Problem 4-1

A borrower makes a fully amortizing CPM mortgage loan.

Principal	=	\$125,000
Interest	=	11.00%
Term	=	10 years

CPM Payment:

The monthly payment for a CPM is found using the following formula:

Monthly payment	=	PMT (n,i,PV, FV)
Monthly payment	=	PMT (10 yrs, 11%,\$125,000, \$0)
Payment	=	\$1,721.88

If the loan maturity is increased to 30 years the payment would be:

Monthly payment	=	PMT (n,i,PV, FV)
Monthly payment	=	PMT (30 yrs, 11%,\$125,000, \$0)
Payment	=	\$1,190.40

Problem 4-2

(a) Monthly payment (PMT $(n,i,PV,FV) = 515.44	1
Solution:	

n	=	25x12 or 300
i	=	6%/12 or .50
PV	=	\$80,000
FV	=	0

Solve for payment: PMT = -\$515.44

(b) Month 1:

interest payment:		
\$80,000 x (6%/12)) = 5	\$400
principal payment:		
\$515.44 - \$400	=	\$115.44

(c) Entire 25 Year Period:

total payments:	
$515.44 \times 300 =$	\$154,632
total principal payment:	\$80,000
total interest payments:	
\$154,632 - \$80,000 =	\$74,632

(d) Outstanding loan balance if repaid at end of ten years = 66,191.38

Solution:

n	=	120 (pay off period)
i	=	6%/12 or 0.50
PMT	=	\$515.44
PV	=	\$80,000
Solve for FV:		
FV	=	\$61,081.77

(e) Trough ten years: total payments: \$515.44 x 120 = \$61,852.80 total principal payment (principal reduction): \$80,000 - 61,081.77* = \$18,918.23
*PV of loan at the end of year 10 total interest payment: \$61,852.80 - \$18,918.23 = \$42,934.57

(f) Step 1, Solve for loan balance at the end of month 49:

n	=	49	
i	=	6%/12 or 0.50	
PMT	=	\$515.44	
PV	=	- \$80,000	
Solve for loan balance:			
PV	=	\$73,608.28	

Step 2, Solve for the interest payment at month 50: interest payment: \$73,608.28 x (.06/12)=\$368.04 principal payment: \$515.44 - \$368.04 = \$147.40

Problem 4-3

Sol

(a) Monthly payment PMT (n,i,PV, FV) = \$599.55Solution: n = 30x12 or 360

n	=	30x12 or 360		
i	=	6%/12 or 0.50		
PV	=	-\$100,000		
FV	=	0		
Solve for payment:				
PMT	=	\$599.55		

(b) Quarterly Payment PMT (n,i,PV, FV) = \$1,801.85 Solution:

n	=	30x4 or 120		
i	=	6%/4 or 1.50		
PV	=	-\$100,000		
FV	=	0		
ve for payment:				
PMT	=	\$1,801.85		

(c) Annual Payment PMT (n,i,PV, FV) = \$7,264.89 Solution:

Donand	<i>,</i> ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,		
	n	=	30
	i	=	6%
	PV	=	-\$100,000
	FV	=	0
Solve t	for payme	ent:	
	PMT	=	\$7,264.89

(d) Weekly Payment (n,i,PV, FV) = 138.26					
	Solution:				
	I	1	=	52x30 o	r 1,560
	i		=	6%/52	or 0.12
	I	PV	=	-\$100,0	00
	I	FV	=	0	
	Solve for	paymer	nt:		
		PMT	=	\$138.26	i
Problem	4-4				
Monthly:					
total principal payment:				\$100,000	
total interest:					
$($599.55 \times 360) - $100,000 = $$				\$115,838	
Quarterly	:				
total principal payment:			\$100,000		
total interest:					
	(\$1,80	1.85 x	120)-\$10	0,000=	\$116,222
Annually	:				
total principal payment: \$			\$100,000		
t	total inter	est:			
	(\$7,26	4.89 x 1	30) - \$10	0,000=	\$117,946.7
Weekly:					

	(\$7,264.89 x 30) - \$100,000=	\$117,946.70
eekly:		
5	total principal payment:	\$100,000
	total interest:	
	(\$138.26 x 1560)-\$100,000 =	\$115,685.60

The greatest amount of interest payable is with the Annual Payment Plan because you are making payments less frequently. Therefore, the balance is reduced slower and interest is paid on a larger loan balance each period.

