Solutions to Problems - Chapter 6
Mortgages: Additional Concepts, Analysis, and Applications

INTRODUCTION

The following solutions were obtained using an HP 12C financial calculator. Answers may differ slightly due to rounding or use of the financial tables to approximate the answers. As pointed out in the chapter, there is often more than one way of approaching the solution to the problems in this chapter. Thus "alternative solutions" are shown were appropriate.

Problem 6-1

(a)

Because the amount of the loan does not matter in this case, it is easiest to assume some arbitrary dollar amount that is easy to work with. Therefore we will assume that the purchase price of the home is $100,000. Thus the choice is between an 80 percent loan for $80,000 or a 90 percent loan for $90,000. The loan information and calculated payments are as follows:

Alternative	Interest Rate	Loan Term	Loan Amount	Monthly Payments
90% Loan	8.5%	25 yrs.	$90,000	$724.70
80% Loan	8.0%	25 yrs.	80,000	617.45
Difference			$10,000	$107.25

i(n,PV,PMT,FV)

n = 25x12 or 300
PMT = $107.25
PV = -$10,000
FV = 0

Solve for the *annual* interest rate:

i = 12.26%

Solving for the interest rate with a financial calculator we obtain an incremental borrowing cost of 12.3 percent. (Note: Be sure to solve for the interest rate assuming *monthly* payments. With an HP12C you will first solve for the monthly interest rate, then multiply the monthly rate by 12 to obtain the nominal annual rate.)

(b)

Alternative	Loan Amount	Points	Net Proceeds	Monthly Payments
90% Loan	$90,000	$1,800	$88,200	$724.70
80% Loan	80,000	0	80,000	617.45
Difference			$8,200	$107.25

$107.25 x (MPVIFA, ?%, 25 yrs..) = $8,200

i(n,PV,PMT,FV)

n = 25x12 or 300
PMT = $107.25
PV = -$8,200
FV = 0

Solve for the *annual* interest rate:

i = 15.35%

Solving for the interest rate with a financial calculator we now obtain an incremental borrowing cost of 15.35 percent.

(c)
We now need the loan balance after 5 years.

Alternative	Loan Amount	Monthly Payments	Loan Balance
90% Loan	$90,000	$724.70	$83,508.62
80% Loan	80,000	617.45	73,819.37
Difference	$10,000	$107.25	$9,689.37

Note that the net proceeds of the loan is still $8,200 as in Part b. Thus we have:

i(n,PV,PMT,FV)

n = 25x12 or 300
PMT = $107.25
PV = -$8,200
FV = $9,689.37
Solve for the *annual* interest rate:
i = 17.96%

Solving for the interest rate with a financial calculator we now obtain an incremental borrowing cost of 17.96 percent.

Problem 6-2
(a)
For this problem we need to know the effective cost of the $180,000 loan at 9% combined with the $40,000 loan at 13%

	Loan Amount	Interest Rate	Loan Term	Monthly Payments
	$180,000	9%	20 yrs..	$1619.51
	40,000	13%	20 yrs..	468.63
Combined	$220,000			$2,088.14

i(n,PV,PMT,FV)

n = 240
PMT = $2,088.14
PV = -$220,000
FV = $0
Solve for the *annual* interest rate:
i = 9.76%

Solving for the effective cost of the combined loans we obtain 9.76%. This is greater than the 9.5% rate on the single $220,000 loan. Thus the $220,000 loan is preferable.

(b) **REV**
We now need the loan balance after 5 years

	Loan Amount	Interest Rate	Loan Term	Monthly Payments	Loan Balance
	$180,000	9%	20 yrs..	$1619.51	$159,672.44
	40,000	13%	20 yrs..	468.63	37,038.81
Combined	$220,000			$2,088.14	**$196,711.25**

i(n,PV,PMT,FV)

n = 60
PMT = $2,088.14
PV = -$220,000
FV = $196,711.25
Solve for the *annual* interest rate:
i = 9.74%

Solving for the interest rate, which represents the combined cost, we obtain 9.74%. The effective cost of the single $220,000 would still be 9.5% even if it is repaid after 5 years because there were no points or prepayment penalties. Thus the $220,000 loan is still better.

