Optimal Forest Rotation When Stumpage Prices Follow a Diffusion Process

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ABSTRACT. *A key assumption in the Faustmann rule for financial maturity is that stumpage prices are constant over time. Timber price series, however, exhibit wide fluctuations over time which this paper models as a lognormal diffusion process. Comparing the diffusion results modeled here to the fixed-price Faustmann results show: (1) the prescribed rotation length is generally longer; (2) computed stand values are higher using the diffusion model with the greatest divergence occurring when a stand is about the midpoint of a rotation; (3) as the stumpage price volatility increases, the gain in computed NPV increases, though in a nonlinear fashion.*

I. INTRODUCTION

Classic questions in forest management are when to harvest a stand that is currently immature, and whether to regenerate a new timber stand after harvest. Faustmann (1849) presented what is now widely accepted as the correct methodology for determining timber stand financial maturity when stumpage prices, forest management costs, and timber growth remain constant over time. Forest mensuration studies have determined forest growth over time. It seems reasonable to assume that forest management costs do not vary much over time. Stumpage prices, however, exhibit large variation from year to year; thus, an important consideration in studying financial maturity is how managers should adapt their timber harvests in response to changing stumpage prices. Recent articles in this journal have addressed the non-constant stumpage price consideration. McConnell, Daberkow, and Hardie (1983) note that both prices and forest management costs fluctuate which leads to changes in optimal rotation lengths and possibly the conversion of timber stands into agricultural uses. Newman, Gilbert, and Hyde (1985) find that exponentially increasing prices decrease rotation length until an equilibrium rotation length is achieved. Sandhu and Phillips (1991) add that demand induced price changes may occur at the beginning of a forest rotation resulting in variable rotation lengths. These papers assume that timber price changes are deterministic. Timber price series, however, exhibit continual stochastic changes rather than discrete predictable changes; thus, a more appropriate model should recognize this stochastic nature. Some recent work assumes that stumpage prices can be modeled as annual independent draws from a known distribution (Brazee and Mendelsohn 1988; Lohmander 1988; Haight 1990, 1991). Washburn and Binkley (1990a), however, note that the independent draw assumption is inconsistent with informationally efficient stumpage markets and use time series tests to reject the validity of such models. Teeter and Caulfield (1991) construct a Markov model of stumpage prices assumed to follow an autoregressive process (AR(1)) to assess optimal thinning regimes. In their study the rotation age is fixed but they note their technique could be used to study the optimal rotation age. This paper assumes that stumpage prices follow a lognormal diffusion process and that trees grow according to well-established yield tables or growth models. The numerical solutions are computed using the familiar technique of dynamic programming.

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The typically maintained assumption in financial economics is that stock prices follow a lognormal diffusion, a process popularized as the underpinning of the Black and Scholes (1973) option pricing formula. Such a process is consistent with both informationally efficient markets and fluctuating prices. This paper models the variable stumpage prices using a discrete form of a diffusion process. The stock price diffusion process underlying the Black-Scholes model is:

$$dP = \mu P dt + \sigma P dz \quad [1]$$

where:

P = stock price,
t = time,
μ = drift rate in stock price,
σ = volatility of stock prices (instantaneous standard deviation),
dz = the increment of a Wiener process (or Brownian motion).[1]

When this diffusion process is melded with several other financial market assumptions, Black and Scholes (1973) solve the closed form solution to the option pricing problem. If the forestry problem could be put in a similar framework, the rotation length problem could be solved using this closed form solution. Thomson (1991) shows that the forest rotation problem cannot be solved using the Black-Scholes solution because not all of the requisite inputs are available for a corresponding timber valuation problem.

Morck, Schwartz, and Stangeland (1989), in an extension to the natural resource valuation approach of Brennan and Schwartz (1985), apply diffusion models to valuing a mature forest resource that must be harvested during the next ten years. They assume that stumpage prices follow equation [1] and that growth in inventory follows Brownian motion with drift. They derive the resulting nonlinear partial differential equation that they solve numerically to value the forest stand. Their solution values the optimal depletion path for a mature forest given that it must be harvested during the terms of the lease or abandoned. They note, "It also would be of interest to extend the problem over a much longer time horizon in order to consider issues such as optimal reforestation policies." Their model is not appropriate for such extensions as the long time-horizon problem is not easily handled using their numerical evaluation technique, and more important, tree growth does not follow Brownian motion with drift.

Clarke and Reed (1989) and Reed and Clarke (1990) make a more realistic stochastic growth assumption by making the stochastic growth a function of age or size (rather than simply a Brownian motion with constant drift) and use a lognormal diffusion for stumpage prices. They do not consider any costs of forest management; hence, they show that the Faustmann results are appropriate.

II. THE BINOMIAL OPTION PRICING MODEL

When the financial market assumptions required for the Black-Scholes solution are not present, it is common to use numerical methods to solve option valuation problems. Cox, Ross, and Rubinstein (1979) and Rendleman and Bartter (1979) independently developed a two-state option pricing model, that is commonly called the binomial option pricing model. It is pedagogically instructive and is a popular tool for the numerical analysis of option values. In the two-state option pricing model, the continuous time stochastic process described in equation [1] is replaced with a discrete state, up or down price movement. The binomial option pricing model is described as follows. Suppose the stock price starts at P. In the next period it will either rise to $u * P$ (up state) or fall to $d * P$ (down state).

[1]The discrete version of the model, $dP = \sigma dz$, is commonly called the random walk model. When a drift term of μdt is added it is called a random walk with drift. Dividing both sides of equation [1] by P shows that the change in price divided by the current price follows a random walk with drift. It is also called geometric Brownian motion.

