Chapter Six: Transportation, Transshipment, and Assignment Problems

**PROBLEM SUMMARY**

1. Balanced transportation
2. Balanced transportation
3. Balanced transportation
4. Unbalanced transportation
5. Unbalanced transportation
6. Unbalanced transportation
7. Unbalanced transportation, multiple optimal
8. Sensitivity analysis (6–7)
9. Unbalanced transportation, multiple optimal
10. Unbalanced transportation
11. Unbalanced transportation
12. Balanced transportation
13. Balanced transportation
14. Sensitivity analysis (6–13)
15. Unbalanced transportation, multiple optimal
16. Sensitivity analysis (6–15)
17. Shortage costs (6–15)
18. Unbalanced transportation
19. Unbalanced transportation, multiple optimal
20. Balanced transportation
21. Unbalanced transportation, multiple optimal
22. Sensitivity analysis (6–21)
23. Unbalanced transportation
24. Sensitivity analysis (6–23)
25. Sensitivity analysis (6–23)
26. Unbalanced transportation
27. Unbalanced transportation
28. Unbalanced transportation
29. Unbalanced transportation, production scheduling
30. Unbalanced transportation
31. Sensitivity analysis (6–30)
32. Shortage costs
33. Multiperiod scheduling
34. Balanced transportation
35. Transshipment
36. Transshipment
37. Transshipment
38. Transshipment
39. Transshipment
40. Unbalanced assignment, LP formulation
41. Assignment
42. Assignment
43. Assignment
44. Unbalanced assignment, multiple optimal
45. Assignment, multiple optimal
46. Assignment
47. Unbalanced assignment, multiple optimal
48. Assignment or transportation
49. Prohibited routes (6–48)
50. Unbalanced assignment
51. Unbalanced assignment, multiple optimal
52. Unbalanced assignment
53. Unbalanced assignment (maximization)
54. Unbalanced assignment
55. Assignment
56. Unbalanced assignment

**PROBLEM SOLUTIONS**

1. St. Louis - Chicago = 250
   Richmond - Chicago = 50
   Richmond - Atlanta = 350
2. \[ x_{13} = 2 \\
\quad x_{14} = 10 \\
\quad x_{22} = 9 \\
\quad x_{23} = 8 \\
\quad x_{31} = 10 \\
\quad x_{32} = 1 \]

3. \[ x_{A3} = 100 \\
\quad x_{B1} = 135 \\
\quad x_{B2} = 45 \\
\quad x_{C2} = 130 \\
\quad x_{C3} = 70 \\
Z = \$2,345 \]

4. Minimize \[ Z = 6x_{A1} + 9x_{A2} + 100x_{A3} + 12x_{B1} + 3x_{B2} + 5x_{B3} + 4x_{C1} + 8x_{C2} + 11x_{C3} \]

subject to
\[
\begin{align*}
x_{A1} + x_{A2} + x_{A3} & \leq 130 \\
x_{B1} + x_{B2} + x_{B3} & \leq 70 \\
x_{C1} + x_{C2} + x_{C3} & \leq 100 \\
x_{A1} + x_{B1} + x_{C1} & = 80 \\
x_{A2} + x_{B2} + x_{C2} & = 110 \\
x_{A3} + x_{B3} + x_{C3} & = 60 \\
x_{ij} & \geq 0
\end{align*}
\]

\[ x_{A2} = 80 \\
\quad x_{B2} = 10 \\
\quad x_{B3} = 60 \\
\quad x_{C1} = 80 \\
\quad x_{C2} = 20 \\
Z = \$1,530 \]

5. \[ x_{11} = 70 \\
\quad x_{13} = 20 \\
\quad x_{22} = 10 \\
\quad x_{31} = 100 \\
\quad x_{32} = 100 \\
Z = \$1,240 \]

6. \[ x_{A2} = 20 \\
\quad x_{A3} = 60 \\
\quad x_{B2} = 70 \\
\quad x_{C1} = 80 \\
\quad x_{C2} = 20 \\
Z = \$1,290 \]

