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Point of View

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Long-Term Portfolio Returns from Timber and Financial Assets

Executive Summary. *This study estimates inflation adjusted multiperiod portfolio returns from direct real estate investments in Douglas fir and Southern pine timber stands combined with common stocks, corporate bonds, U.S. Government bonds, and U.S. Government Treasury bills over the fifty-eight-year period 1937–1994. A theoretical timber returns benchmark is created that is highly correlated to historical timber prices. The wealth accumulation one may have realized during the 1937–1994 period from investments in Douglas fir, Southern pine and common stocks is presented. The wealth accumulation is substantial for each of these assets, but the timber assets display both a higher return and higher variability. An empirical risk-return relationship for the financial market investments is developed using a multiperiod portfolio optimization technique. Timber asset returns are then included with security returns as input for the portfolio optimization routine. When the optimization model allowed unrestricted choice among assets, it was common to include timber assets in the portfolio. Timber assets were in some cases the only components of the portfolio. The long-run risk-return results, however, were unfavourable. Employing a strategy of holding a fixed portfolio allocation (%) of timber assets while rebalancing one's holding of financial assets, however, appears favorable. Holding a fixed 10% of a portfolio in timber showed about a 1% higher rate-of-return across portfolios with no increase in risk. Higher timber proportions indicated higher returns, but with higher risk.*

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by Thomas A. Thomson

INTRODUCTION

While forest land covers a larger portion of the United States than agricultural or urban land, their investment potential has not received the same level of scrutiny or interest as other investment opportunities. Norman et al. (1995) summarize numerous studies that show over certain periods of time that real estate has offered higher returns and lower risk than stocks or bonds. In a recent survey of pension funds, Harris et al. (1989) find that timberland investments were present in some pension portfolios, though the number was small. Pension funds that invested in timberland, however, were expecting to hold these investments over a ten-to-fifteen-year period and perhaps to increase their total timberland holdings. Other financial institutions including John Hancock and Wachovia Bank, are increasing their investment in timberlands.[1]

Parallel with similar work in real estate, recent timber investment studies have incorporated the methods of modern portfolio theory (MPT) and the capital asset pricing model (CAPM) into the evaluation of timber assets. A typical finding of studies using the CAPM (cf. Redmond and Cubbage, 1988; Thomson, 1987; Washburn and Binkley, 1990) is that the computed timber *beta* is not statistically significant suggesting that the hypothesis of the timber *beta* equals zero cannot be rejected. Since timber investment *betas* tend to be small or zero, timber investments can be expected to be desirable components of investment portfolios. These studies, however, do not determine the proportions of timber investments that will optimize an investment portfolio.

Several studies (cf. Conroy and Miles, 1989; Mills and Hoover, 1982; Thomson, 1987, 1992b; Zinkhan and Mitchell, 1990) use single-period portfolio models to demonstrate that adding timber assets to a portfolio of assets is a wise decision for at least some investors. This result is not surprising since if an asset with a reasonable rate-of-return and an imperfect correlation is added to a portfolio, it can be expected to push out the single-period efficient frontier. On an ex ante basis, it appears wise to add timber assets to the portfolio in some proportion. If a timber asset has the highest expected rate-of-return it will be the sole asset held by a risk-neutral investor. Thomson (1991) demonstrates that a single timber asset (and some other high variance portfolios) when held in constant proportions over a long period, may have a lower compounded rate-of-return, however, than some less risky portfolios. This result suggests that strict adherence to single-period results may lead to poor investment strategies for long-term investors. Also, a reasonable approach to long-term investing may be to adjust one's portfolio over time in response to actual investment realizations.

Samuelson (1989) notes the importance of portfolio rebalancing, that is, adjusting one's portfolio as information about returns realized to date is available so that one holds the appropriate mix of assets for the next period. If portfolio rebalancing is not done, after several periods the asset proportions in one's portfolio may be quite different than is consistent with the risk tolerance of the investor. When making portfolio decisions, investors typically use past returns to infer the distribution of future realizations. Using a portfolio optimization model, and the historical returns information, a set of portfolio weights are chosen that are expected to maximize the utility of the investor. Associated with this set of portfolio weights are (ex ante) an expected return and standard deviation. After holding this portfolio one period, a realized (ex post) return will be earned on that portfolio that will in general not be that expected ex ante. The asset weights now held will no longer coincide with the desired portfolio weights, and it is appropriate to rebalance the portfolio to the optimal weights. Also, with the new realized return, the investor may wish to update the assessment of expected future returns and thus compute a different set of portfolio weights to hold in the next period.[2] As pointed out by Grauer and Hakansson (1995), such a dynamic update is important if the asset return distributions change over time. The purpose of this study is to examine the historical long-term results that a portfolio rebalancing strategy would provide over the long run, given that one was investing in financial and timber assets.

The portfolio model employed is that presented by Grauer and Hakansson (1982) in their study of the historical returns under active portfolio management for common stocks, corporate bonds, U.S. Government bonds, and U.S. Government Treasury bills. This study adds direct timber investments of Douglas fir and Southern pine to the domain of assets from which portfolio choices are made. Douglas fir and Southern pine are chosen to study the importance of timber assets due to their importance as timber species, and due to the availability of long-term "stumpage" price[3] data for these species. A USDA Forest Service (1958) publication notes that Southern pine and Douglas fir are the timber species with the highest growth in the U.S. The timber harvest of these two species provided over 50% of the total timber harvest in the nation. More recent figures (Waddell et al. 1989) show these species contribute more than 50% of the recent softwood roundwood harvest and make up more than half of the current standing volume of softwood trees.

THE DATA

A standard method for computing security returns, as used by Ibbotson and Sinquefield (1982), is:

$$r_t = \left(\frac{V_t + D_t}{V_{t-1}} \right) - 1, \tag{1}$$

where:

r_t = the rate-of-return earned in period t;
V_t = the value (price) of the security in period t;
V_{t-1} = the value (price) of the security in the previous period, $t-1$; and
D_t = the dividend received in period t.

In period $t-1$ one invests the value of the security, V_{t-1}, and receives after one period the current value of that security V_t plus the value of the dividend, D_t. These values are used in equation

(1) to determine the return over that period. For real estate investments, modifications of this approach are used to create indices of return such as the Frank Russell-NCREIF Property Index. The basic approach of this Index is to collect data from real estate investors, and to aggregate this data into an index. The advantage of the approach is that it is based on specific building data. A disadvantage is that a given investor may have an experience that is quite different from the index, perhaps because the real estate is somewhat unique. Vacancies will also have an important effect on individual building returns.

This study takes a somewhat different approach and establishes a theoretical benchmark rate-of-return for timber investing. Taking an approach similar to the Frank Russell-NCREIF Property Index is not possible for the long-term nature of this study, since individual forest property data sources have only recently become available. Fortunately for timber, there is more reason to believe that results will be highly related to a theoretical benchmark. Whereas office buildings are subject to local market conditions, harvesting a stand of Douglas fir will bring similar prices regardless of where the stand is located as the ultimate product, say 2×4's, are sold into national markets. There is no equivalent of local market vacancy—all effects of supply and demand feed directly into price, and all producers will face about the same price for timber.

Some modifications of equation (1) are required in order to use a similar approach for computing direct timber investment returns. Instead of purchasing one security, this study assumes the purchase of a one-acre fully regulated forest.[4] To purchase this forest one must pay the value of the land (*LV*) plus the value of the growing stock at time $t-1$. Annually the investor receives a dividend in the form of a timber sale from a forever sustainable timber harvest. After a year of growth, the stands that were one year short of maturity will now be ready for harvest, and all other stands will be one year older. In this fashion, the growing stock will replenish itself prior to the next harvest. The value of the dividend is reduced by the cost of managing the forest for one year, so the net dividend is the value of the timber sold, minus the cost of managing the forest for one year. To compute a timber return the value of the one-acre fully regulated forest (land plus timber) must be determined annually. This value depends on current stumpage prices. Although the growing stock of a fully regulated forest is unchanged from period to period, its value fluctuates with stumpage prices. The *LV* of the one-acre forest will also change as stumpage prices change. These statements, of course, are directly analogous to saying that the value of commercial land, and the buildings on them, fluctuate in value as the rental rate of the investment properties changes. The real annual return for growing a fully regulated timber stand can be estimated as:

$$f_t = \left(\frac{LV_t + P_t G + P_t H - C}{LV_{t-1} + P_{t-1} G} \right) - 1, \qquad (2)$$

where:

f_t = real annual return from the forest asset;
LV_t = real land value after harvest of the trees in period t;
P_t = real stumpage price in period t;
H = constant annual harvest volume from the mature trees;
G = the growing stock index, a measure of the (immature) standing tree volume, which is constant from year to year for a fully regulated forest; and
C = constant annual real cost of managing the forest including all tree planting and cultivation expenditures, land taxes and other costs associated with forest management.