The probability of an up move is π, thus, the probability of a down move is $1 - \pi$. The following model parameters will converge to equation [1] as $\Delta t \rightarrow 0$:[2]

$$u = \exp(\sigma \sqrt{\Delta t}) \qquad [2]$$

$$d = \frac{1}{u} \qquad [3]$$

$$\pi = \frac{\exp(\mu \Delta t) - d}{u - d}. \qquad [4]$$

From this basic structure, a tree can be constructed which shows a time path of prices and their probabilities.

The standard discounted value maximization criterion asks whether the value from immediate harvest exceeds the value of the stand discounted one period, and thus computes the value of a timber stand as:

$$V[P, Q] = \mathrm{Max}\left\{P_{t,i}Q_{t,j} + V[P_{t,i}, Q_{t,0}],\ \frac{V[P_{t+1,i}, Q_{t+1,j+1}] - C}{1 + r}\right\} \qquad [5]$$

where:

$V[P, Q]$ = the value of the forest stand which depends on the stumpage price and volume.

t = calendar time in periods, $t = 0, 1, 2, 3, \ldots$

j = age of the timber stand in periods, $j = 0, 1, 2, 3, \ldots$

$P_{t,i}$ = stumpage price i realized at time t.

$Q_{t,j}$ = stumpage volume in period t for a stand of age j.

$V[P_{t,i}, Q_{t,0}]$ = the land value (LV) in period t for realized stumpage price i, prior to establishing a tree crop (i.e., $Q_{t,0} = 0$). When forest management will be practised this is the discounted value of the costs and revenues from managing timber stands ad infinitum. Alternatively, the LV may be what someone would pay for the bare land to employ it in some other use.

$V[P_{t+1,i}, Q_{t+1,j+1}]$ = the value of the timber stand one period from now. This may be either its harvest plus land value in $t + 1$ or its discounted future value.

C = per period forest management costs.

r = appropriate one-period discount rate.

The term $P_{t,i}Q_{t,j}$ is the harvest value of the standing inventory at time t for a timber stand that is age j, and $V[P_{t,i}, Q_{t,0}]$ is the value of bare land after the harvest. When added together they comprise the total stand value received if the stand is harvested today. This value is compared to the discounted value that will be received one period from now if harvest is delayed. To hold the timber stand another period requires the expenditure of a cost for land management, protection, and property taxes which must be subtracted from the next period value. If stumpage prices, management costs, and growth rates are constant over time, solving equation [5] will provide the Faustmann results, and $V[P_{t,i}, Q_{t,0}]$ is the Faustmann land expectation value (LEV).

If the stumpage price is stochastic, one can only exactly determine the current harvest value of the stand as today's stumpage price and volume is known. $V[P_{t,i}, Q_{t,0}]$ and $V[P_{t+1,i}, Q_{t+1,j+1}]$ depend on the realization of uncertain prices through time. At any point in time t, the stochastic process can be described by two states, the current stumpage price ($P_{t,i}$) and the current stand volume ($Q_{t,j}$). The binomial option pricing model specifies the price process; thus, the current harvest value and expected future values can be explicitly determined at discrete nodes as long as the growth of stands is known. This paper models the two states separately at each stage using a dynamic programming approach proposed by Amin (1991). The price state follows the binomial options pricing model and the growth state

[2] The model parameters described below apply for convergence to the Black-Scholes lognormal price diffusion process of equation [1]. Nelson and Ramaswamy (1990) show that the two-state modeling approach can also be used for alternative diffusion processes by choosing an appropriate set of model parameters.

follows a typical timber yield function. This approach endogenously computes the LV for each price state which is then used in making both the harvest and regeneration decision.

In this dynamic programming formulation, one evaluates the harvest decision for the current node by noting that in the next period the stumpage price will proceed to one of two nodes (up state or down state) and the trees will grow by a one-period increment. For each of the two next period price nodes (for a given volume) the decision maker must evaluate whether the stand is financially mature (in which case it will be harvested) or whether to let the tree growth continue. The probability of reaching each of the two next period price nodes is π and $1 - \pi$. The probability of reaching the next period volume node is 1.0, for at the decision maker's discretion the stand either grows for one year or is harvested. The value of reaching either of the next period nodes will be the greater of the harvest plus land value for that node, or the discounted value of all future harvests at that node. The next period managerial action (harvest or wait) with the highest value will be chosen for that period and its value discounted one period to determine if the present managerial action of harvest exceeds the value of waiting. When the value of immediate harvest exceeds the discounted value of waiting, the stand should be harvested, and a new stand established if the LV at that price node indicates forest management is the best land use. If prices are too low to justify the establishment of a new stand, then the forestry enterprise will be abandoned. For risk neutral valuation, the following equation states the appropriate recursive financial maturity condition at any price and quantity node in the binomial tree:

$$V[P,Q] = \mathrm{Max}\left\{P_{t,i}Q_{t,j} + V[P_{t,i}, Q_{t,0}], \frac{\pi * V[u * P_{t,i}, Q_{t+1,j+1}] + (1-\pi) * V[d * P_{t,i}, Q_{t+1,j+1}] - C}{1+r}\right\} \quad [6]$$

where $V[u * P_{t,i}, Q_{t+1,j+1}]$ is the value of a stand one year from now given that the up state has occurred. This value can be expressed as:

$\mathrm{Max}\{Harvest_u, Wait_u\}$

where:

$Harvest_u = u * P_{t,i}Q_{t+1,j+1} + V[u * P_{t,i}, Q_{t+1,0}]$ which is the harvest plus land value after an up state if the decision to harvest is chosen. (Note that the price the next period equals $u * P_{t,i}$ if the up state occurs. The j subscript on Q is increased by 1 as the stand volume has increased by one period of growth.)