(c)
Assuming the loan is held for the full term (to compare with Part a:)

	Loan Amount	Interest Rate	Loan Term	Monthly Payments
	$180,000	9%	20 yrs..	$1619.51
	40,000	13%	10 yrs..	597.24
Combined	$220,000			$2,216.75

The combined payments are made for the *first 10 years* only. After that, only the payment on the $180,000 loan is made.

(c) **IRR(CF1, CF2, ….CFn)**

CF_j	n_j
-$220,000.00	
2,216.75	n = 12
2,216.75	n = 12
2,216.75	n = 12
2,216.75	n = 12
2,216.75	n = 12
2,216.75	n = 12
2,216.75	n = 12
2,216.75	n = 12
2,216.75	n = 12
2,216.75	n = 12
1,619.51	n = 12
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1,619.51	n = 12
1,619.51	n = 12
1,619.51	n = 12
1,619.51	n = 12

Solve for the IRR:

= 0.79% x 12 = 9.49% (annual rate, compounded monthly)

Note that the payment of $1,619.51 is first discounted as a 10 year annuity (years 11 to 20) and further discounted as a lump sum for 10 years to recognized the fact that the annuity does not start until year 10. When calculating the IRR in excel input the monthly payment (annuity) in each cell for each period as opposed to one lump annual payment amount.

Solving for the cost we obtain 9.49%. This is less than 9.5% rate for the single $220,000 loan. Thus, the combined loans are preferred.

Assuming the loan is held for 5 years (to compare with Part b):
We now need the loan balance after 5 years.

	Loan Amount	Interest Rate	Loan Term	Monthly Payments	Loan Balance
	$180,000	9%	20 yrs..	$1619.51	$159,672.68
	40,000	13%	10 yrs..	597.24	26,248.89
Combined	$220,000			$2,216.75	$185,921.57

i(n,PV,PMT,FV)

n = 60
PMT = $2,216.75
PV = -$220,000
FV = $185,921.57
Solve for the *annual* interest rate:
i = 9.67%

We now obtain 9.67%. This is greater than the 9.5% rate for a single loan.

Problem 6-3
Preliminary calculation:

The existing loan is for $95,000 at a 11% interest rate for 30 years (monthly payments). The monthly payment is $904.71. The balance of the loan after 5 years is $92,306.41.

Payment on a new loan for $92,306.41 at a 10% rate with a 25 year term are $838.79.

(a)

Alternative	Interest Rate	Loan Term	Loan Amount	Monthly Payments
Old loan	11%	30 yrs..	$95,000	$904.71
New loan	10%	25 yrs..	92,306	838.79
Savings				$65.92

Cost of refinancing are $2,000 + (.03 x $92,306.41) = $4,769.19. Considering the $4,769.19 as an "investment" necessary to take advantage of the lower payments resulting from refinancing

i(n,PV,PMT,FV)
n = 300
PMT = $65.92
PV = -$4,769.19
FV = $0
Solve for the *annual* interest rate:
i = 16.30%

Solving for the rate we obtain 16.30%. It is desirable to refinance if the investor can not get a higher yield than 16.30% on alternative investments.

Alternate solution:

Amount of new loan	$92,306.00
Cost of refinancing	$4,769.19
Net proceeds	$87,536.81

The net proceeds can be compared with the payment on the new loan to obtain an effective cost of the new loan. We have:

i(n,PV,PMT,FV)

n	=	300
PMT	=	$838.79
PV	=	-$87,536.81
FV	=	$0

Solve for the *annual* interest rate:

i	=	10.70%

Solving for the effective cost we obtain 10.70%. Because the effective cost is *less* than the cost of the existing loan (11%) the conclusion is to refinance.