Stumpage price, harvest volume and management cost information are needed to compute the annual return for each timber investment. P_tG is the current value of the growing stock. The sum of P_tG and LV_t is the value of the growing stock plus land in period t and thus is analogous to the value of a security, V_t. The sum of P_{t-1} G and LV_{t-1} is the value of the timber plus growing stock in period $t-1$ and thus is analogous to the value of a security in period $t-1$, (V_{t-1}). The one-period earning from managing the forest is the value of the timber harvest, P_tH minus the annual forest management costs, C, so P_tH-C is analogous to the dividend, D_t, of a financial security.

Capital theory implies that the value of the growing stock is the discounted value of its future harvest rather than its immediate harvest value. The measure of growing stock, G, is calculated as the sum of the discounted timber volume for each age class of the regulated forest. If trees are harvested at age thirty, the growing stock index for twenty-nine-year-old trees is the harvest volume from a thirty-year-old forest discounted by one year ($H/1.05$ for a 5% real discount rate[5]). The growing stock index for twenty-eight-year-old trees is the growing stock index of twenty-nine-year-old trees discounted one year, which is the harvest volume of a thirty-year-old forest discounted two years ($H/(1.05)^2$). The growing stock indices for each age are summed to determine the growing stock index, G, for the fully regulated acre. Assuming trees are harvested when they are N years old, this may be expressed as:

$$G = \sum_{n=1}^{N} \frac{H}{(1.05)^n}. \qquad (3)$$

Because a one-acre fully regulated forest is assumed, the number of acres harvested each year is the inverse of the harvest age (N). The per-acre timber harvest at maturity is divided by the harvest age to calculate the annual harvest, H, as only $1/N$ acres can be harvested each year. Similarly, per acre regeneration and cultivation costs are also divided by the harvest age as only $1/N$ acres will be treated each year. Annual taxes and administrative costs are included on a per acre basis as these costs must be paid for the entire acre whereas planting and cultivation activities are done on only a portion of the acre in each year. The forest management cost, incurred annually for a one-acre fully regulated forest, is the sum of the planting and cultivating costs plus the administration and tax costs. This cost is computed in real terms and assumed constant throughout the analysis period.

Employing equation (1) is difficult, however, as a lot of information must be collected or inferred. For this study, the cost figures for managing Southern pine are from Straka et al. (1989). All stands are assumed to be planted. Other cultivation costs such as preparing the site for planting, weed control and prescribed fires are assumed to be done in proportion to the acreage of each reported in Straka et al. (1989). The seedling cost, which is not included in Straka et al. (1989) is $28 per thousand, the 1990 price of loblolly pine seedlings from the State of Virginia, Waverly nursery. The cost figures for managing Douglas fir are from Bare (1979). The planting site is prepared prior to planting, a herbicide treatment is used early for weed tree control in young stands, and excess trees are removed at age fifteen. Computerized projections show a harvestable tree volume for loblolly pine at age thirty is 12.6 thousand board feet per acre, and for Douglas fir at age fifty, 45 thousand board feet per acre.[6]

The land value (LV), which depends on the real stumpage price, P_t, was calculated using the Faustmann (1849) method.[7] A 5% real discount rate was used in the formula:

$$LV_t = \frac{P_t Y_N - \sum_{n=0}^{N} S_n (1.05)^{N-n}}{(1.05)^N - 1} - \frac{A}{0.05}, \qquad (4)$$

where:

- Y_N = per acre timber volume of the forest at harvest age N (equals H^*N);
- S_n = per acre real tree cultivation costs in year n such as tree planting or timber stand improvement; and
- A = real per acre annual overhead costs of forest management such as taxes and administrative costs.

and other variables are as defined earlier. The computed LV figures for the early years were often negative due to low stumpage prices. In such cases a value of 0 was used.[8] An implicit assumption in the LV calculations is that growing continuous timber crops of the species under consideration is the financially optimal use of the forest land.

The other data required to compute rates of returns are historical stumpage prices. Because growing timber is a long-term endeavor, and because this study sought to use the longest term timber price data available, it uses stumpage prices for Douglas fir and Southern pine from U.S. National Forest timber sales over the 1926–1994 period. Data for 1926–1957 is from Potter and Christie (1962), for 1950–1979 from the USDA Forest Service (1982) and for 1959–1994 is compiled from an annual USDA Forest Service series of stumpage prices.[9] Douglas fir timber prices increased (in nominal terms) from $2.10 per thousand board feet in 1936, to $652.34 in 1994. Corresponding figures for Southern pine are \$5.20 and \$265.85, respectively. The correlations between annual changes in the CPI and annual changes in nominal timber prices are 0.17 for Douglas fir and 0.04 for Southern pine. The nominal prices given in these reports were adjusted to the common base year of 1967 using the consumer price index as reported in Ibbotson Associates (1995) to provide real stumpage prices. The real compound price growth for timber over the 1936–94 period was 6.0% for Douglas fir and 2.7% for Southern pine. The real stumpage prices, real *LVs*, and real forest management costs were input, along with the harvest volume and growing stock indices, into equation (2) to compute the annual real rate-of-return for each timber investment.[10] These data are in Exhibit 1.

Inflation-adjusted monthly market return indices through 1994 from common stocks, corporate bonds, U.S. Government bonds, and U.S. Treasury bills from Exhibits B-11, 13, 14, and 16, respectively, of Ibbotson Associates (1995). These data serve as the basis for computing an annual return for each of these financial investments. The annual stumpage price series used to compute the timber returns incorporates the result of averaging timber sale prices recorded over the year. The security market returns given in Ibbotson Associates, however, are first trading day of the month to first trading day of the following month, thus they represent the return over a precise period. To adjust for this difference in data periodicity, the Ibbotson Associates monthly real return indices were averaged to compute the average return index for that year.[11] The change in the average return index from one year to the next determines annual rate-of-return earned by that security. Exhibit 1 provides the annual real rate-of-return earned by each of the four financial investments.

METHODS

In order to estimate the wealth accumulation that could be realized from investing in securities or timber growing, a wealth index, w_{it}, for security or tree species, i, by year, t, given its real annual return, r_{it}, is constructed. This index is similar to results presented by Ibbotson Associates (1995), but adds the investments of Douglas fir and

Exhibit 1: Annual Real Rate-of-Return for Financial Assets (Common Stocks, Corporate Bonds, U.S. Government Bonds, U.S. Government Treasury Bills) and Timber Investments (Douglas Fir, Southern Pine)