$Wait_u$ = stand value in the next period if an up state occurs but the decision to not harvest is chosen. It is the discounted expected value of the stand one more period in the future ($t + 2$).

$V[d * P_{t,i}, Q_{t+1,j+1}]$ = the value of a stand one year from now given that the down state has occurred. Its value is computed in a manner analogous to that for the up state.

Other variables are as noted earlier. If at any node, the value of the stand under the harvest decision exceeds the value of letting the growth continue, the stand should be harvested and a new stand regenerated if its land value is positive. If not, the forestry enterprise should be abandoned at this time.

The LV's used in the model are computed as follows. If the stand is harvested this period, it will be replaced by a one-period-old stand the next period unless the land is converted to another use or abandoned. If the value of having a one-period-old stand in the next period exceeds its regeneration costs plus one period of management costs, the timber LV is the value of a one-period-old forest minus the management costs discounted one period, less the immediate regeneration cost. If this computation yields a lower value than that of an alternative use, then the value of the alternative use is the LV which signals a conversion from forestry. If no other use is possible for this forest stand the alternative use value is zero so choosing the alternative land use in this case means abandoning the forestry enterprise and the land. The computed results, therefore, include the option value of abandoning the land or converting to an alternate use if the stumpage price is too low to justify sustainable forestry.[3] Formally the LV is computed as:

$$V[P_{t,i}, Q_{t,0}] = \text{Max}\left\{\frac{\pi * V[u * P_{t,i}, Q_{t+1,1}] + (1-\pi) * V[d * P_{t,i}, Q_{t+1,1}] - C}{1+r} - R, Alt\right\} \quad [7]$$

where R is the regeneration cost and Alt is the value of the land if converted to an alternative use. $Alt = 0$ if no alternative land use has a positive value.

Equations [6] and [7] show that the LV and the expected value of delaying harvest depend on the next period values; thus, future periods must first be evaluated. Consequently, the dynamic nature of the problem is appropriately solved by backward recursion. Furthermore, the problem is an infinite horizon problem. To solve we replace the infinite horizon problem with a finite horizon problem so that $t = 1, 2, 3, \ldots T$, where T is the final stage of the problem. Because no value will be gained by waiting an additional period beyond T, the optimal decision at time T is to harvest all stands, regardless of price and age and to sell the land for the alternative use value. For the results presented below, time horizons were chosen to ensure that increasing T did not affect the solution by more than \$0.005/acre. The recursion proceeds backwards from stage T and evaluates at each price and volume node whether the current harvest value of a stand plus the LV exceeds the discounted expected value of the next period's best decision and thus the recursion continues until the present.

The boundary condition is the value of the forest in the final stage, that is, the product of the stumpage price and timber volume, plus the value of the alternative use, for each price and volume node:

$$V[P_{T,i}Q_{T,j}] = P_{T,i}Q_{T,j} + Alt \qquad i = 1, 3, 5, \ldots, I \quad j = 1, 2, 3, \ldots, J \quad [8]$$

where I is the total number of price nodes (equals $2T - 1$) and J is the maximum age stand (in periods) that will be allowed in the model. J should be chosen such that the computed results are invariant with J unless there is some alternate consideration such as maximum tree size that can be processed which forces an upper bound on J.

The backward recursion proceeds as follows. Equation [8] is used to compute the harvest values for all P_i and Q_j at $t = T$. Then equation [6] is evaluated for all P_i and Q_j values at stage $t = T - 1$. The choice of the highest valued strategy at this stage embeds the optimal decisions of harvest or wait, and, if the harvest option is chosen, regenerate or convert. If the stand is currently age J, then the decision to harvest must be chosen. The recursion continues via equation [6] until $t = 0$. At $t = 0$ there is only one price node (the starting price) and J quantity nodes. Each node has a value, $V[P_0, Q_j]$, which is the net present value (NPV) of the timber plus land for the current price computed using the binomial diffusion approximation. The same dynamic algorithm is used to compute the Faustmann values except that the stumpage price remains fixed over time. The differences in computed NPV between the two

[3]The conversion results embedded in this modeling approach are very different than that taken by Zinkhan (1991). Zinkhan assumes that the timber rotation length is given, but that the alternative use value is uncertain. He assumes conversion can take place only on a specified date. This paper assumes the conversion value is known, but the timber values are stochastic and conversion can take place at any date depending on the timber price realized at that date.

models are thus due to the volatile price assumption.

A FORTRAN program was written to solve the model described in equations [6]–[8]. The model has been solved using IBM compatible microcomputers running MS-DOS. The problem size (i.e., number of time periods and number points on the volume function) is limited by the memory available using DOS; thus, the program truncates the binomial price process after 125 stages. Extensive testing has demonstrated that the numerical solutions in the presented results remain accurate to $0.01/acre. The model computes the $t = 0$ values, $V[P_0, Q_j]$, which are the NPV's per acre for a stand of age j, given the current price is P_0, and notes if the stand is financially mature (i.e., harvest value exceeds wait value) given the current stumpage price and age. The dynamic program also computes Faustmann harvest age and NPV's. The difference in NPV between the diffusion model and the Faustmann model is computed and recorded.