7. Minimize \[ Z = 14x_{A1} + 9x_{A2} + 16x_{A3} + 18x_{A4} + 11x_{B1} + 8x_{B2} + Mx_{B3} + 16x_{B4} + 16x_{C1} + 12x_{C2} + 10x_{C3} + 22x_{C4} \]

subject to
\[
\begin{align*}
x_{A1} + x_{A2} + x_{A3} + x_{A4} & \leq 150 \\
x_{B1} + x_{B2} + x_{B3} + x_{B4} & \leq 210 \\
x_{C1} + x_{C2} + x_{C3} + x_{C4} & \leq 320 \\
x_{A1} + x_{B1} + x_{C1} & = 130 \\
x_{A2} + x_{B2} + x_{C2} & = 70 \\
x_{A3} + x_{B3} + x_{C3} & = 180 \\
x_{A4} + x_{B4} + x_{C4} & = 240 \\
x_{ij} & \geq 0
\end{align*}
\]

\[ x_{A2} = 70 \\
\quad x_{A4} = 80 \\
\quad x_{B1} = 50 \\
\quad x_{B4} = 160 \\
\quad x_{C1} = 80 \\
\quad x_{C3} = 180 \\
Z = \$8,260 \]

8. There is no effect. The Gary mill has 60 tons left over as surplus with the current solution to Problem 11. Reducing the capacity at Gary to 30 still leaves a surplus of 30 tons.

9. Minimize \[ Z = 100x_{A1} + 10x_{A2} + 5x_{A3} + 12x_{B1} + 9x_{B2} + 4x_{B3} + 7x_{C1} + 3x_{C2} + 11x_{C3} + 9x_{D1} + 5x_{D2} + 7x_{D3} \]

subject to
\[
\begin{align*}
x_{A1} + x_{A2} + x_{A3} & = 90 \\
x_{B1} + x_{B2} + x_{B3} & = 50 \\
x_{C1} + x_{C2} + x_{C3} & = 80 \\
x_{D1} + x_{D2} + x_{D3} & \leq 60 \\
x_{A1} + x_{B1} + x_{C1} + x_{D1} & \leq 120 \\
x_{A2} + x_{B2} + x_{C2} + x_{D2} & \leq 100 \\
x_{A3} + x_{B3} + x_{C3} + x_{D3} & \leq 110 \\
x_{ij} & \geq 0
\end{align*}
\]

\[ x_{A1} = 90 \\
\quad x_{B1} = 30 \\
\quad x_{B3} = 20 \\
\quad x_{C2} = 80 \\
\quad x_{D1} = 40 \\
\quad x_{D2} = 20 \\
Z = \$1,590 \]

10. Minimize \[ Z = 9x_{TN} + 14x_{TP} + 12x_{TC} + 17x_{TB} + 11x_{MN} + 10x_{MP} + 100x_{MC} + 10x_{MB} + 12x_{SN} + 8x_{SP} + 15x_{SC} + 7x_{SB} \]

subject to
\[
\begin{align*}
x_{TN} + x_{TP} + x_{TC} + x_{TB} & \leq 200 \\
x_{MN} + x_{MP} + x_{MC} + x_{MB} & \leq 200 \\
x_{TN} + x_{TP} + x_{TC} + x_{TB} & \leq 200 \\
x_{TN} + x_{MN} + x_{SN} & = 130 \\
x_{TP} + x_{MP} + x_{SP} & = 170 \\
x_{TC} + x_{MC} + x_{SC} & = 100 \\
x_{TB} + x_{MB} + x_{SB} & = 150 \\
x_{ij} & \geq 0
\end{align*}
\]

78
Tampa - NY = 100
Tampa - Chicago = 100
Miami - NY = 30
Miami - Philadelphia = 120
Fresno - Philadelphia = 50
Fresno - Boston = 50
Z = $5,080