Year	Common Stock	Corp. Bonds	Gov. Bonds	T-Bills	Douglas Fir	Southern Pine
1927	0.3167	0.0924	0.1119	0.0534	0.1118	−0.0217
1928	0.3970	0.0609	0.0447	0.0442	0.1725	0.0596
1929	0.3240	0.0236	−0.0039	0.0390	−0.0614	0.0004
1930	−0.1424	0.1101	0.1028	0.0646	0.2212	−0.0820
1931	−0.2664	0.1704	0.1165	0.1165	−0.1096	0.0928
1932	−0.3804	0.0845	0.1663	0.1277	−0.3867	−0.0980
1933	0.4873	0.2036	0.1319	0.0588	−0.2744	0.0689
1934	0.1067	0.0932	0.0192	−0.0314	0.2628	0.1473
1935	0.1195	0.0839	0.0376	−0.0241	0.0698	0.5566
1936	0.5009	0.0593	0.0418	−0.0081	0.1840	0.1649
1937	−0.0103	−0.0035	−0.0181	−0.0333	−0.2869	0.0283
1938	−0.1871	0.0683	0.0719	0.0214	0.5131	0.4005
1939	0.1077	0.0666	0.0723	0.0127	−0.1341	−0.1761
1940	−0.0338	0.0321	0.0357	−0.0076	0.0583	−0.1820
1941	−0.0947	−0.0129	−0.0007	−0.0464	0.5847	1.5213
1942	−0.1356	−0.0783	−0.0774	−0.0971	0.4368	−0.1788
1943	0.3372	−0.0280	−0.0361	−0.0549	−0.0205	−0.1076
1944	0.1179	0.0129	0.0060	−0.0126	−0.1409	0.2335
1945	0.2551	0.0235	0.0434	−0.0192	−0.0425	−0.1276
1946	0.0770	−0.0416	−0.0319	−0.0714	0.3256	−0.0223
1947	−0.1880	−0.1267	−0.1232	−0.1259	0.4412	0.1891
1948	0.0044	−0.0771	−0.0821	−0.0647	0.7552	0.3229
1949	0.0600	0.0595	0.0649	0.0194	−0.4647	0.1755
1950	0.2909	0.0082	0.0143	0.0010	0.5549	0.4491

Exhibit 1: Annual Real Rate-of-Return for Financial Assets (Common Stocks, Corporate Bonds, U.S. Government Bonds, U.S. Government Treasury Bills) and Timber Investments (Douglas Fir, Southern Pine) (continued)

Year	Common Stock	Corp. Bonds	Gov. Bonds	T-Bills	Douglas Fir	Southern Pine
1951	0.2091	−0.0833	−0.0927	−0.0611	0.5944	0.4012
1952	0.1450	−0.0039	−0.0230	−0.0070	−0.0330	0.0739
1953	0.0522	−0.0084	−0.0194	0.0101	−0.1960	−0.1180
1954	0.2846	0.0799	0.0929	0.0090	−0.1795	−0.0954
1955	0.4246	0.0110	−0.0033	0.0138	0.8622	0.1369
1956	0.1650	−0.0263	−0.0206	0.0067	0.4087	0.2616
1957	−0.0435	−0.0660	−0.0564	−0.0053	−0.3100	−0.1437
1958	0.0719	0.0422	0.0104	−0.0045	−0.1650	0.0047
1959	0.2654	−0.0399	−0.0570	0.0125	0.7218	0.1544
1960	−0.0204	0.0472	0.0466	0.0161	−0.1116	0.0226
1961	0.2253	0.0492	0.0512	0.0109	−0.1109	−0.1918
1962	−0.0524	0.0516	0.0289	0.0126	−0.0748	0.0088
1963	0.1584	0.0344	0.0195	0.0162	0.1598	0.0033
1964	0.1825	0.0181	0.0091	0.0203	0.4195	0.1524
1965	0.1009	0.0164	0.0176	0.0209	0.1819	0.1889
1966	−0.0427	−0.0599	−0.0481	0.0130	0.2311	0.2586
1967	0.0993	−0.0132	−0.0224	0.0175	−0.1946	0.0156
1968	0.0582	−0.0368	−0.0488	0.0044	0.5881	0.1294
1969	−0.0302	−0.0921	−0.1113	0.0041	0.3888	0.3024
1970	−0.1631	−0.0567	−0.0489	0.0092	−0.5586	−0.1903
1971	0.1690	0.1138	0.1149	0.0081	0.1590	0.1789
1972	0.1139	0.0599	0.0430	0.0070	0.5654	0.3705
1973	−0.0582	−0.0196	−0.0568	−0.0077	1.0827	0.5824
1974	−0.2826	−0.1406	−0.1106	−0.0288	0.4592	−0.2631
1975	0.0227	0.0123	0.0141	−0.0218	−0.2405	−0.3632
1976	0.1607	0.0826	0.0618	−0.0031	−0.0227	0.5892
1977	−0.0705	0.0411	0.0217	−0.0146	0.2751	0.1786
1978	−0.0418	−0.0564	−0.0730	−0.0136	0.0713	0.3927
1979	0.0258	−0.0892	−0.0874	−0.0203	0.5465	0.1302
1980	0.0744	−0.1958	−0.1845	−0.0225	0.0151	−0.0899
1981	0.0192	−0.1298	−0.1190	0.0260	−0.2851	−0.0003
1982	−0.0623	0.1503	0.1590	0.0632	−0.7374	−0.4212
1983	0.3588	0.2273	0.1617	0.0559	0.4053	0.1650
1984	0.0012	0.0040	−0.0001	0.0495	−0.2052	−0.0036
1985	0.1922	0.2286	0.2159	0.0502	−0.0684	−0.3582
1986	0.2888	0.2691	0.3407	0.0497	0.3431	0.1610
1987	0.1923	0.0114	−0.0116	0.0213	0.2153	0.3327
1988	−0.0676	0.0452	0.0224	0.0173	0.4588	0.0633
1989	0.2028	0.0808	0.0889	0.0234	0.4972	−0.0694
1990	0.0027	0.0363	−0.0121	0.0311	0.1854	−0.0354
1991	0.1375	0.1031	0.0981	0.0225	−0.0032	0.2978
1992	0.0941	0.1085	0.1049	0.0137	0.0107	0.2968
1993	0.0864	0.1231	0.1637	0.0012	−0.3645	0.2333
1994	0.0182	−0.0399	−0.0465	0.0061	1.2021	0.2275
For 1937–1994						
Average	0.0736	0.0133	0.0102	−0.0008	0.1684	0.1120
Std Dev.	0.1461	0.0880	0.0901	0.0351	0.3985	0.2950

Correlation Coefficients (1937–1994)

	Common Stock	Corp. Bonds	Gov. Bonds	T-Bills	Douglas Fir	Southern Pine
Common Stock	1.00					
Corp. Bonds	0.42	1.00				
Gov. Bonds	0.39	0.98	1.00			
T-Bills	0.29	0.60	0.57	1.00		
Douglas Fir	0.07	−0·21	−0·25	−0·25	1.00	
Southern Pine	−0·01	−0·05	−0·08	−0·16	0.48	1.00

Southern pine to the financial market investments, and uses the fifty-eight-year period, 1937–1994, via the equation:

$$w_{it} = \prod_{t=1937}^{1994}(1 + r_{it}). \quad (5)$$

In addition to simply showing the wealth accumulation from timber or financial market investing, it is natural to ask what timber, in conjunction with financial market investments, would earn. Many portfolio models are solved using a Markowitz approach, a well-known single-period model. The Markowitz approach of minimizing the portfolio variance while achieving a specified expected rate-of-return during the next period has been shown to be equivalent to maximizing a quadratic utility function. The shortcomings of a quadratic utility function are well known (Copeland and Weston, 1988). The objective of long-run portfolio analysis is to maximize the utility of terminal wealth through maximizing the utility of a risk-averse investor.[12] Focusing on terminal wealth avoids the potential pitfall of choosing portfolios that may maximize single-period expected value, but lead to suboptimal long-run wealth accumulation. Latane (1959) showed that maximizing the logarithm of portfolio returns in each period leads to the highest wealth accumulation in the long run. This approach is often called "growth optimal" investing. There has been some criticism of the simple growth optimal model (Fama and MacBeth, 1974; Merton and Samuelson, 1974) because not all investors focus on long-time horizons and thus may achieve a higher utility using a single-period utility criterion. Also, varying levels of risk aversion are not accounted for in the simple growth optimal model. Grauer and Hakansson (1982) address these problems by solving the long horizon problem using a power utility function. Hakansson (1979) shows the power utility function is myopic (only considers the next period) while being consistent with multiperiod optimization. Friend and Blume (1975) conclude that observed portfolio choices are consistent with a power utility function. The optimization problem developed by Grauer and Hakansson (1982) is:

$$Max\ z = \sum_{s=1}^{S} p_{ts} \frac{1}{u}\left(1 + \sum_{i=1}^{I} x_{it} r_{its}\right)^{u}, \quad (6)$$

subject to:

$$\sum_{i=1}^{I} x_{it} = 1 \quad (7)$$

$$x_{it} \geq 0 \quad for\ all\ i,t, \quad (8)$$

where:

z = the power utility function to be maximized;
x_{it} = the proportion of investment in each of the I investments, in each period, t;
r_{its} = the return earned on the i^{th} investment in state, s, in period, t;
u = a parameter that remains fixed over time ($u \leq 1$, $u \neq 0$); and
p_{ts} = the probability of state s occurring in period t.