Problem 4-5 (a) Monthly P

ODICIII 4- 3					
) Monthly Pa	ayment PN	AT (n,i,P	V,FV):		
Soluti	on:				
	n	=	20x12 or 240		
	i	=	6%/12 or 0.50		
	PV	=	-\$100,000		
	FV	=	0		
Solve for payment:					
	PMT	=	\$716.43		

(b) Entire Period:

Monthly Payment PMT (n,i,PV,FV): total payment: $716.43 \times 240 = 171,943.45$ total principal payment: \$100,000 total interest: \$171,943.45-100,000 = \$71,943.45

(c) Outstanding loan balance if repaid at end of year eight = \$73,415.98 Solution:

bolution.					
n	=	96			
i	=	6%/12 or 0.50			
PMT	=	-\$716.43			
PV	=	\$100,000			
Solve for mortgage balance:					
FV	=	\$73,416.22			

Total interest collected: total payment + mortgage balance - principal \$716.43 x (8x12) + \$73,416.22 - 100,000 total interest collected = \$42,193.50

(d) Step 1, Solve for the loan balance at the end of year 8:

96 n = 6%/12 or 0.50 i = -\$716.43 PMT = PV \$100,000 = Solve for loan balance: FV \$73,416.22 =

After reducing the loan by \$5,000, the balance is: \$73,416.22 - 5,000 = \$68,416.22

(e) The new loan maturity will be 78 months after the loan is reduced at the end of year 8.

Solution:

6%/12 or 0.50 i =PMT -\$716.43 = ΡV \$68,416.22 = FV = 0 Solve for maturity: n = 78.26 (months)

(f) The new payment would be \$667.64

Solution:				
i	=	6%/12	or 0.50	
n	=	12 x12	or 144	
PV	=	\$68,416	5.22	
FV	=	0		
Solve for payment:				
PMT	=	-\$667.6	4	

Problem 4-6

Step 1, Solve for the original monthly payment:

	i	=	6%/12 or 0.50
	n	=	30x12 or 360
	PV	=	-\$75,000
	FV	=	0
Solve for	payment	:	
	PMT	=	\$449.66

Step 2, Solve for current balance: 6%/12 or 0.50 i = 20x12 or 240 = n -\$75.000 PV = \$658.18 PMT = Solve for mortgage balance: FV \$68,203.24 = (a) New Monthly Payment = \$561.67 Solution: 10%/12 or 0.83 i = n = 12x20 or 240 PV = \$58,203.24* FV = 0 Solve for payment: PMT = \$561.67 (b) New Loan Maturity = 161 months Solution: 10%/12 or 0.83 i = -\$658.18 PMT = PV \$58,203.24* = FV 0 = Solve for maturity: n = 161 *\$68,203.24 - 10,000

Problem 4-7

The loan will be repaid in 145 months. Solution: n (PMT,i,PV,FV) 6.5%/12 or 0.54 i = PMT = \$1.000 \$100,000 PV = FV = 0 Solve for maturity: 144.42 n =

Problem 4-8

The interest rate on the loan is 12.96%. Solution:

25x12 or 300 = n -\$900 PMT = PV \$80,000 = FV 0 = Solve for the *annual* interest rate: 1.08 (x12) or 12.96% i =

Problem 4-9

(a)	Monthly Paym Solution:	ents = \$	656.70	
	1	ı	=	10x12 or 120
	i		=	7%/12 or 0.58

	PV FV	=	-\$60,000 \$20,000
Solve f	for month	ily pay	ment:
	PMT	=	\$581.10
(b) Loan baland		end of	year five = \$43,454.81

n	=	5x12 or 60		
i	=	7%/12 or 0.58		
PMT	=	\$581.10		
FV	=	\$20,000		
Solve for the loan balance:				
PV	=	-\$43,454.81		

Problem 4-10) Monthl-D

	n	=	10x12 or 120
	i	=	10%/12 or 0.83333
	PV	=	-\$80,000
	FV	=	\$80,000
	Solve for month	ily pay	ments:
	PMT	=	\$666.67
(b)	Loan balance $=$ \$80,	000	

Solution:				
	n	=	12x5 or 60	
	i	=	10%/12 or 0.83333	
	PV	=	-\$80,000	
	PMT	=	\$666.67	
Solve for loan balance:				
	FV	=	\$80,000	

The solution does not have to be calculated because the loan balance will be the same as initial loan amount. This is because it is an interest only loan and there is no loan amortization or reduction of principal.