An important managerial choice is the timing of the harvest. The Faustmann approach prescribes a single harvest age for a fixed price scenario. The binomial diffusion model evaluates the harvest decision based on the node that is attained rather than stand age that has been reached. This allows managers to be flexible in their choices of both when to harvest the existing stand and whether to establish another stand. For the current stumpage price, the model notes the age at which financial maturity is reached. If the stand age is greater than the age of maturity, it should be harvested; otherwise, it should be left to grow. One period from now the model would be rerun with the newly observed stumpage price and an updated evaluation of financial maturity made. This process would continue until it was optimal to harvest the stand. At that point a second managerial decision would be made, that of whether to continue the forestry enterprise by regenerating a new stand. If the timber LV for the stumpage price realized at that date is greater than the land value in an alternative use, then regeneration should take place. Otherwise, the forestry enterprise should be converted to the alternative use (or abandoned if the alternative use is abandonment). The two managerial decisions, when to harvest the current stand and whether to regenerate a new stand, are thus explicitly determined using the model.

An increase in stand NPV is typically computed by the diffusion model. This increase is the value of flexibility in decision making as the Faustmann computed value is correct when no flexibility in decision making is allowed. In other words, this increase in value is the value of the option to modify one's harvest and regeneration decisions as the stumpage price path unfolds. Two options are thus recognized; the option to vary the harvest age and the option to abandon forestry. The model described above solves the value of the combined options. By modifying equation [7] to choose the discounted value of a one-year-old stand even when it has negative value, one disallows the abandonment option; thus, the option value computed is the value of harvest age flexibility. The diffusion and Faustmann models will compute the same NPV only when there is no flexibility in the managerial decision.[4] This occurs for mature stands when stumpage price is so low that it is optimal (by either model) to harvest the current stand and then abandon forestry.

III. EMPIRICAL TRENDS IN HISTORICAL DOUGLAS FIR STUMPAGE PRICES

The diffusion model requires as an input the drift and volatility of prices. National Forest timber sale stumpage price data was compiled for the period 1926–86. Data for 1926–57 are from Potter and Christie (1962), for 1950–79 from USDA Forest Ser-

[4]Because the Clarke and Reed (1989) model does not include costs, their optimal rotation age is the Faustmann age; thus, no option values are present and their computed values are the same as the Faustmann values (for the particular stumpage price under consideration). If a 0 regeneration and management cost is used in the dynamic programming model of this paper, the Faustmann results are obtained.

vice (1982), and for 1959–86 from an annual series of stumpage prices compiled by the USDA Forest Service. It was confirmed they represent a single series as the overlapping values are the same among sources. These prices were deflated to a constant base year and maximum likelihood estimates of the drift and volatility were computed.[5] The deflated prices were then detrended by the computed drift rate and the logarithms of these deflated, detrended prices were computed and subjected to a Lilliefors test which could not reject a null hypothesis of normality at the usual levels of statistical significance (p value = 0.668). Thus, a hypothesis of lognormality of stumpage prices cannot be rejected so use of the price generating process of equation [1] appears reasonable. Annual volatility (σ) was estimated at 0.338.

IV. AN EXAMPLE

The dynamic programming model requires the following parameters: beginning stumpage price, timber yield over time, per period forest management costs, risk-free discount rate, the value of the alternative land use, stumpage price volatility (σ), drift (μ), and step size (Δt). This analysis is done in real terms thus costs are assumed constant over time and stumpage prices are real prices over time. Consider a Site II Douglas-fir stand that will be harvested when financially mature. The initial stumpage price is \$120/1,000 board feet (Mbf) with $\mu = 0$, $\sigma = 0.30$, $\Delta t = 1$ year,[6] $C =$ \$4/acre/year, $R =$ \$250/acre, and $Alt = 0$. Yield was simulated using the SPS model (Arney 1985) and the simulated results were then fitted using nonlinear regression to the functional form of Payandeh (1973) which provided the yield equation:

$$Vol_a = \exp(12.05 - 68.88/a) \qquad [9]$$

where Vol_a is timber yield in Mbf/acre at age a.

Table 1, Panel A, shows a table of NPV's computed using the diffusion model for several current stumpage prices and for several of the stand ages that were evaluated. Also noted is the age when the stand becomes financially mature for that stumpage price. In other words, an asterisk indicates that for a stand of the specified age and stumpage price, the stand value is higher if one harvests immediately rather than waiting. Following in Panel B is the gain in NPV, that is, the NPV computed using the diffusion model minus the NPV computed by the Faustmann model. Also noted in Panel B is the age at which the stand becomes Faustmann mature for the given stumpage price.

Harvesting the stand at a relatively young age may be optimal for one of two reasons. Either the price is so high that the opportunity cost of forgoing the next rotation is high or the price is so low that it makes sense to abandon the forestry enterprise early rather than to continue investing the annual management fees. Figure 1 shows this relationship by plotting the optimal rotation age as a function of the stumpage price observed at that future time.[7] Both the Faustmann and diffusion models prescribe a longer rotation age at a rela-

[5] Reed and Clarke (1990) note that the ML estimates for the drift and volatility are the mean and standard deviation of the time series $\ln(P_{t+1}/P_t)$. Washburn and Binkley (1990b) show that the true volatility will be underestimated using this approach as the historical stumpage prices series are period average prices.

[6] To get a more precise evaluation of the diffusion process requires using a smaller Δt. The model was rerun using $\Delta t = 0.5$ years and $\Delta t = 0.25$ years. The values reported below typically increase by \$0.22 and \$0.25/acre, respectively; thus, it appears reasonable to report results only for $\Delta t = 1$ year.

[7] In the stochastic process literature such a figure is often called the optimal stopping boundary. The results shown in Figure 1 were developed with $\Delta t = 0.5$ year to more accurately represent the age of financial maturity. The figure shows the age and price states where harvest is the optimal decision. It does not show, for example, that with today's stumpage price of \$120, a stand planted today, should be harvested at age 39. The optimal harvest age for a stand planted today will depend on the price path that occurs over time, i.e., when the diffusion path hits the stopping boundary shown. This figure does show, however, if you have a 39-year-old (or older) stand today and the current price is \$120, you should harvest it. If you have a 38-year-old (or younger) stand, you should wait. Whether you will harvest the 38-year-old stand one year from now will depend on the price next period.