Equation (6) is the power utility function to be maximized. Equation (7) requires that the dollar proportion invested in each alternative sum to 1, the total investment. Equation (8) requires that proportions invested in each alternative be non-negative.

The power utility parameter, u, varies among investors. If $u=1$, the concern of the investor is to maximize expected return (risk neutrality). Where u is very small (say -40) the investor obtains a large negative utility from any loss in wealth, but only a small gain in utility from any increase, or in other words, the primary concern is risk minimization.[13] To solve for the efficient trade-off between return and risk, equations (6)–(8) are solved parametrically, adjusting the u coefficient to trace out the result. Calculations were performed on an IBM compatible personal computer using a custom written FORTRAN program which links to a nonlinear programming optimization library, MP6 from SCI computing (Saigal, 1986).

In the limit as $u \to 0$, the objective function becomes the simple growth optimal strategy which can be shown using L'Hôpital's rule (Ingersoll, 1987:40) to be:

$$Max\ z = \sum_{s=1}^{S} p_{ts} \ln\left(1 + \sum_{i=1}^{I} x_{it} r_{its}\right), \quad (9)$$

where ln is the natural logarithm. Equations (7) and (8) complete this programming specification. The growth optimal policy has the greatest likelihood of achieving the highest wealth accumulation over the long run. This study refers to the logarithmic form of the power utility maximization as the $u=0$ case.

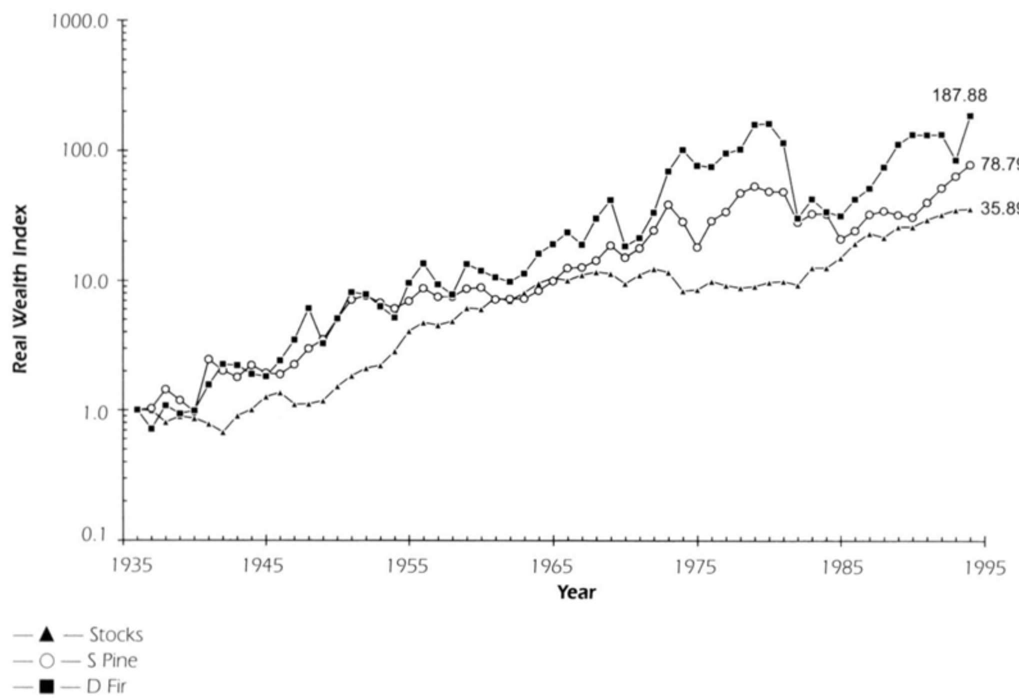
Grauer and Hakansson (1982) use this approach to determine the optimal portfolios of common stocks, corporate bonds, Government bonds, and Treasury bills for each u-parameter for each year in their historical analysis. They use the observed (ex post) returns from 1926–1935 to compute the portfolio (ex ante) weights to hold during 1936. Then they use the ex post returns of 1936 to compute the realized portfolio return for 1936. Next they use the ex post returns from 1927–1936 to compute portfolio (ex ante) weights to hold during 1937, and then use the ex post returns of 1937 to compute the realized portfolio return that 1937 and so on. They use the "simple probability approach" of assuming that each of the previous ten yearly returns is equally likely to occur in the following year ($p_{ts}=0.1$ for all t, s).

The initial portfolio analysis in this study adopts the Grauer and Hakansson (1982) approach with the following extensions. First, the timber assets of Douglas fir and Southern pine are added to the choice of investments. Second, the analysis is done using real rates of return rather than nominal rates of return. Third, the time period is 1937–1994. The return series starts one year later as the 1926 inflation data is needed to compute the real stumpage price for that year. Therefore 1927 is the first year for which a timber return can be computed. The length of the horizon is increased as more recent financial market and timber assets returns data are available. Fourth, this study also reports on the results expected ex ante by retaining this information for comparison with ex post results. This can also be seen as an extension of the approach reported by Pagliari et al. (1995) who use five years of quarterly real estate data to estimate optimal holdings from a Markowitz model, and then look at the results for the following five-year period. The process employed here uses annual updates and ex post realizations and thus is able to evaluate the use of a portfolio process over a long horizon.

RESULTS

Exhibit 2 presents the real wealth over time that would be realized from a $1 investment at the end of 1936 in Douglas fir, Southern pine and

Exhibit 2: Real Wealth Indices by Year (year-end 1936 = 1)



common stocks. The wealth accumulations from corporate bonds, Government bonds and Treasury bills are too small to be easily seen on this figure. These results assume that all returns earned by each asset are reinvested and allowed to accumulate. It shows that the highest wealth accumulation is from Douglas fir, followed by Southern pine, then common stocks. The return variability is seen to be much higher for the timber investments. A $1 investment at the end of 1936 in Douglas fir, Southern pine and common stocks would have grown (in real terms) to be $187.88, $78.79 and $35.89, respectively, in 1994. The compound real rates-of-return indicated by these wealth indexes are 9.45%, 7.82% and 6.37%. The substantial gains posted for timber investing are partly explained by their considerable long-term price appreciation.[14] The timber investments reached their high during the late 1970s, after which they declined and have more recently shown renewed growth. The corresponding wealth accumulation figures for corporate bonds, Government bonds and Treasury bills are $1.78, $1.42 and $0.92, respectively.

When the portfolio model is solved, its solution is a set of portfolio weights x_{iut} for each asset i, for each u-parameter for year t. The 1927–1936 period data is used in the first portfolio optimization calculation. Eighteen sets of portfolio weights (x_{iut}) are computed, corresponding to the eighteen u-parameters used (1, 0.75, 0.5, 0.25, 0, −0.5, −1, −2, −3, −4, −5, −7, −10, −15, −20, −25, −30, −40). The ex post returns earned by each of these eighteen portfolio's (r_{ut}) are then calculated for t=1937 using the 1937 return data of Exhibit 1 and the equation:

$$r_{ut} = \sum_{i=1}^{I} x_{iut} r_{it} \quad u = 1, 0.75, 0.5, \ldots, -40. \qquad (10)$$

The return data for the 1928–1937 period is then used in the portfolio optimization routine to compute a new set of optimal portfolio weights from which the ex post return earned in 1938 is computed. This process repeats itself, finally using the 1984–1993 data to compute portfolio weights held during 1994. The result of these computations is a series of ex post historical rates of return earned over the 1937–1994 period for each portfolio of parameter u.