(b)
For a 5-year holding period we must also consider the balance of the old and new loan after 5 years. We have:

Alternative	Loan Amount	Interest Rate	Loan Term	Monthly Payments	Loan Balance
Old loan	$95,000	11%	30 yrs..	$904.71	$87,648.82*
New loan	92,306	10%	25 yrs..	838.79	86,918.44
				65.92	$730.38

* Balance after 5 *additional* years or 10 years total.

Looking at the refinancing cash outflows as an investment we have:

i(n,PV,PMT,FV)

n	=	60
PMT	=	$65.92
PV	=	-$4,769.19
FV	=	$730.38

Solve for the *annual* interest rate:

i	=	-0.60%

Solving for the IRR we obtain -0.60%.

The negative return tells you this is a *bad investment* if the new loan is paid off so quickly. The reason for the negative return, is that you pay for the refinancing up-front, but do not benefit from the lower monthly payments on the new financing for a period of time long enough to cover and/or justify the cost of refinancing.

Alternative solution:
The effective cost is now as follows:

i(n,PV,PMT,FV)

n	=	60
PMT	=	$838.79
PV	=	-$87,536.81
FV	=	$86,918.44

Solve for the *annual* interest rate:

i	=	11.39%

Solving for the effective cost we obtain 11.39%. The effective cost is now higher than the rate of return on the old loan (11%) so refinancing is not desirable.

Problem 6-4

Payments on the $140,000 loan at 10%, 30 years are $1,228.60 per month.

(a)

Note that there are 25 years remaining.

The balance after 5 years can be found by discounting the remaining payments as follows:

PV (n,i,PMT,FV)

n	=	300
PMT	=	$1,228.60
i	=	10%
FV	=	$0

Solve for the *annual* interest rate:

PV	=	$135,204.03

The market value of the loan can be found by discounting the payments of $1,228.60 for 25 years (monthly) using the required rate of 11%. We have:

PV (n,i,PMT,FV)

n	=	300
PMT	=	$1,228.60
i	=	11%/12
FV	=	$0

Solve for the *annual* interest rate:

PV	=	$125,352.88

This is *lower* than the balance of the loan because payments are discounted at a higher rate than the contract rate on the loan.

(b)

The balance of the original loan after five *additional* years (10 years from origination) is $127,313.21.

To calculate the market value assuming the loan is repaid after 5 additional years, we have:

PV (n,i,PMT,FV)

n	=	300
PMT	=	$1,228.60
i	=	11%
FV	=	$127,313.21

Solve for the *annual* interest rate:

PV	=	$130,144.64

Problem 6-5

(a)

Alternative 1: Purchase of $150,000 home:

	Interest Rate	Loan Term	Loan Amount	Monthly Payments
First mortgage	10.5%	20 yrs..	$120,000	$1,198.06

or

Alternative 2: Purchase of $160,000 home:

	Interest Rate	Loan Term	Loan Amount	Monthly Payments
Assumption	9%	20 yrs..	$100,000	$899.73
Second mortgage	13%	20 yrs..	20,000	234.32

			$120,000	$1,134.05

The loan amounts are the same under the two alternatives. The second alternative has lower total payments resulting in savings of $64.01 per month ($1,198.06 - $1,134.05), but requires an additional $10,000 cash outflow as an additional down payment.

i(n,PV,PMT,FV)

n	=	240
PMT	=	$64.01
PV	=	-$10,000
FV	=	$0

Solve for the *annual* interest rate:

i	=	4.64%

The IRR is 4.64%. This does not make sense if the investor can earn more than this on the $10,000. This appears to be too low to justify the additional $10,000 equity - especially with mortgage interest rates at 10.5%. Note that the borrower could take the $150,000 home and use the extra $10,000 to borrow less money, e.g. $110,000 instead of $120,000 which results in interest savings of 10.5% (the rate on the loan).