TABLE 1

DIFFUSION MODEL NPV AND INCREASE IN NPV AS A FUNCTION OF INITIAL STUMPAGE PRICE AND STAND AGE

	Current Stumpage Price ($/Mbf)					
Age	240.00	180.00	120.00	60.00	45.00	30.00
Panel A: Diffusion Model NPV ($/ac) by Stumpage Price and Stand Age						
LV	1,571.56	1,088.44	607.50	131.98	15.17	0.00
1	1,897.56	1,395.04	894.98	400.41	278.94	159.72
2	1,976.46	1,453.94	933.81	419.58	293.22	169.17
37	7,953.72*	5,877.13	3,812.51	1,789.24	1,304.61	838.18
38	8,274.17*	6,115.39*	3,960.17	1,856.03	1,352.35	869.90
39	8,593.04*	6,354.55*	4,118.24*	1,923.27	1,401.71	902.78
40	8,910.01*	6,592.28*	4,276.73*	1,993.13	1,451.50	935.62
41	9,224.79*	6,828.36*	4,434.12*	2,064.13	1,502.09	970.05
42	9,537.13*	7,062.62*	4,590.29*	2,134.74	1,553.12	1,004.51
43	9,846.80*	7,294.87*	4,745.12*	2,205.95	1,604.65	1,039.28
44	10,153.62*	7,524.98*	4,898.53*	2,277.50*	1,657.42	1,074.07
45	10,457.42*	7,752.84*	5,050.43*	2,353.45*	1,709.87	1,110.73*
55	13,308.78*	9,891.35*	6,476.11*	3,066.29*	2,216.48	1,467.15*
56	13,574.22*	10,090.44*	6,608.83*	3,132.65*	2,265.67*	1,500.33*
57	13,836.04*	10,286.80*	6,739.74*	3,198.10*	2,314.76*	1,533.06*
Panel B: Increase in NPV ($/ac) by Stumpage Price and Stand Age						
LEV	45.15	49.31	59.90	65.09	57.75	0.00
1	46.10	50.35	57.05	66.84	59.21	39.35
2	46.94	51.46	58.37	68.67	60.71	39.99
37	45.05	48.21	63.32	109.91	92.48	33.80
38	45.15†	49.31†	57.01†	105.53	87.75	29.34
39	45.15†	49.31†	55.65†	98.74	82.52	24.60
40	45.15†	49.31†	55.65†	91.63†	75.54†	18.31†
41	45.15†	49.31†	55.65†	83.93†	67.10†	13.40†
42	45.15†	49.31†	55.65†	76.45†	59.58†	8.81†
43	45.15†	49.31†	55.65†	70.25†	53.04†	4.88†
44	45.15†	49.31†	55.65†	65.09†	48.28†	1.31†
45	45.15†	49.31†	55.65†	65.09†	43.77†	.00†
55	45.15†	49.31†	55.65†	65.09†	15.76†	.00†
56	45.15†	49.31†	55.65†	65.09†	15.17†	.00†
57	45.15†	49.31†	55.65†	65.09†	15.17†	.00†

Note: Model parameters: Drift = 0.000, Regeneration Cost = $250/ac, Risk Free Interest Rate = 0.04, Volatility = 0.30, Value of Alternative Use = 0, Annual Costs = $4/ac/yr, Step Size = 1 year.

*Indicates stand is financially mature.

†Indicates stand is Faustmann mature.

tively moderate price. For very low prices, the Faustmann rotation is somewhat shorter than the diffusion model rotation. For stumpage prices above $130/Mbf, the rotation length prescribed by either model is about the same.[8] A large difference in prescribed rotation length occurs if stumpage prices are about $45/Mbf. In this range, the rotation length prescribed by the diffusion approach is 65 years whereas the Faustmann rotation is 40. At first inclination this result suggests that the optimal harvest age may be quite different between the models. The probability of attaining a 65-year harvest, however, is near zero as

[8] As noted earlier, Reed and Clarke (1990) show the diffusion model rotation is the same as the Faustmann model when there are no costs; a result also noted in Newman et al. (1985). Figure 1 shows that when timber prices are high enough, costs do not significantly affect the harvest decision.