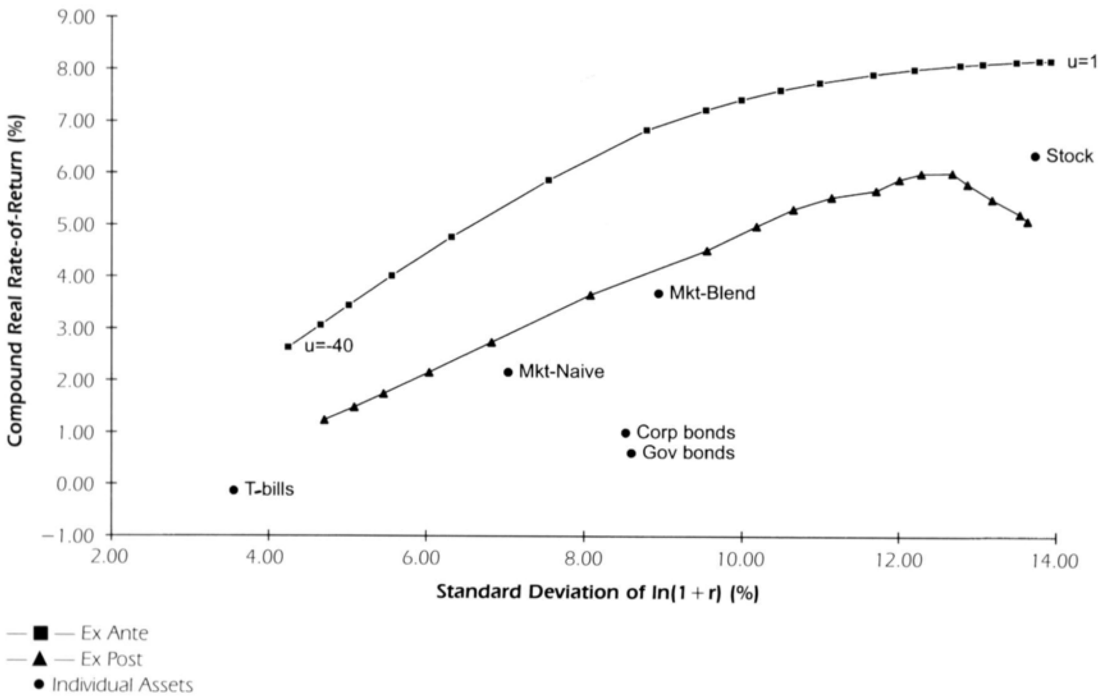
The long-run portfolio rate-of-return for each u is computed as the compound (also called geometric) rate-of-return for that portfolio strategy which is:

$$g_u \left(\prod_{t=1}^{58} (1 + r_{ut}) \right)^{1/58} = 1 \quad u = 1, 0.75, 0.5, \ldots, -40. \ (11)$$

The compound rate-of-return from holding each asset individually was also computed for comparison purposes. In addition, the standard deviation of the natural log of one plus the annual returns[15] was calculated for each portfolio (with parameter u) and for the individual assets.

The above procedure was first applied to the financial market assets. The output of a portfolio model includes the ex ante expected return and standard deviation of each portfolio over the next period. The ex ante results were recorded for each portfolio parameter, and these fifty-eight annual results were used to compute the '*Ex Ante*' line shown on Exhibit 3. The point labeled $u=-40$ shows the ex ante results for the portfolios derived using this parameter. Each subsequent point indicates an increasingly risk-tolerant u-parameter through the $u=1$ point which simply chooses a portfolio consisting of the asset with the highest return during the previous ten years. Also plotted on Exhibit 3 are the ex post results, that is, the results when one determines the portfolio weights using the previous ten years returns, but replaces the ex ante expected return with the ex post actual return. The ex post line retains the same basic shape as the ex ante line, but it is displaced downwards about two percentage points. Ex post the portfolio optimization routine allows for selection of portfolios with about the same risk as expected ex ante, but with about a 2% lower rate-of-return. The ex post results also show that if the individual assets were purchased on a buy-and-hold strategy, an investor would earn a lower rate-of-return for each level of risk except for holding common stock. Common stock had a somewhat higher return than that achieved using the portfolio strategy, but it also had a higher standard deviation. A naïve portfolio strategy of simply placing one quarter of one's wealth in each of the four financial assets in each period provides risk reduction, but not to the degree that using the portfolio optimization approach provides (see asset labeled as *Mkt-Naive* on Exhibit 3). A market blend

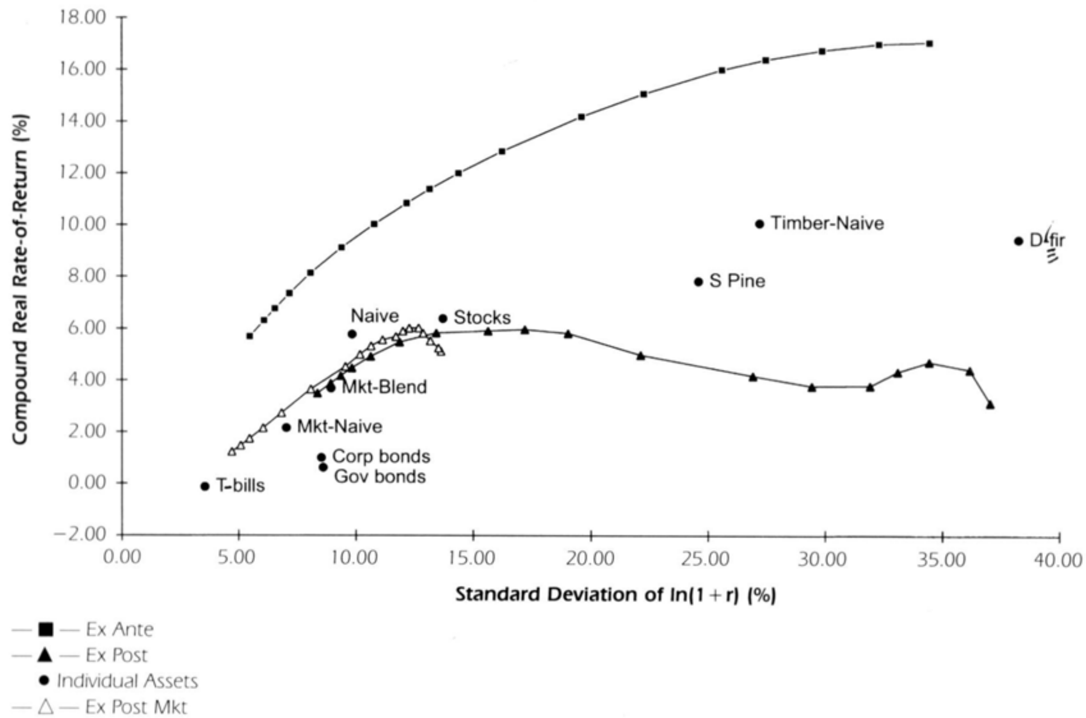
Exhibit 3: Compound Return versus Standard Deviation for Financial Market Investment Portfolios and Individual Assets



of 50% stock, 20% corporate bonds, 15% Government bonds, and 15% Treasury bills also lies below the portfolio strategy results (see asset labeled as *Mkt-Blend* on Exhibit 3). Naïve portfolio diversification did provide, however, a higher return and lower standard deviation than investing in either corporate or Government bonds and the blended approach yielded a higher return than bonds with only a small increase in risk. The corporate and Government bonds had less than a 1% return over this period with a standard deviation of 9%, whereas the portfolio strategy with $u=-10$ had about the same risk, and greater than a 3% rate-of-return. The growth optimal portfolio strategy ($u=0$) provided the maximum long-run portfolio return, although a higher return was achieved (with a higher standard deviation) from investing solely in common stocks. As expected, portfolios with u-parameters above the growth optimal parameter (i.e., $0<u<1$) achieved a lower return and higher standard deviation than some of those with lower u-parameters. Treasury bills, typically considered risk-free assets, had a negative return (−0.14%), with a somewhat lower standard deviation (3.6% versus 4.7%) than the $u=-40$ portfolio strategy which provided a positive real rate-of-return of 1.2%.