The point is that the investor's opportunity cost is 10.5%, which is higher than the 4.64% that would be earned by taking the second alternative.

Note:
It is informative to calculate exactly how much more the borrower could pay for alternative 2. This is found by discounting the payment savings at 10.5%. We have:

PV (n,i,PMT,FV)

n	=	240
PMT	=	$64.01
i	=	10.50%/12
FV	=	$0

Solve for the *annual* interest rate:

PV	=	$6,411.39

Thus, the borrower would be indifferent between alternative 1 and 2 if the price of the home for alternative 2 was $156,411.

(b)
With the homeowner providing the second mortgage for the additional $20,000 at 9% (purchase money mortgage) we have:

Alternative 2: Purchase of $160,000 home:

	Interest Rate	Loan Term	Loan Amount	Monthly Payments
Assumption	9%	20 yrs..	$100,000	$899.73
Second mortgage	9%	20 yrs..	20,000	179.95
			$120,000	$1,079.68

Savings are now $1,198.06 - $1,079.68 = $118.38 per month. An additional down payment of $10,000 is still required.

The IRR is now 13.17%

(c)
Alternative 2: Purchase of $160,000 home:

	Interest Rate	Loan Term	Loan Amount	Monthly Payments
Assumption	9%	20 yrs..	$100,000	$899.73
Second mortgage	9%	20 yrs..	30,000	269.92
			$130,000	$1,169.65

The savings are now $1,198.06 - $1,169.65 or $28.41 per month. Because of the additional amount of the second mortgage, there is no additional down payment even though $10,000 more is paid for the home. Thus, the borrower saves $28.41 under alternative 2 with no additional cash outlay- which is clearly desirable.

Problem 6-6

Loan	Amount	Payment	Term
Wraparound	$150,000	$1,800.25	15 yrs.
Existing	100,000	1,100.25	15 yrs. (remaining)
Difference	$50,000	$700.25	

$700.25 x (MPVIFA, ?%, 15 yrs..) = $50,000

i(n,PV,PMT,FV)

n = 180
PMT = $700.25
PV = -$50,000
FV = $0
Solve for the *annual* interest rate:
i = 15.01%

Solving for the IRR we obtain 15.01%. This is the incremental return on the wraparound. Because this is greater than the 14% rate on a second mortgage, the second mortgage is better.

Alternative solution:

Loan	Amount	Payment	Term
Second mortgage	$50,000	$665.87	15 yrs.
Existing loan	100,000	1,100.00	15 yrs. (remaining)
Total	$150,000	$1765.87	

The total payments on the existing loan plus a second mortgage is $1,765.87, which is less than the payments on the wraparound. Furthermore, the effective cost of the combined loans is as follows:

i(n,PV,PMT,FV)

n = 180
PMT = $1,765.87
PV = -$150,000
FV = $0
Solve for the *annual* interest rate:
i = 11.64%

The IRR is 11.64%. Thus, the effective cost of the combined loans is less than the wraparound.

Thus, the combined loans are better. Note: we can only compare payments when the loan terms are the same. However, we can compare effective costs when they differ. As a result, the effective cost is more general than simply comparing payments.

Problem 6-7

(a)

Payments on a $100,000 loan at 9% for 25 years is $839.20.

The present value of $839.20 at 9.5% for 25 years is $96,051.64.

The difference between the contract loan amount ($100,000) and the value of the loan ($96,051.64) is $3,948. This must be added on to the home price. Thus, the home would have to be sold for $110,000 + $3,948 or $113,948.

Alternative solution:

Payments on a loan for $100,000 at 9.5%:	$873.70
Payments on a loan for $100,000 at 9%:	839.20
Savings by getting the loan at 9%:	$34.50

Present value of the saving discounted at 9.5%:

PV (n,i,PMT,FV)

n	=	300
PMT	=	$34.50
i	=	9.50%
FV	=	$0

Solve for the *annual* interest rate:

PV	=	$3,948

This is the amount that has to be added to the home as before.

