Chapter 9 - Interest Rates

Bank Warranty Rate: An interest rate that "predicts" future interest and is used as a benchmark.

Prime Rate: It is the rate that banks charge their "prime" or most creditworthy clients. It's a short-term interest rate. This is a Bank Warranty rate. **Currently 4%**

Federal Funds Rate: It is the rate banks charge other banks for overnight loans. Banks borrow or lend to meet capital requirements. (This rate is market determined, though the Fed sets a range for this rate) 
**Currently 1%**

Discount Rate: the rate set by the Fed for overnight loans to member banks 
**Currently 2%**
Money Market Instruments

US Treasury Bills - the largest ($value) and most liquid money market debt instrument.

- It's a short term debt obligation of the US Treasury.

- Like other money market instruments, it's sold at a discount and redeemed at par. (The interest amount earned is the difference between its Face Value and the Per Chose Price. Currently 1.25-2%, depending on Maturity.)
Term Structure Theories

1. Expectation Theory - The term structure reflects the market's prediction (or expectation) of future interest rates.

   Problem - The term structure is typically upward sloping, but interest rates are not constantly increasing.

2. Maturity Preference
   - Says that lenders need to be rewarded for holding long-term bonds because long-term bonds are riskier, i.e., there is more interest rate risk in long-term bonds. (This suggests the term structure should always be upward sloping.)

3. Market Segmentation
   - The rate for each maturity is determined in a separate market.

   Problem - We expect most borrowers & lenders to make trade-offs between rate & maturity.
Commercial Paper - is a short term debt obligation of the Treasury of a "credit worthy" corporation, often a financial intermediary.

"Prime" customers can generally borrow below prime rate as they could issue commercial paper.

Money Market Instruments are typically quoted on a discount basis.

Analogy: At your favorite store there is an item on sale that is regularly priced at $50. It is 10% off (i.e. a 10% discount). What is the price.

\[
\text{SALE} = \text{REGULAR} \times (1 - \text{DISCOUNT}) \\
\text{PRICE} = \text{PRICE} \\
= 50(1-0.1) = 50(0.9) = 45
\]

This same approach is used for money market securities, except the discount is quoted on an annual basis.
(360 day year) so an adjustment has to be made in the formula.

\[
\text{Price} = \frac{\text{FACE}}{\text{VALUE}} \left(1 - \frac{\text{Days to Maturity} \times \text{Discount}}{360}\right)
\]

A multipart example.

You can buy a $10,000 face value security that matures in 60 days at a 6% discount.

1. **What is its price?**

\[
\text{Price} = 10,000 \left(1 - \frac{60}{360} \times 0.06\right)
\]

\[
= 10,000 \left(1 - 0.01\right)
\]

\[
= 10,000 \times 0.99 = 9900
\]

2. **What is the hpr?**

\[
\text{hpr} = \frac{\text{EV} - \text{BV}}{\text{BV}} = \frac{10,000 - 9900}{9900} = 0.010101
\]

3. **What is the APR?**

\[
\text{APR} = \text{hpr} \times m
\]

\[
m = \frac{365}{60} = 6.083333
\]

\[
\text{APR} = 0.010101 \times 6.083333 = 0.061448
\]

\[
\approx 6.14\%
\]
Note: The BEY (Bond Equivalent Yield) is an APR, so this is also the BEY.

Note: The text shows BEY can be directly computed using the following formula (P 294):

\[ \text{BEY} = \frac{365 \times \text{Discount}}{360 - \text{Days to Maturity} \times \text{Discount}} \]

\[ = \frac{365 \times 0.06}{360 - (60 \times 0.06)} = \frac{21.9}{360 - 3.6} \]

\[ = \frac{21.9}{356.40} = 0.061448 \]

\[ = 6.14\% \]

4. What is the EAR?

\[ \text{EAR} = \left(1 + \text{hpr} \right)^m - 1 \]

\[ = \left(1.01\right)^{0.08333} - 1 \]

\[ = 1.062401 - 1 \]

\[ = 0.062401 \]

\[ = 6.24\% \]
**Basis Point (bp)**

One hundredth of 1%, i.e., there are 100 b.p. in 1%

So, 90 day T-Bills are at 1.25%.

1 year T-Bills are at 2.00%.

So we would say there is a 75 b.p. yield spread.

---

Nominal versus Real Interest Rates

The rate quoted on Bonds & Bills is not adjusted for inflation, and is called a nominal rate. (An exception is the rate on TIPS which are inflation protected Treasury Securities)

Real rates are the result of adjusting the nominal rate for inflation. The real rate measure gain in purchasing power.
Note: EAR > BEY > DISCOUNT (APR)
6.24% > 6.14% > 6.00%

The EAR measures your true annualized yield for this security.

The Treasury Yield Curve
- This is a plot of the yield to maturity (YTM) of US Treasury STRIPS versus maturity

STRIPS: Separate Trading of Registered Interest and Principal of Securities

(Note: "Coupon Stripping" was originally done by securities firms, and then the US Treasury decided to do it).
**Coupon Stripping**

If you bought 20, 10 year, 10% bonds, what would your CF's be?