Except for the modifications discussed earlier, results similar to those in Exhibit 3 are presented in Grauer and Hakansson (1982). Of particular interest to this study is what happens as timber assets are included in the portfolio choices. The above analysis was redone adding the two timber investments. Exhibit 4 shows these results, including the returns to investing in Douglas fir or Southern pine individually. The timber assets are shown to be very risky if held singly, which is not surprising given the results presented in Exhibits 1 and 2. Exhibit 4 also reveals a very different pattern than may be expected for the portfolio investments.[16] First, there is a very marked difference between the ex ante and ex post results. The ex post portfolio choices are shown to be somewhat riskier than expected ex ante. More importantly though, there is a very large difference between ex ante expected return, and actual ex post performance. The ex post line lies as much as 13% below the ex ante line. Also, when timber assets are included in the portfolio optimization routine, the process delimits the risk-return

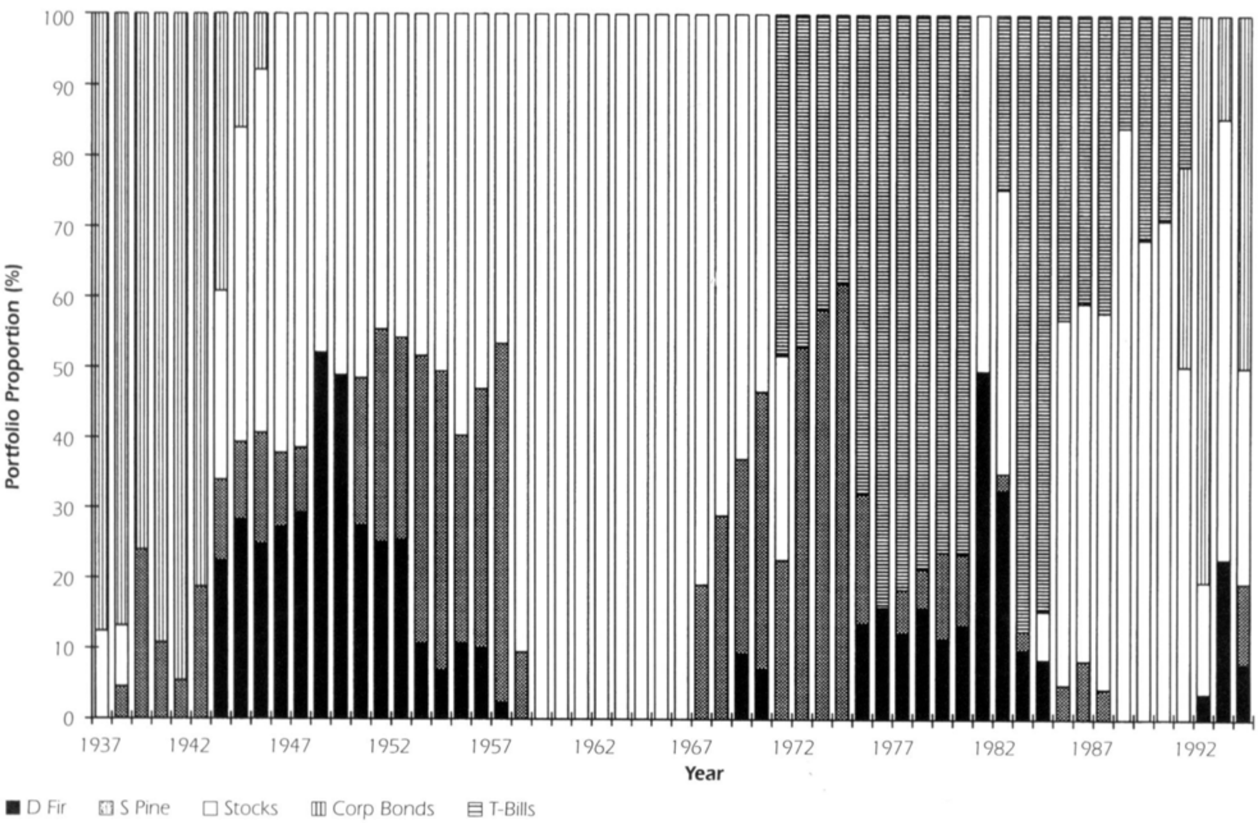
Exhibit 4: Compound Return versus Standard Deviation for Timber and Financial Asset Portfolios and Individual Assets



minimizing result for only relatively risk-averse investors ($u<-4$). For higher u-values, risk is increased, but return is not. A higher return and lower risk can be achieved by holding common stocks or the timber assets singly, rather than following the portfolio strategy. While the single-period portfolio models cited earlier all show the benefit of including timber in one's portfolio, the repeated multiperiod optimization solved here questions such a conclusion as the ex post risk-return frontier generated when timber assets are excluded, dominate the frontier that includes timber. Naïve diversification (holding one sixth of one's wealth in each asset in each period) provides a higher return (5.8%) than is achieved with the timber and financial asset portfolio strategy (4.5%) with $u=-20$ even though they have about the same risk (standard deviation=9.8%). This naïve diversification also dominates a portion of the market only portfolio results (i.e., either higher return for a given level of risk, or a lower risk for a given level of return, or both) suggesting that including timber assets in portfolios may be a viable long-run investment strategy if an alternate portfolio strategy is used. A simple naïve diversification in timber (one half in each timber species in each period) provides a return of 10.1% with a standard deviation of 27.2% (see point labeled *Timber-Naive*). This represents a higher return than achieved by Douglas fir with a standard deviation closer to that of Southern pine, demonstrating the desirability of diversifying between timber species.

Exhibit 5 shows the asset portfolio weights over time that are chosen by the optimization strategy for $u=-7$ over the period of analysis. Following this portfolio strategy yielded a compound return of 5.8% with a standard deviation of 13.4%. Exhibit 5 shows that prior to 1943, little timber was chosen for the portfolio, which was primarily invested in corporate bonds. During the 1943–1957 period it was common to have close to half of one's portfolio invested in timber, with both species typically being present. The remainder was placed in stocks. From 1959–1966 common stocks were the only asset in the portfolio,[17] after which timber was again chosen, first in conjunction with common stocks, and then with Treasury bills. In the mid-1980s timber again lost out to a mix of common