(c) Yield to the lender i(n,PV,PMT,FV) = 10%

Solution:

	n	=	12x5 or 60	
	PMT	=	\$666.67	
	PV	=	-\$80,000	
	FV	=	\$80,000	
Solve for the <i>annual</i> yield:				
	i	=	0.83333 (x12) or 10%	

(d) Yield to the lender i(n,PV,PMT,FV) = 10%Solution:

12x10 or 120 = n \$666.67 PMT = PV = -\$80,000 FV \$80,000 = Solve for the annual yield: 0.83333 (x12) or 10% i =

Problem 4-11 Monthly Payments PMT (n,i,PV,FV) = \$877.14 Solution: 10x12 or 120 n =6%/12 or 0.50 i = ΡV \$90,000 = FV = -\$20,000 Solve for monthly payments: PMT \$877.14 = Yield to the lender i(n,PV,PMT,FV) = 6.39%Solution: n = 12x10 or 120 PMT = \$877.14 PV -\$88,200* = \$20,000 FV = Solve for the *annual* yield: 6.39% i = *-\$90,000 x (100-2)% = -\$88,200 (amount disbursed) Step 1, Solve the loan balance if repaid in four years: Solution: 4x12 or 48 n = i = 6%/12 or 0.50 - \$90,000 PV = PMT = \$877.14 Solve for the loan balance: FV \$66,892.65 = Step 2, Solve for the yield: Solution: = 12x4 or 48 n PMT \$877.14 = PV -\$88,200* = FV \$66,892.65 = Solve for the annual yield: i(n,PV,PMT,FV) i = i = 6.64% $*-\$90,000 \ge (100-2)\% = -\$88,200$ Problem 4-12 (a) At the end of year ten \$94,622.86 will be due: Solution: 12x10 or 120 n = 8%/12 or 0.67 i = PV -\$50,000 = PMT = 0

Solve for loan balance: FV =\$110,982.01 (b) Step 1, the loan yield remains 8%, this can be "proved" by solving for loan balance at end of year eight. Solution:

Solution.			
n	=	8x12 or 96	
i	=	8%/12 or 0.67	
PV	=	-\$50,000	
PMT	=	0	
Solve for loan balance:			
FV	=	\$94,622.86	

Step 2, Solve for the yield:

Solution:

n = 8x12 or 96 PMT = 0 PV = -\$50,000 FV = \$94,622.86 Solve for the *annual* yield: .67 (x12) or 8% i =

Note: because there were no points, the yield must be the same as the initial interest rate of 8% so no calculations were really necessary.

(c) Yield to lender with one point charged = 8.13%

Solution:

8x12 or 96 = n 0 PMT = PV -\$49.500* = FV = \$94,622.86 Solve for the *annual* yield: .68 (x12) or 8.13% (annual rate, compounded monthly) i =

 $*-\$50,000 \ge (100-1)\% = -\$49,500$

Problem 4-13

(a)

Property value	=	\$105,000
Principal	=	\$84,000
Interest rate	=	8.00%
Maturity	=	30 years
Loan origination fee	=	\$3,500

Lender will disburse \$84,000.00 less the loan origination fee of \$3,500.00 or \$80,500.00

(b) Monthly payments are based on the loan amount of \$84,000 and would be PMT (n,i,PV,FV):

Monthly Pa	yment		=	PMT (n,i,PV,FV)
n	=	360		
i	=	8% ÷ 12		
FV	=	0		
PV	=	-\$84,000		

Monthly Payment	=	\$616.36
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Fin 5413: Chapter 04 - Fixed Interest Rate Mortgage Loans

The effective interest rate would be:

Effective Inte	erest rate		=	i(n,PV,PMT,FV)
n	=	360		
PMT	=	616.36		
FV	=	0		
PV	=	\$80,500		
Effective Inte	erest rate		=	.7045 * 12=8.45%

(c) Assuming the loan payoff occurs after 5 years, determine the mortgage balance:

Mortgage balance = PV of 300 monthly payments of \$616.36 discounted at 8.00%

		PV =	PV (n,i,PMT,FV)
n	=	60	
PMT	=	616.36	
FV	=	0	
i	=	8÷12	
		PV =	\$79,858.39

The effective interest rate would be:

n	=	60
PMT	=	\$616.36
PV	=	-\$80,500
FV	=	\$79,858.39
i	=	i(n,PV,PMT,FV)
i	=	.755 * 12 = 9.06%

The effective interest rate in this part is different from the APR because the loan origination fee is amortized over a much shorter period (5 years instead of 30 years).

