risk - The risk that interest rates will change which will induce a change in the bond price.
Malkiel's Bond Theorems

1. Bond prices and yields are inverse
   
   \[
   \text{Price} \downarrow \text{negative slope}
   \]
   
   \text{YTM}

2. Longer Term Bonds Have a Steeper Slope
   
   \[
   \text{Long Term} \downarrow \text{Short Term}
   \]

3. For a given change in interest rates, there is a declining effect on price as the term to maturity increases.

4. The slope changes as the YTM changes
   
   \[
   \text{Price} \downarrow \text{steep slope} \quad \text{YTM}
   \]
   
   \text{Flatter slope}
Convexity: For a given interest rate change, the price increase will be larger than the corresponding price decrease for an opposite change in interest rates.

\[ \Delta P_2 > \Delta P_1 \]

Duration: Captures the effects of the bond THEOREMS

- It's measured in years.
- It's a measure of the riskiness of a bond, more specifically - a measure of how sensitive bond prices are to interest rate changes.
- It measures the effective maturity of a bond, by considering how long on average it takes to repay your purchase price, in a PV context.
- The duration of a zero coupon bond is the maturity of the bond.
- The higher the coupon on a bond, the shorter its duration ceteris paribus.

- Duration is related to the slope on the Price vs. YTM graph.

\[
\text{Price} \quad \text{Duration is related to this slope.}
\]

Using Duration to Predict Price Change

\[
\frac{\Delta P}{P} = -\text{Duration} \times \frac{\Delta \text{YTM}}{1 + \text{YTM}^2}
\]

This says that price change is a function of duration. The higher the duration, the greater the price change for a given shift in interest rates.
By algebra, we can also write
\[
\frac{\Delta P}{P} = -\text{Duration} \times \Delta \text{YTM} \times \left(1 + \frac{\text{YTM}}{2}\right)
\]
\[
\text{Modified Duration}
\]

**Bond Risk Management**

1. **Dedicated Portfolio** - Create a portfolio to pay off a future liability.
2. **Duration Matching** - Keep the duration of your liabilities equal to the duration of your assets. You will have an immunized portfolio.

**Reinvestment Risk** - If interest rates fall, you will reinvest your coupons at a smaller rate.

**Price Risk** - If interest rates fall, the price of my bonds will rise.

If the duration of your assets equals the duration of your liabilities, the effects will cancel \(\Rightarrow\) Immunized Portolio.
Dynamic Immunization - As YTM's change, and as time advances, the duration of your assets and liabilities change, which may require a rebalancing of your portfolio to keep it immunized.
Coupon = 9.5%

21 years to maturity

Current Price = 1289

What is YTM? Current Yield?

P/V = 2

\[ \text{Y} = 5 \times 9.5 \]

INR (N = 42, PV = -1289, PMT = 4750, FV = 1000)

a) \[ \text{Y} = 6.88\% \]

b) Current Yield = \[ \frac{\text{Annual Coupon}}{\text{Current Price}} = \frac{95}{1289} \]

= 0.0737 = 7.37\%