Exhibit 5: Portfolio Weights for Individual Assets by Year for u = −7

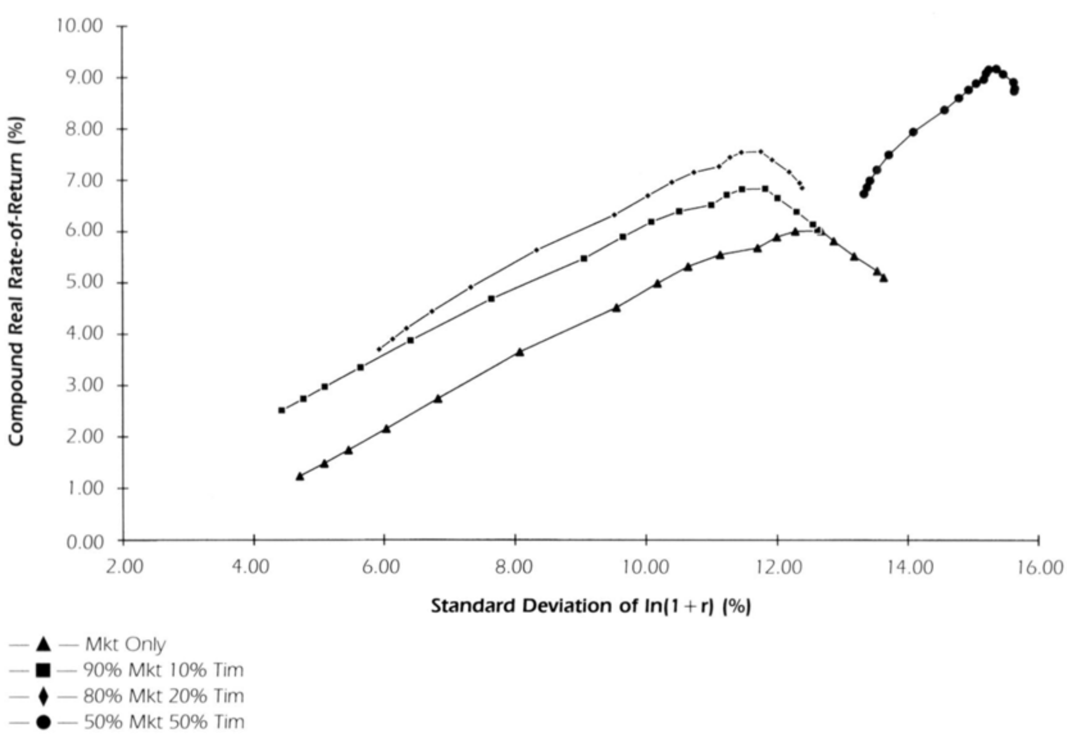


stocks and Treasury bills. Similar timber investment patterns were observed for more and less risk-averse investors. The growth optimal strategy ($u=0$) showed higher proportions of timber investments with the 1943–1957 period having the portfolio dominated by timber, as was also true for the 1968–1983 period. The balance of the portfolios consisted primarily of common stocks. For the more risk-averse ($u=-40$) strategy, timber was present during the same periods, but with much smaller proportions, especially in the 1968–1983 period. Some common stock was present, but corporate bonds in the first decade and Treasury bills after 1958 were the largest components of the portfolios. Timber investments thus were present for much of the period across risk parameter, though higher percentages were common in the portfolios of those willing to assume more risk.

Because timber assets offer a higher rate-of-return on average, and because the capital asset pricing model (CAPM) studies cited earlier have shown that timber returns are not correlated to the returns on common stocks, one would expect timber investments to be valuable additions to a portfolio. The CAPM model, and the previously reported portfolio models all used past realized returns to develop an ex ante expected risk-return relationship between timber and financial market investments. The portfolio optimization reported here uses the returns of the previous ten years to choose a strategy that is expected to maximize (for a given level of risk) the returns in the next period, under the assumption that the next period will be like the previous periods. The historically realized ex post return, however, is used to compute the results, rather than the expected return. When timber investments have shown a high rate-of-return over the past ten periods, the model may choose high amounts of timber for the next period. Because the return on timber investments swings widely, it appears that the returns earned when timber was included were more often lower than expected resulting in the poor risk-return results. One possible way to adjust for this problem is to use a longer returns horizon for making portfolio decisions. The analysis was repeated using 1947 as the first period of portfolio formulation so that twenty years of returns data could be used for portfolio formation. For comparison, ten-year portfolio formation data was also used beginning at this point. The resulting risk-return frontiers were qualitatively similar to those shown in Exhibit 4. For financial assets only and financial assets combined with timber assets, the results, when using the previous twenty years of data for portfolio formation, were somewhat worse than when using the previous ten years of data.

Nevertheless one would expect that timber investments could still be valuable additions to portfolios, given their high returns when held for long periods. Using historical return data, however, did not help in choosing the optimal amount of timber to include period by period in one's portfolio as it did for the financial assets. Also, because timber is a physical rather than a financial asset, it would be difficult to pursue a rebalancing strategy on an annual basis even if this appeared advantageous. An alternate approach to including timber in the investment portfolio is to choose a fixed proportion of timber to hold, and then use the portfolio techniques described here to choose the mix of financial assets that will be held in the remainder of the portfolio. The results of such a strategy were computed which showed timber to be a desirable addition to a portfolio when added as a fixed proportion to a financial asset portfolio. For example, when 10% of the portfolio is in timber (5% in Douglas fir and 5% in Southern pine) the risk return profile is shifted up about 1%. In other words, for any level of risk, the return increases by about 1% (Exhibit 6). As more timber is added to the portfolio, higher returns can be earned, but at increased risk. When an equal mix of timber and financial assets are chosen, the standard deviation exceeds that of financial assets portfolios, but higher returns can be earned. The general message of Exhibit 6 is that investors can benefit with some addition of timber to their portfolios, as long as its

Exhibit 6: Compound Return versus Standard Deviation When Timber Investments Are Held to the Constant Proportions Noted and Financial Assets are Rebalanced Annually: 1937–1994



held as a constant portfolio proportion over time. Risk-averse investors should hold small amounts of timber, perhaps 10% of the portfolio. Risk-tolerant investors could hold a higher proportion, and their increased risk tolerance would be rewarded with a higher return.

SUMMARY

This study has estimated the historical long-run wealth-building capability of investing in Douglas fir and Southern pine in fully regulated forests, along with the standard financial investments of common stocks, corporate bonds, U.S. Government bonds, and U.S. Treasury bills. The maximum asset return earned over the 1937–1994 period was from holding Douglas fir (9.4%) but the standard deviation was also high (38.2%). A low portfolio risk appears possible though investing in a limited and constant proportion of timber species along with a rebalanced portfolio of low-risk financial assets. The lowest risk portfolio was constructed with each timber species held at 5% with the remaining 90% of the portfolio determined using a portfolio optimization routine with $u=-40$. Its estimated compound real return of 2.5% is substantially larger than the −0.14% earned by Treasury bills, though it has a somewhat higher standard deviation (4.4% vs. 3.6%). This portfolio has a higher return and lower standard deviation than the minimum risk portfolio with no timber assets. A portfolio with timber equal to 5% in each species and the remainder invested in rebalanced financial assets and a growth optimal portfolio parameter of $u=0$ showed a return of 6.8% and standard deviation of 11.8%. This outperformed common stocks, which provided a return of 6.4% with a standard deviation of 13.8% over the 1937–1994 period. A 50–50 mix of (fixed) timber and (rebalanced) financial investments with the growth optimal strategy showed a return of 9.2% and a standard deviation of 15.4%. This standard deviation is near half of that of the Douglas fir asset with a small decrease in return of 0.2%.

Several cautions remain for applying these results. First, numerous computations and projections were necessary to compute the timber returns, as there is no long-term reliable transaction data that can be used. Second, much of the timber return was derived from timber price appreciation. If timber prices stabilize, the returns to timber will not be so appealing though timber's lack of correlation with financial assets should remain a benefit. This paper limits the analysis to timber and standard financial assets, because long-term return data was attainable. Further work should compare timber to other high-risk asset classes, and also to more traditional real estate investment classes to determine whether timber possesses unique return characteristics, or whether other assets could just as well fill its apparent niche.

In summary, this study has shown that over the 1937–1994 period, timber investments have been valuable either held singly, or as additions to financial asset portfolios. When held singly they exhibit a high standard deviation of returns which tends to be negatively correlated with other financial assets. When held as part of an overall portfolio strategy, they should be held in fixed proportions, with the proportion held increasing as one's risk tolerance increases. The remainder of the portfolio can be held in an annually rebalanced financial assets portfolio.

NOTES

1. According to its ForesTree Investor newsletter, the John Hancock Timber Resource Group managed 1.35 million acres in 1993 with an estimated value of $1.1 billion, and in 1995 it managed over 2 million acres with an estimated value of approximately $2 billion.
2. This may be more clearly shown by employing an example that is drawn from results to be presented later. Exhibit 5 shows the portfolio weights chosen for $u=-7$ during 1937 were 0.12 for stocks and 0.88 for corporate bonds. From Exhibit 1, the expected return (computed using past returns from 1927–36) for stocks is 0.1463 and for corporate bonds is 0.0982. The expected portfolio return (ex ante) thus is: 0.12*(0.1463)+ 0.88*(0.0982)=0.1040. Using the 1937 return data (which was unknown when the portfolio choices were made at the end of 1936) one sees stocks earned −0.0103 and corporate bonds earned −0.0035, providing a realized return (ex post) on this portfolio of: 0.12*(−0.00103)+0.88*(−0.0035)=−0.0043. While this investor expected a return (ex ante) of a little more than 10%, the actual return (ex post) was a loss of about ½%. The investor now knowing the 1937 rate-of-return, uses this information along with the previous nine years of returns to choose a portfolio to hold in 1938. Exhibit 5 shows the updated portfolio to consist of 0.04 Southern pine, 0.09 stocks, and 0.87 corporate bonds. This process repeats itself through the realized return computed for 1994.
3. Stumpage is the forestry term for harvestable trees standing in the forest. Stumpage price is the price a buyer agrees to pay for trees, "on the stump," i.e., trees standing in the forest. The

buyer would be responsible for harvesting the tree and would sell or use the logs he produced.