(d) With a prepayment penalty of 2% on the outstanding loan balance of \$79,858.39, the penalty would be \$1,597.17.

The effective interest cost would be:

n	=	60
PMT	=	\$616.36
PV	=	-\$80,500
FV	=	\$81,455.56 (\$79,858.39+\$1,597.17)
i	=	i(n,PV,PMT,FV)
i	=	9.37%

This rate is different from the APR because penalty points are not used in the calculation of the APR.

Note: Penalty equals 79858.39 *.02 = \$1597.17

Fin 5413: Chapter 04 - Fixed Interest Rate Mortgage Loans

Problem 4-14

Points required to achieve a yield to 10% for the 25 year loan.

Monthly payments PMT (n,i,PV,FV):

n = 300 8% ÷ 12 i = PV = \$95,000 FV \$0 =Solve for monthly payments: PMT = \$733.23

PV (n,i,PMT,FV) of 300 payments of \$733.23 discounted at 10% = \$80,689.93

Subtracting \$80,689.93 from \$95,000.00, we get \$14,310.07

The loan origination fee should be \$14,310.07 if the loan is to be repaid after 25 years and the lender requires a 10% yield.

If the loan is expected to be repaid after 10 years, the loan balance at the end of 10 years must be determined:

n	=	180
i	=	8%
PMT	=	\$733.23
PV	=	\$95,000
Solve for FV:		
FV	=	\$83,423.67

Loan balance after 10 years = \$83,423.67

Discounting \$733.23 monthly for 120 months and \$83,423.67 at the end of the 120th month by the desired yield of 10% gives:

Present value = \$86,301.65

Subtracting \$86,301.65 from \$95,000.00, we get \$8,698.35.

The loan origination fee should be \$8,698.35 if the loan is to be repaid after 10 years, and the lender requires a yield of 10%.

Problem 4-15

(a) In order to find which loan is the better choice after 20 years, the effective interest rate for each loan must be calculated.

	Loan A	Loan B
Principal	\$75,000	\$75,000
Nominal interest rate	6.00%	7.00%
Term (years)	30	30
Points	6	2
Payment	\$449.66	\$498.98
Loan Balance after 20 years	\$40,502.43	\$42,975.33
Loan Balance after 5 years	\$69,790.32	\$70,599.14

Loan A

n	=	240
PMT	=	\$449.66
PV	=	-\$70,500
FV	=	\$40,502.43
i	=	i(n,PV,PMT,FV)
i	=	.5525% * 12 = 6.63%

Loan B

n	=	240
PMT	=	\$498.98
PV	=	-\$73,500
FV	=	\$42,975.33
i	=	i(n,PV,PMT,FV)
i	=	.6008% * 12 = 7.21%

Loan A is the better alternative if the loan is repaid after 20 years.

(b) This part is solved the same as (a) except using the assumption that the loan is repaid after 5 years.

Loan A

			Note: Balance at the end of $60 \text{ months} = \$69,790.32$
n	=	60	
PMT	=	\$449.66	
PV	=	-\$70,500	
FV	=	\$69,790.32	
i	=	i(n,PV,PMT,FV)	
i	=	.623917% * 12 = 7.49%	
			Note: Balance at the end
			of 60 months $=$ \$70,599.14

Loan B

n	=	60
PMT	=	\$498.98
PV	=	-\$73,500
FV	=	\$70,599.14
i	=	i(n,PV,PMT,FV)
i	=	.624417 * 12 = 7.49%

The borrower would be indifferent between the two loans if the repayment period is 5 years.

Problem 4-16

(a) Monthly Payments = \$1,382.50 to be made to the borrower Solution: 10x12 or 120 n = 11%/12 or 0.92 i = PV = 0 FV = -\$300,000 Solve for monthly payments:

PMT = \$1,382.50 (b) The borrower will have received monthly payments of 1,382.50 during months 1 to 36

Solve for loan balance at the end of month 36

Solution:				
n	=	36		
i	=	11%/12 or 0.92		
PV	=	0		
PMT	=	\$1,382.50		
Solve for loan balance*:				
FV	=	-\$58,649.97		

*Note that this is equivalent to finding the Future Value of a \$1382.50 monthly ordinary annuity at an annual rate of 11%, compounded monthly.