11. Minimize \( Z = 7x_{1A} + 8x_{1B} + 5x_{1C} + 6x_{2A} + 100x_{2B} + 6x_{3C} + 10x_{3A} + 4x_{3B} + 5x_{3C} + 3x_{4A} + 9x_{4B} + 100x_{4C} \)
subject to
\[
\begin{align*}
    x_{1A} + x_{1B} + x_{1C} &\leq 5 \\
x_{2A} + x_{2B} + x_{2C} &\leq 25 \\
x_{3A} + x_{3B} + x_{3C} &\leq 20 \\
x_{4A} + x_{4B} + x_{4C} &\leq 25 \\
x_{1A} + x_{2A} + x_{3A} + x_{4A} & = 10 \\
x_{1B} + x_{2B} + x_{3B} + x_{4B} & = 20 \\
x_{1C} + x_{2C} + x_{3C} + x_{4C} & = 15 \\
\end{align*}
\]
\( x_{ij} \geq 0 \)
\( x_{1C} = 5 \\
x_{2C} = 10 \\
x_{3B} = 20 \\
x_{4A} = 10 \\
Z = $195

12. \( x_{1A} = 70 \\
x_{2B} = 20 \\
x_{2C} = 10 \\
x_{3A} = 20 \\
x_{3B} = 100 \\
x_{3D} = 100 \\
Z = $13,200

13. \( x_{A2} = 1,800 \\
x_{A4} = 950 \\
x_{A6} = 750 \\
x_{B1} = 1,600 \\
x_{B3} = 1,500 \\
x_{B5} = 1,250 \\
x_{B6} = 650 \\
Z = $3,292.50

14. No effect

15. \( x_{1B} = 250 \\
x_{1D} = 170 \\
x_{2A} = 520 \\
x_{2C} = 90 \\
x_{3C} = 130 \\
x_{3D} = 210 \\
Z = $21,930

16.(1) \( x_{1B} = 250 \)
\( x_{1D} = 350 \)
\( x_{2A} = 520 \)
\( x_{2C} = 90 \)
\( x_{3C} = 310 \)
\( x_{3D} = 30 \)
\( x_{4D} = 300 \)
\( Z = $29,130 \)
\( Z = $24,930 \)

Select alternative 2; add a warehouse at Charlotte

17. \( x_{1B} = 250 \)
\( x_{1D} = 170 \)
\( x_{2A} = 520 \)
\( x_{2C} = 90 \)
\( x_{3C} = 130 \)
\( x_{3D} = 210 \)
\( x_{4C} = 180 \)
\( Z = $26,430 \)

Total transportation cost = $21,930
Total shortage cost = $4,500

18. GA - 1 to NC - W = 2
GA - 1 to VA - SW = 10
GA - 2 to NC - SW = 6
GA - 2 to VA - C = 4
SC - 1 to NC - SW = 1
SC - 1 to NC - P = 6
\( Z = $841,000 \)
\( Z = $841,000 \)

19. \( x_{1B} = 60 \)
\( x_{2A} = 45 \)
\( x_{2B} = 25 \)
\( x_{2C} = 35 \)
\( x_{3B} = 5 \)
\( Z = $1,605 \)

20. \( x_{11} = 30 \)
\( x_{12} = 5 \)
\( x_{14} = 2 \)
\( x_{22} = 20 \)
\( x_{33} = 14 \)
\( x_{44} = 26 \)
\( Z = 364 \text{ miles} \)

21. North A = 250
South B = 200
South C = 40
East A = 150
East C = 160
West D = 210
Central B = 100
Central D = 190
\( Z = 20,700 \text{ min.} \)

79
22. North A = 250  
   South B = 200  
   South C = 140  
   East A = 100  
   East C = 210  
   West D = 210  
   Central B = 150  
   Central D = 140  
   Z = 21,200 min.  

The overall travel time increased by 500 minutes, which divided by all 1,400 students is only an increase of .357 minutes per student. This does not seem to be a significantly large increase.

23. A - 3 = 8  
    A - 4 = 18  
    B - 3 = 12  
    B - 5 = 27  
    D - 3 = 5  
    D - 6 = 35  
    E - 1 = 25  
    E - 2 = 15  
    E - 3 = 4  
    Z = $1,528 (multiple optimal)

24. If Easy Time purchased all the baby food demanded at each store from the distributor total profit would be $1,246, which is less than buying it from the other locations as determined in problem 23. This profit is computed by multiplying the profit at each store by the demand. In order to determine if some of the demand should be met by the distributor a new source (F) must be added to problem 23. This source represents the distributor and has an available supply of 150 cases, the total demand from all the stores. The new optimal solution is shown as follows.