<table>
<thead>
<tr>
<th>t</th>
<th>Years</th>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>1000</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>1000</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>21,000</td>
</tr>
</tbody>
</table>

From these proceed you could sell 40 zero coupon bonds with a $1000 face value. The first bond would mature in 6 mo, the second in 1yr and so on until at year 10, 21 bonds would mature. This is an example of "unbundling" cash flows.
Real Rate = Nominal Rate - Inflation Rate

**Exact Relationship**

\[
(1 + \text{Real}) = \frac{(1 + \text{Nominal rate})}{(1 + \text{inflation rate})}
\]

**Fisher Hypothesis** - Says that nominal interest rates follow inflation (i.e. the real interest rate does not fluctuate much)

**Rule of 72** - To determine the period for an investment to double, take:

\[
72 \div \text{Interest Rate} \% = \# \text{ of years to double}
\]

Example: At a 7.2% interest rate, it will take 10 years for an investment to double in value.
FORWARD INTEREST RATES

Forward rates interest rates implied by the term structure.

Example: Say 1 year STRIPS yield 3% and 2 year year STRIPS yield 4%.

What is the implied 1 year rate, one year from now?

\[ f_{x,y} \text{ forward rate } x \text{ years from now for } y \text{ years} \]

So \( f_{1,1} \) is the 1 year forward rate 1 year from now.

\[ r_1 = 3\% \]

\[ f_{1,1} \]

\[ r_2 = 4\% \]

So be equally well off choosing \( r_1 \) followed by \( f_{1,1} \) versus choosing \( r_2 \) for two years, what must \( f_{1,1} \) equal?
4% per year versus 3% followed by f_{1,1}

\[(1.04)^2 = (1.03)(1+f_{1,1})\]

\[\frac{(1.04)^2}{1.03} = 1 + f_{1,1}\]

\[a \quad f_{1,1} = \frac{(1.04)^2}{1.03} - 1\]

\[= 1.050597 - 1\]

\[= 5.01\%\]

As a formula:

\[(1+f_{1,1}) = \frac{(1+r_2)^2}{1+r_1}\]

\[a \quad f_{1,1} = \frac{(1+r_2)^2}{1+r_1} - 1\]

By analogy can apply to other time periods:

\[f_{2,1} = \frac{(1+r_3)^3}{(1+r_2)^2} - 1\]
#7 P 3/16 STRIPS

- Matures in 10 yr
  - Current price = 502.57
  - What is its YTM?

Recall for bond calculations

\[
P/\text{YR} = 2
\]

\[N = \# \text{ of 6 month periods}\]

Face Value generally is 1000

- PV is the price.

\[
P/\text{YR} = 2
\]

\[I/\text{YR}(N=20, PV = -502.57, FV = 1000) = 7.0\%\]
<table>
<thead>
<tr>
<th>Maturity</th>
<th>N</th>
<th>Quote</th>
<th>Price</th>
<th>YTM</th>
<th>I/YR</th>
<th>EAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb '05</td>
<td>2</td>
<td>96:05</td>
<td>961.563</td>
<td>3.96</td>
<td>4.00 %</td>
<td>4.00 %</td>
</tr>
<tr>
<td>Feb '06</td>
<td>4</td>
<td>91:31</td>
<td>947.6875</td>
<td>4.23</td>
<td>4.27 %</td>
<td>4.27 %</td>
</tr>
<tr>
<td>Feb '07</td>
<td>6</td>
<td>86:18</td>
<td>865.625</td>
<td>4.87 %</td>
<td>4.92 %</td>
<td>4.92 %</td>
</tr>
</tbody>
</table>

For Treasury Notes & Bonds, a quote is read as a percent of face value.

XX : YY

The part following the colon is the number of 32nds.

96 : 05 means 96 $\frac{5}{32}$ percent of Face Value.

$5/32 = 0.1563$

96.1563% of Face Value

For 1000 Face Value, this is

961.563

P/IYR = 2

$I/YR(N=2, PV=-961.563, FV=1000) = 3.96\%$
I/YR \( (N=4, PV=-919.688, FV=1000) = 4.23 \% \)

I/YR \( (N=6, PV=-865.625, FV=1000) = 4.87 \% \)

Observation: Spot interest rates are increasing \( \Rightarrow \) Upward Sloping Term Structure

\[
\text{EAR} = (1 + \frac{\text{APR}}{2})^2 - 1 = \left(1 + \frac{0.0423}{2}\right)^2 - 1 = 0.0427 = 4.27\% 
\]

16) Asking for \( f_{1,1} \)

\[
f_{1,1} = \frac{(1+r_2)^2}{(1+r_1)} - 1 = \left(1 + \frac{0.0423}{2}\right)^2 \left(1 + \frac{0.0396}{2}\right) - 1 = 1.0445 - 1 
\]

\[
= 0.0450 = 4.5\% 
\]

We can use this rate to compute the price of a 1 year pure discount bond one year from now. It's simply the PV computed using this rate.
By Calculator: \( P/YR = 2 \)

\[ PV(n = 2, FV = 1000, I/YR = 4.5) = 956.47 \]

or by formula

\[ PV = \frac{1000}{(1 + \frac{0.045}{2})^2} = 956.47 \]

Conclusion: From prices for US Treasury STRIPS we can compute the yield curve and implied future interest rates and STRIPS prices.