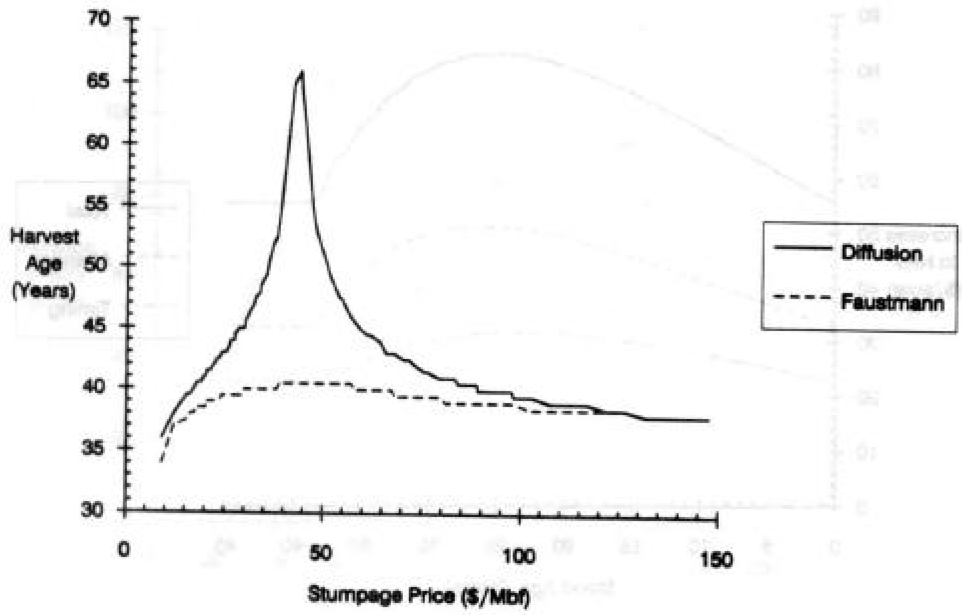

FIGURE 1
HARVEST AGE AS A FUNCTION OF STUMPAGE PRICE FOR THE DIFFUSION AND FAUSTMANN MODELS

prices are volatile. Time appears on the vertical axis, so one can think of a growing tree as proceeding vertically on the figure. The price keeps changing as the stand grows. When this diffusion hits the boundary (a price and age point) shown on Figure 1, the stand should be harvested. It is very unlikely with a stochastic stumpage price diffusion to not hit the boundary before age 65, thus it is likely that the realized rotation age will not vary greatly between models.[9] The diffusion model specifies growing the stand somewhat longer unless the stumpage price is greater than $130/Mbf near the time of financial maturity. The expected rotation age, therefore, will be somewhat longer.

The difference in NPV between the two models, which measures the value of managerial flexibility, varies by stand age (Figure 2). For a financially mature stand, the gain in NPV is the gain in LV as the current stand should be harvested and replaced with a new stand. The greatest divergence in value is for a stand near the middle of a rotation. This gain in value is decomposed into its two components—the value of harvest age flexibility and the value of abandonment of forestry if stumpage price is too low when the stand is harvested. The lower line on Figure 2 is the value of rotation age flexibility and the line above it is the value of forestry abandonment if stumpage prices are too low after harvest.

It is well known that the value of an option increases as the price volatility of the underlying security increases. Of interest to forest decision makers is the magnitude of the change in the timber stand NPV as the stumpage price volatility changes. The base case model was run with a series of stumpage price volatilities. Figure 3 demonstrates, for bare land and for a 20-year-old stand, the increase in NPV as the volatility of stumpage prices increases. This result is nonlinear. For a small stumpage price volatility, there is a small increase in NPV but the rate of increase in NPV becomes more pronounced as a volatility of 0.2 is achieved. Beyond a volatility of 0.6 the rate at which NPV increases fails. Because the

[9]A stochastic simulation of a discrete form of equation [1] using monthly price changes showed that for a starting price of $40, the probability of the price remaining in the interval [30, 50] over a five-year (60-month) period is 0.00328.

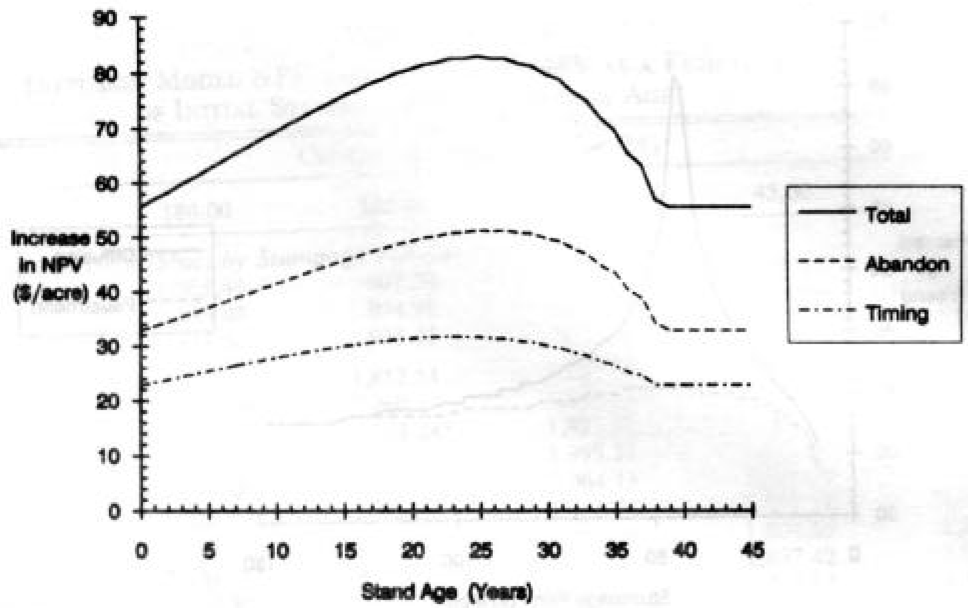

FIGURE 2

TOTAL INCREASE IN NPV, INCREASE DUE TO THE FORESTRY ABANDONMENT OPTION, AND INCREASE DUE TO THE HARVEST AGE TIMING OPTION AS A FUNCTION OF STAND AGE

