EXAM 1: SOLUTION

Read carefully all the questions and do not spend too much time in any single question.
Show all your work to receive credit.

1. A biologist studying the possible effects of pollution on the health of frogs of a certain kind collected ten frogs near a lake and measured their weights (in grams). The collected data were:
15.2 14.1 13.7 15.2 15.0 14.5 13.8 16.5 13.2 14.0

Compute:
(a) Sample mean 14.52
(b) Sample median 14.3
(c) Sample standard deviation 0.97
(d) Interquartile range 1.4

2. Consider the histograms in Figure 1.
(a) For each of these histograms decide whether the sample mean \( \bar{y} \) is larger, smaller or about the same when compared to the sample median \( \tilde{y} \)
(A) \( \bar{y} < \tilde{y} \)  (B) \( \bar{y} \approx \tilde{y} \)  (C) \( \bar{y} > \tilde{y} \)  (D) \( \bar{y} > \tilde{y} \)
(b) For histogram (A), give a reasonable guess for the sample median 1.25
(c) For histogram (C), give a reasonable guess for the number of observations between 0.8 and 1.0 22

Figure 1:
3. A random sample of size 36 is taken from a population with mean 120 and standard deviation 39. The mean and standard deviation of the sample mean (i.e. \( \bar{X} \)) are, respectively: **120 and 6.5** (1 point)

4. From a sample of 200 items, 12 items are defective. The point estimate of the population proportion of defectives is: **0.06** (1 point)

5. When the population standard deviation \( \sigma = 8 \), the confidence interval for the population mean \( \mu \) is 40.52 ± 3.24. Had \( \sigma \) equaled 16, the confidence interval for \( \mu \) would be: **40.52 ± 6.48** (1 point)

6. Determine whether the following statements are true or false.
   (a) All binomial distributions can be approximated closely by normal distributions. **false** (1 point)
   (b) The spread of the distribution of the sample mean is less than the spread of the sampled population. **true** (1 point)

7. The breakdown voltage of a randomly chosen diode of a certain type is known to be normally distributed with mean 40 V and standard deviation 1.5 V.
   (a) What is the probability that the voltage of a single diode is between 39 and 42 ? (2 points)
   (b) What value is such that 15% of all diodes have voltages exceeding that value ? (2 points)
   (c) If a random sample of 49 such diodes is selected, what is the probability that the sample mean exceeds 40.5 V ? (2 points)

   **Answer.**
   (a) \[ P(39 < X < 42) = P\left(\frac{39-40}{1.5} < Z < \frac{42-40}{1.5}\right) = \Phi(1.33) - \Phi(-0.67) = 0.9082 - 0.2514 = 0.6568. \]
   (b) 0.15 = \( P(X > a) = P(Z > \frac{a-40}{1.5}) \), so \( \Phi\left(\frac{a-40}{1.5}\right) = 0.85 \). Then, \( \frac{a-40}{1.5} = 1.04 \) and \( a = 41.56. \)
   (c) \[ P(\bar{X} > 40.5) = P\left(Z > \frac{40.5-40}{1.5/\sqrt{49}}\right) = 1 - \Phi(2.33) = 1 - 0.9901 = 0.0099. \]

8. Each of the following is a confidence interval for \( \mu \), the population mean: (114.4, 115.6) , (114.1, 115.9).
   (a) What is the value of the sample mean ? (1 point)
   (b) Both intervals were calculated from the same data. The confidence level of one of the intervals is 90% and for the other is 99%. Which of the intervals has the 90% confidence level ? (1 point)

   **Answer.**
   (a) 115 (the midpoint of the interval)
   (b) (114.4, 115.6) is the one with 90% confidence level because it is narrower than the other.

9. A study conducted in a hospital involved monitoring a random sample of 75 patients. For this sample, it took an average of 3 cc of tranquilizer to put a patient to sleep before surgery, with sample standard deviation of 0.2 cc.
   (a) Calculate a 95% confidence interval for the true mean amount of tranquilizer to put a patient to sleep (\( \mu \)). (2 points)
   (b) Suppose that you had believed a priori that the population standard deviation was about 0.4 cc. Based on this supposition, how many patients would be required in a random sample to obtain a 95% confidence interval for \( \mu \) with a width of 0.01 ? (2 points)

   **Answer.**
   (a) \[ \bar{x} \pm \frac{2 \cdot 0.675 \cdot 3}{\sqrt{75}} = 3 \pm \frac{(1.96)(0.2)}{\sqrt{75}} = (2.95, 3.05). \]
   (b) \[ (2(1.96)(0.4)/0.01)^2 = 24586.24, \text{ so } n = 24587. \]