(b)

The balance of the $100,000 loan (9%, 25 yrs.) after 10 years is $82,739.23. We now discount the payments on the $100,000 loan which are $839.20 and the balance after 10 years which is 82,739.23. Both are discounted at the market rate of 9.5%. We have:

PV (n,i,PMT,FV)

n	=	300
PMT	=	$839.20
i	=	9.50%
FV	=	$82,739.23

Solve for the *annual* interest rate:

PV	=	$96,973

Subtracting this from the loan amount of $100,000 we have $100,000 - $96,973.69 or $3,027. This is the amount that must be added to the home price. Thus, the home price must be $110,000 + $3,027 or $113, 027. Not as much has to be added relative to (a) because the borrower would not have to be given the interest savings for as many years.

Alternative solution:

The difference in payments for a $100,000 loan at 9% and $100,000 at 9.5% is $34.50 (same as alternative solution to part a.) We must also consider the difference in loan balances after 10 years.

Balance of $100,000 loan at 9.5% after 10 years:	$83,668.75
Balance of $100,000 loan at 9% after 10 years:	82,739.23
Savings	$929.52

We now discount the payment savings and the savings after 10 years.

PV (n,i,PMT,FV)

n	=	120
PMT	=	$34.50
i	=	9.50%
FV	=	$929.52

Solve for the *annual* interest rate:

PV	=	$3,027

Thus $3,027 must be added to the home price as above.

Problem 6-8

(a)

	Amount of Reduction	Payment will be	Months
Monthly payment reduction during the first year (50% of $726.96):	$363.48	$363.48	12
Monthly payment reduction during the second year (25% of $726.96):	$181.74	$545.22	12
Discounting the payment reduction at 10% per annum (10%/12 per month)	-	$726.96	276

Solve for PV of all future monthly payments:

PV (i,PMT,n,FV)

CF_j	=	363.48
n_j	=	12
CF_j	=	181.74
n_j	=	12

Discount at i = 10% ÷ 12

And find PV = $6005.66

Note that the second year payment reduction is an annuity that starts after one year i.e. period 13.

Thus, the builder would have to give the bank $6,005.66 up front.

(b)

Based on the results from (a), the buydown loan is worth $6,005.66 in present value terms. We would expect the home to sell for $6,005.66 more than a comparable home that did not have this loan available. Thus, if the home could be purchased for $5,000 more, the borrower would gain in present value terms by $6,006 - $5,000 or $1,006.

Problem 6-9

(a) Step 1, Calculate the dollar monthly difference between the two financing options.

Original loan payment:

PV	=	-$140,000
i	=	7/12 or 0.58
n	=	15x12 or 180
FV	=	0

Solve for the payment:

PMT	=	$1,258.36

Find present value of the payments at the market rate of 8%

i	=	8%/12
n	=	15x12 or 180
FV	=	0
PMT	=	$1,258.36

Solve for PV:

PV	=	$131,675.49

This is the market (cash equivalent) value of the loan.

The buyer made a cash down payment of $60,000.

Cash equivalent value of loan	$131,675.49
Cash down payment	60,000.00
Cash equivalent value of property	$191,675.49

(b) If it is assumed that the buyer only expected to benefit from the favorable financing for five years:

Loan balance after 5 years is $108,378

Find present value of payments for 5 years and loan balance at the end of the 5th year.

i	=	8%/12
n	=	5x12 or 60
FV	=	$108,378
PMT	=	$1,258.36

Solve for PV:

PV	=	$134,804.72

Cash equivalent value of loan	$134,804.72
Cash down payment	60,000.00
Cash equivalent value of property	$194,804.72

The cash equivalent value is higher because the buyer was not assumed to have discounted the loan by as much.