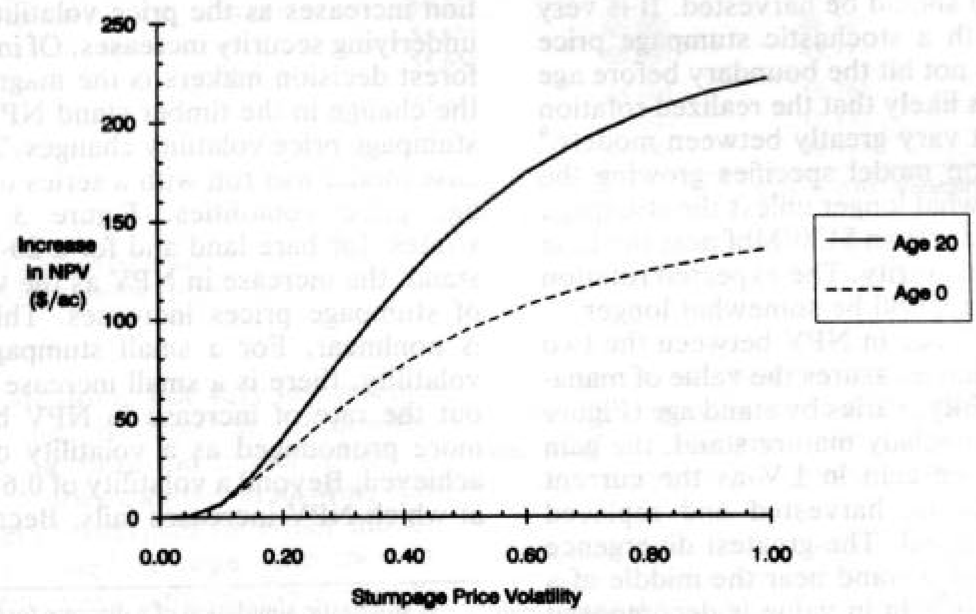

FIGURE 3

INCREASE IN NPV COMPUTED USING THE DIFFUSION MODEL VERSUS THE FAUSTMANN MODEL AS A FUNCTION OF STUMPAGE PRICE VOLATILITY

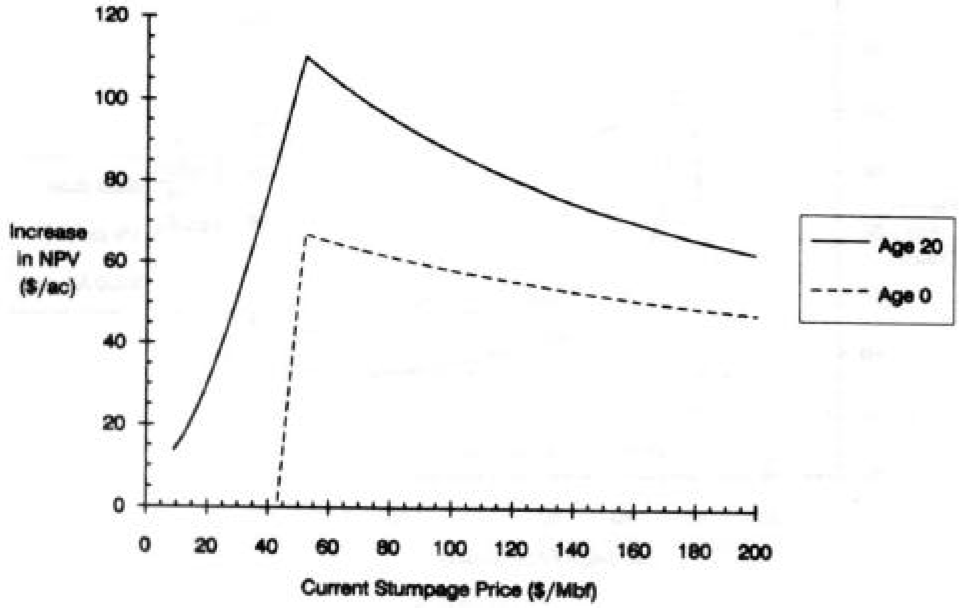

FIGURE 4
INCREASE IN NPV COMPUTED USING THE DIFFUSION MODEL VERSUS THE FAUSTMANN MODEL AS A FUNCTION OF CURRENT STUMPAGE PRICE

historically estimated volatility is about 0.3, the gain in NPV near this range seems most relevant.

The diffusion model allows the evaluation of both how the current stumpage price affects stand value, and how it affects the computed difference in stand value between the Faustmann and the diffusion models. Such a comparison can be made for any age stand, but for illustration will be made for a stand of age zero (i.e., the difference in LV) and age 20 (near mid-rotation). A series of model runs was made at various initial prices with $\sigma = 0.30$ and the difference in value between the diffusion and Faustmann solutions are plotted on Figure 4. At very low stumpage prices, there is no difference in LV as it is optimal in either model to abandon forestry; thus, the computed LV is zero. As the stumpage price rises, a price region begins where the LV computed using the diffusion model is positive, but Faustmann LEV is zero. This disparity occurs because the diffusion model evaluates price states where forestry is a good investment, in addition to states where forestry is a poor investment. The Faustmann approach, however, evaluates a constant price state where forestry is a poor investment. The stumpage price for which the Faustmann model is indifferent between continued forestry and abandonment (i.e., stumpage price = $51.44 and Faustmann LEV = $0.01/acre) is where the difference in computed LV between the two models is the highest (illustrated by the peak shown in Figure 4). For stumpage prices above this level, the gain observed using the diffusion approach declines. The results for an age 20 stand follow a similar trend, that is, the difference in value peaks at the stumpage price for which the Faustmann calculation shows indifference between continuing the forest enterprise or abandoning it. For stumpage prices below this point, the difference in computed NPV's remains strictly positive. The gain in value for an age 20 stand is always higher.