25. Solve the model as a linear programming model to obtain the shadow prices. Among the 5 purchase locations, the store at Albany has the highest shadow price of $3. The sensitivity range for supply at Albany is $25 \leq q_{1} \leq 43$. Thus, as much as 17 additional cases can be purchased from Albany which would increase profit by $51 for a total of $1,579.

26. Charlotte - Atlanta = 30  
   Memphis - St. Louis = 30  
   Louisville - NY = 30  
   Z = 159,000

27. 1 - C = 2  
    1 - E = 5  
    2 - C = 10  
    3 - E = 5  
    4 - D = 8  
    5 - A = 9  
    6 - B = 6  
    Z = 1,275

28. Jan - Jan = 180  
    Jan - May = 30  
    Feb - Feb = 260  
    Feb - March = 40  
    March - March = 300  
    April - April = 210  
    April - May = 90  
    May - May = 280  
    May - June = 20  
    June - June = 300  
    Z = $180,645

29. R_{J} - Jan = 300  
    O_{J} - Jan = 110  
    R_{F} - Feb = 300  
    O_{F} - Feb = 20  
    O_{F} - March = 120  
    R_{M} - March = 180  
    R_{M} - April = 120  
    O_{M} - March = 200  
    R_{A} - April = 300  
    O_{A} - April = 200  
    R_{M} - May = 300  
    O_{M} - May = 130  
    R_{J} - June = 300  
    O_{J} - June = 80  
    Z = $301,004
30. Sacramento - St. Paul = 13  
Sacramento - Topeka = 5  
Bakersfield - Denver = 8  
Bakersfield - St. Paul = 2  
San Antonio - Topeka = 10  
Montgomery - Denver = 12  
Jacksonville - Akron = 15  
Jacksonville - Topeka = 5  
Ocala - Louisville = 15  

Z = $278,000  

It is cheaper for National Foods to continue to operate its own trucking firm.

31. Increasing the supply at Sacramento, Jacksonville and Ocala to 25 tons would have little effect, reducing the overall monthly shipping cost to $276,000, which is still higher than the $245,000 the company is currently spending with its own trucks.

32. L.A. - Singapore = 150  
L.A. - Taipei = 300  
Savannah - Hong Kong = 400  
Savannah - Taipei = 200  
Galveston - Singapore = 350  
Order shortage in Hong Kong = 200  

Z = $723,500  
Penalty cost = $160,000

33. 

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Z = $1,198,500 (multiple optimal solutions)
34. Al - Eagles (2) and Bengals (5)
Barbara - Saints (5) and Jets (1)
Carol - Cowboys (1) and Packers (2)
Dave - Redskins (1) and Cardinals (7)
Z = 24

Multiple optimal solutions exist

Carol seems to have received the best allocation but overall the allocation seems relatively fair.

35. \[ x_{14} \text{ (Hamburg - Norfolk)} = 42 \]
\[ x_{59} \text{ (NY - Chicago)} = 50 \]
\[ x_{26} \text{ (Marseilles - Savannah)} = 63 \]
\[ x_{35} \text{ (Liverpool - NY)} = 37 \]
\[ x_{48} \text{ (Norfolk - St. Louis)} = 42 \]
\[ x_{15} \text{ (Hamburg - NY)} = 13 \]
\[ x_{67} \text{ (Savannah - Dallas)} = 60 \]
\[ x_{68} \text{ (Savannah - St. Louis)} = 3 \]

\[ Z = 77,362 \]

HND = 38
HNS = 17
MSD = 22

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European Ports           U.S. Warehouses          Distribution Centers

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36. \[ x_{16} \text{ (Mexico - Houston)} = 18 \]
\[ x_{24} \text{ (Puerto Rico - Miami)} = 11 \]
\[ x_{34} \text{ (Haiti - Miami)} = 23 \]
\[ x_{47} \text{ (Miami - NY)} = 20 \]
\[ x_{48} \text{ (Miami - St. Louis)} = 12 \]
\[ x_{49} \text{ (Miami - LA)} = 2 \]
\[ x_{69} \text{ (Houston - LA)} = 18 \]

\[ Z = \$479 \text{ or } \$479,000 \]