4. A fully regulated forest is a forest that has an equal land area in each age class. When a harvest is made from the oldest age class, that age class becomes the youngest age class, and all age classes increase by one over the next year. The volume of immature trees (which foresters call growing stock) for the overall forest remains the same year to year while the age of the timber at any particular location increases until it is harvested.

5. Berck (1979) finds that a 5% real discount rate is consistent with the harvest patterns observed on corporate timber lands. The classic paper on valuing forest land, Faustmann (1849), used a 5% discount rate in examples. A 5% real rate-of-return is commonly used in forestry analysis, almost like a 5% level of significance is used in statistics. Nevertheless, this assumption does have an important effect on the results presented here. While the lack of correlation between timber investments and stock or bonds remains regardless of whether the 5% discount is varied, the computed long-run estimated return on timber does vary. Over the range of 3%–7%, as the discount rate is increased 1%, the compound rate-of-return on timber increases about 1%. As a higher discount rate is applied to the value of growing stock and land, the denominator in equation (1) is reduced which increases the rate-of-return.

6. A board foot of lumber measures 12 in. by 12 in. by 1 in. The pine harvest volume was projected as the cubic foot yield by diameter class using the North Carolina State University Plantation Management Simulator for a site index (age 25) of 65 feet, with 700 planted trees per acre having 85% initial survival. The cubic foot yields at age thirty were converted to board foot yields by diameter class using conversion ratios in Vardaman (1987). The yield at age fifty for Douglas fir was projected by employing the Stand Projection System of Arney (1985) with a site index (breast height age fifty) of 125 and a precommercial thinning at age fifteen to 225 trees/acres. These assumptions presume better than average timber growing sites which is reasonable if they are chosen for timber investment. To the degree that stand simulators do not allow for losses such as insect and disease damage, the yields used here may be over estimated. Alternately one could view these as the correct yields for a somewhat higher quality timber site. Mills (1980) finds catastrophic losses (such as forest fire losses) to be quite small. Thomson and Baumgartner (1996) find annual growth fluctuations in timber stands to be small relative to price fluctuations and conclude that variability in return due to growth fluctuation is so small that it can be safely ignored when making portfolio calculations. This will be especially true for an index that is used to represent the average results from many stands.

7. This equation can be thought of as a modified version of the standard income capitalization approach of real estate appraisal where net cash flows are divided by a capitalization rate to determine value. In this case the incomes are received only when timber harvests are made whereas costs can occur more often. The capitalization rate used here is 5% which is consistent with Berck (1979). If the future price path of timber was known, this *LV* computation would use future rather than current timber prices and this portfolio study would not be required as timber would be a risk-free asset. Clarke and Reed (1989) show that if stumpage prices follow a lognormal diffusion (without drift) and there are no management costs, the expected *LV* is the *LV* that will be computed using the current timber price. Thomson (1992a) assesses forest land values in an options pricing context (i.e., stumpage prices following a log normal diffusion) and shows only a small difference in value between the mechanistic approach taken in the present study and the elaborate options-based model employed in Thomson (1992a). Washburn and Binkley (1990b) provide statistical evidence that timber prices follow a diffusion.

   Washburn (1990), using forest property sale data, concludes that forest land prices depend on ten-year average prices rather than solely on the current price as is predicted using diffusion models. There are several reasons, however, to maintain the assumption used above. First, is that Washburn's data set is for a ten-year period so it may be better to use a theoretical approach for this long horizon study rather than an empirical approach that has a limited time series associated with it. Second, there is a problem in that Washburn did not have previous prices to use in computing his average stumpage price; thus, for all but the final period's observations he is using some ex post prices that were not known at the time the appraisal was made as if they were known at the time of appraisal. Third, the inference may be due to an artifact of the methodology that appraisers use in allocating the sale price between timber and land. If the merchantable timber is valued by appraisers as the current stumpage price times the volume, then by default, the rest of the value is assigned to the land. The land value in Washburn's database, therefore, may reflect the appraisal methodology rather than the true economic value of the land.

8. There is no requirement to own forest land, thus, if the true land value was negative one would observe land abandonment. Land abandonment has not been observed for Douglas fir and Southern pine, demonstrating that the land value has not fallen below zero. The reason this computation yielded a negative land value is that stumpage prices were so low in the early years that one could not afford the planting and stand tending costs. At that time, however, natural regeneration was used in place of planting, and few stand tending investments were made.

9. These price series cover overlapping periods. It was confirmed that all sources encompass a common series as prices were identical between sources during overlapping periods. Where data was missing from one series it could usually be found in one of the other series. Three data points that could not be identified this way were estimated using regression methods with other price series for the same species. The regression had an R^2 value of 0.99 suggesting that the few estimated points were accurately estimated.

10. It is prudent to be concerned with the number of assumptions required to estimate the timber returns. For this reason some researchers, such as Redmond and Cubbage (1988) use only the stumpage prices that are compiled from timber sale data. For estimating a stumpage *beta*, this is a suitable approach. For portfolio analysis, however, one must include the effect of the timber growth and costs in determining the return that will be achieved. The returns developed in this study are highly correlated with the historical price fluctuations which are based on timber sale data; thus, the fluctuations in returns, which are important in portfolio analysis, are not an artifact of the timber management regime assumed. The "returns" generated solely by the historical price data (i.e., $P_t/P_{t-1}-1$, where P_t is real stumpage price in year t) were input as the independent variable in a simple linear regression with the returns given in Exhibit 1 as the dependent variable. The regressions produced highly significant coefficient estimates

and r-square values in excess of 0.98. From this result one may conclude it is the market price data and not the timber growth assumptions that has the most influence in the results presented here. The results presented here, therefore, can be seen as having applicability beyond the two example forests as other Douglas fir and Southern pine forest returns would also be highly correlated to the stumpage price changes.

11. Washburn and Binkley (1990a) note that this approach will adjust for the fact that the individual timber sale prices are averaged over the year to compute the reported timber price, so security market returns must be similarly averaged to make the return period comparable.
12. Grauer (1986) notes that the results of using a Markowitz mean-variance approach are typically very similar to those computed using the power utility approach. Grauer shows differences occur when high leverage is used. Because no leverage is used in this study, the results should be similar to those derived from a Markowitz approach.
13. Values of u smaller than -40 would be representative of even more risk-averse investors. The computing algorithm, however, could not reliably solve the optimization for smaller u-values.
14. Recent evidence of timber returns exceeding stock market returns is documented by the Hancock Timber Resource Group (1994) which reports a (nominal) return of 20.6% after fees, from its 1985 inception to mid-1994. The market return over this period was about 13.4%, showing that timber outpaced stocks by 720 basis points over this period.
15. Grauer and Hakansson (1995) note it is more consistent to use the standard deviation of $\ln(1+r)$ when the returns are presented as compound returns. They also note that the results are very similar to the standard deviation of returns when returns are less than 25%.
16. There are now several studies that suggest the Markowitz approach produces a "fuzzy" efficient frontier (Gold, 1995). The results demonstrated in the present study can be seen as empirical evidence that applying an optimization approach over many periods may sometimes produce results that are of limited appeal.
17. Recall the returns of the prior ten years are used in making the portfolio choices. Exhibit 1 shows stocks enjoyed positive favorable returns during this period; thus stocks were favored during this period.

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