V. COMPARISONS TO OTHER STUDIES

Newman et al. (1985) show that when timber prices are increasing exponentially in a deterministic fashion, the rotation

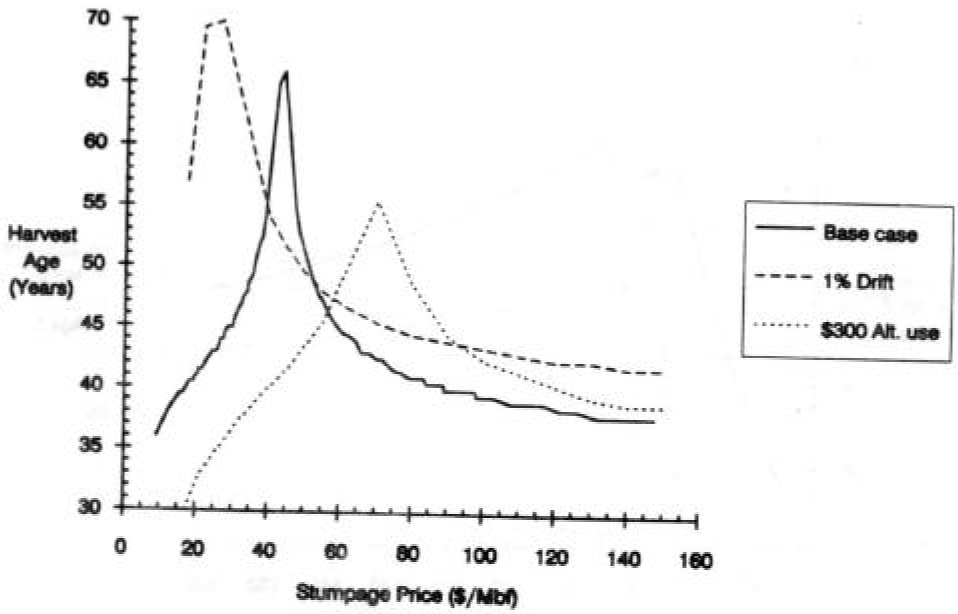

FIGURE 5
HARVEST AGE AS A FUNCTION OF STUMPAGE PRICE FOR THE DIFFUSION MODEL USING THE BASE CASE ASSUMPTIONS, A 1% PER ANNUM STOCHASTIC PRICE DRIFT (μ = 0.01), AND AN ALTERNATIVE USE LAND VALUE OF $300/ACRE

length will be longer than when no price increase is present, but it declines over time to the zero cost rotation length. Figure 5 shows the diffusion model optimal harvest boundary with and without the presence of an exponential price trend ($\mu = 0.01$). The price trend shifts the optimal harvest boundary upwards and to the left. The rotation length in general will be longer with the exponential price trend, but there is a region where the rotation length may be shorter than when no price trend is present.

Zinkhan (1991) notes the importance of the timber land-use conversion option. Figure 5 also shows how the optimal harvest boundary changes in response to an alternate land-use value of $300 per acre. For high stumpage prices there is little change (optimal rotation is slightly longer) as the probability of remaining in forest management is high. For low stumpage prices, harvests will take place sooner to convert to the better use earlier. Rotation length may either be longer or shorter when the conversion option exists and will depend on the stumpage prices realized over time. By varying the value of the alternative use one may compute the LV of the forest land with allowing for conversion of land use when the stumpage price is too low relative to the opportunity the alternative use provided. Figure 6 shows that as the value of the alternative use increases, the LV of land currently slated for timber production increases—recognizing the future potential for land-use conversion.

VI. SUMMARY

This paper has shown how the price diffusion model commonly used in financial economics can be applied to the study of financial maturity of timber stands. The diffusion model allows flexibility in response to an evolving stumpage price for two managerial decisions; choice of rotation age and whether to abandon forestry after the current stand is harvested. Stand NPV computed using the diffusion model is generally higher than the Faustmann NPV as it values the future flexibility of forest management decisions. The example showed that rotation lengths are longer except at high prices where they are the same as the Faustmann rotation. At high stumpage prices, forest management costs are rela-

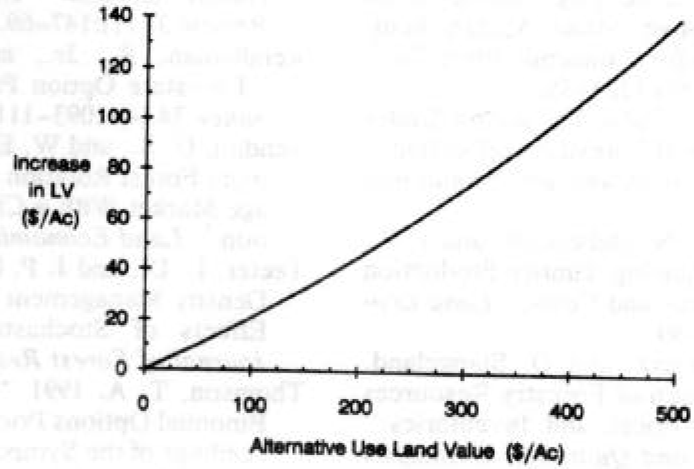

FIGURE 6

INCREASE IN LV AS A FUNCTION OF THE ALTERNATIVE USE LAND VALUE

tively unimportant as forestry is expected to be a good investment; thus, it is not surprising that the chosen rotation is similar to the Faustmann rotation. The increase in computed stand values reaches a maximum for stands in the mid-rotation range. As stumpage price volatility increases, so does the gain from managerial flexibility as the likelihood of both better and poorer price states is increased. In the poorer states, the land can be abandoned (or converted to a higher value use) so there is a smaller loss on the down side than there is gain in the upside resulting in an increasing stand value as volatility increases. The value of managerial flexibility was also shown to be a function of the current stumpage price. The gain in NPV is highest when stumpage prices are so low that the Faustmann LEV is only slightly positive. At this price, the Faustmann model values forestry at a modestly profitable state whereas the diffusion model considers the upside potential of higher prices along with the option to abandon forestry if the downside is realized. When stumpage prices are increasing, rotation lengths will generally be longer though the difference depends on the currently realized stumpage prices. As the value of an alternative land use increases, rotations will be shorter for low price states so that the more profitable alternative land use can be deployed and longer for moderate or high price states.

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