37.(a) \[ x_{15} = 72 \]
\[ x_{24} = 105 \]
\[ x_{35} = 83 \]

\[ x_{46} = 25 \]
\[ x_{47} = 80 \]
\[ x_{56} = 65 \]
\[ x_{58} = 90 \]

\[ Z = \$4,871,000 \]

(b) Adding a capacity constraint at plants in Indiana and Georgia
\[ x_{14} = 15 \]
\[ x_{15} = 57 \]
\[ x_{24} = 105 \]
\[ x_{35} = 83 \]

\[ x_{46} = 40 \]
\[ x_{47} = 80 \]
\[ x_{56} = 50 \]
\[ x_{58} = 90 \]

\[ Z = \$4,886,000 \]
38. \( x_{1C} = 15 \) \( x_{BA} = 40 \) \( x_{2B} = 57 \) \( x_{CB} = 80 \)
\( x_{3A} = 105 \)

\[ Z = 1,490 \text{ or } $14,900 \]

39. \( x_{37} \text{ (Italy - Texas)} = 2.1 \)
\( x_{15} \text{ (Germany - Mexico)} = 5.2 \)
\( x_{26} \text{ (Belgium - Panama)} = 6.3 \)
\( x_{59} \text{ (Mexico - Ohio)} = 5.2 \)
\( x_{68} \text{ (Panama - Virginia)} = 3.7 \)
\( x_{69} \text{ (Panama - Ohio)} = 2.6 \)

\[ Z = $27.12 \text{ million} \]

40. 1 - 1
    2 - 4
    3 - 2
    5 - 3

\[ Z = 78 \]

41. 1 - C
    2 - A
    3 - B
    4 - D

\[ Z = 37 \text{ min.} \]

42.(a) 1 - B
    2 - D
    3 - C
    4 - A

\[ Z = $32 \]

(b) Minimize \[ Z = 12x_{1A} + 11x_{1B} + 8x_{1C} + 14x_{1D} \]
    + 10x_{2A} + 9x_{2B} + 10x_{2C} + 8x_{2D} \]
    + 14x_{3A} + 100x_{3B} + 7x_{3C} + 1lx_{3D} \]
    + 6x_{4A} + 8x_{4B} + 10x_{4C} + 9x_{4D} \]

subject to
\[ x_{1A} + x_{1B} + x_{1C} + x_{1D} = 1 \]
\[ x_{2A} + x_{2B} + x_{2C} + x_{2D} = 1 \]
\[ x_{3A} + x_{3B} + x_{3C} + x_{3D} = 1 \]
\[ x_{4A} + x_{4B} + x_{4C} + x_{4D} = 1 \]
\[ x_{1A} + x_{2A} + x_{3A} + x_{4A} = 1 \]
\[ x_{1B} + x_{2B} + x_{3B} + x_{4B} = 1 \]
\[ x_{1C} + x_{2C} + x_{3C} + x_{4C} = 1 \]
\[ x_{1D} + x_{2D} + x_{3D} + x_{4D} = 1 \]
\[ x_{ij} \geq 0 \]

43. 1 - B
    2 - D
    3 - A
    4 - C
    5 - E

\[ Z = 51 \text{ days} \]

44. 1 - B \text{ or } 1 - E
    2 - E
    3 - A
    4 - C
    5 - D
    6 - F

\[ Z = $36 \]

45. 1 - C \text{ or } 1 - D
    2 - A
    3 - B
    4 - D
    5 - E

\[ Z = $26 \]

46. 1 - C
    2 - F
    3 - E
    4 - A
    5 - D
    6 - B

\[ Z = 85 \text{ defects} \]

47. A - 3 \text{ or } A - 6
    B - 2
    C - 6
    D - 1
    E - 5
    F - 4

\[ Z = 14 \text{ miles} \]

48. 1, 4 and 7 - Columbia
    2, 6 and 8 - Atlanta
    3, 5 and 9 - Nashville

\[ Z = 985 \text{ (multiple optimal solutions)} \]