(c) The borrower will receive \$2,000 per month for 50 months and then will receive monthly payments of \$626.22 during months 51 to 120. This is calculated as follows:

Step 1, Solve for loan balance at the end of month 50

Solution:

n	=	50	
i	=	11%/12 or 0.92	
PV	=	0	
PMT	=	\$2,000	
Solve for loan balance at the end of month 50:			
FV	=	-\$126,139.10	

Step 2, Solve for payments during months 51 to 120

Solution:

n	=	120-50 or 70		
i	=	11%/12 or 0.92		
PV	=	\$126,139.10		
FV	=	-\$300,000		

Solve for monthly payments beginning in month 51 through 120 or for the next 70 months: PMT = \$626.22

Problem 4-17

Find the balance at the end of 5 years for a fully amortizing \$200,000, 10% mortgage with a 25 year amortization schedule:

PV	= -200,000	FV	= 0
i	$= 10\% \div 12$	Solve PMT	= \$1,817.40
n	= 300		

Solve for balance at end of 5 years:

i	= 10%	PMT	= \$1,817.40
n	=240	FV	= 0
		Solve PV	= -188,327.38

Problem 4-18

CAM loan:

(a) Calculate constant monthly amortization:

 $125,000 \div 240 \text{ months} = 520.83 \text{ per month}$

Fin 5413: Chapter 04 - Fixed Interest Rate Mortgage Loans

	Beg.					
Month	Balance	Rate	Interest	Amortization	Total Payment	End Balance
1	125,000	*11%/12	1,145.83	520.83	1,666.66	124,479.17
2	124,479.17	*11%/12	1,141.05	520.83	1,661.88	123,958.34
3	123,958.34	*11%/12	1,136.28	520.83	1,657.11	123,437.51
4	123,437.51	*11%/12	1,131.51	520.83	1,652.34	122,916.68
5	122,916.68	*11%/12	1,126.74	520.83	1,647.57	122,395.85
6	122,395.85	*11%/12	1,121.96	520.83	1,642.79	121,875.02

(b) For a constant payment loan (CPM) we have:

PV	=	-\$125,000		
n	=	240		
i	=	11% ÷12		
FV	=	0		
Solve PMT = \$1,290.24				

(c) <u>In the absence of point and origination fees</u>, the effective interest rates on both loans will be an annual rate of 11%, compounded monthly. This is true regardless of when either of the loans are repaid. Monthly payments are different, however i is the same for both loans.

Problem 4-19

(a) Determine monthly payments based on interest being accrued daily. Solve for interest due at the end of month one:

PV	=	\$50,000
i	=	6% ÷ 365
n	=	30
Solve for FV		
FV	=	\$50,247.16

Because this is an "interest only" loan, payments of \$247.16 will be due at the end of each month for 360 months.

(b) The loan balance will be \$50,000 at the end of each month for the life of the loan. At the end of 30 years it also will be \$50,000.

(c) The equivalent annual rate will be:

FV	/ =	\$50,000
n	=	360
PV	=	-\$50,000
PN	1T =	247.16
Solve for i :	= .4943 * 1	2 = 5.93% (annual rate, compounded monthly)

Or
$$\frac{\$50,247.16 - \$50,000}{\$50,000} = .4943 * 12 = 5.93\%$$

<u>Interpretation</u>: A loan could be made at an annual interest rate of 5.93%, compounded monthly, which would be equivalent to a loan made at an annual rate of 6%, compounded daily.

Problem 4- 20 Comprehensive Review Problem

A.4

Loan = A.	100,000, 12% interest, 20 years Monthly payments if (1) Fully amortizing:					
	P' i	V = -100,000	n <u>Solve</u> PMTs	= 240 = \$1,101.09		
	P' i	artial amortizing: V = -100,000 = 12% V = \$50,000	n <u>Solve</u> PMTs	= 240 = \$1,050.54		
	P' i	tterest only V = 100,000 = 12% V = 100,000	n <u>Solve</u> PMTs	= 240 = \$1,000.00		
	P' i	egative amortization: V = -100,000 = 12% V = 150,000	n <u>Solve</u> PMTs	= 240 = \$949.46		
В.	B. Loan Balances for A.1. – A.4 after 5 years					
	A.1	PMTs = 1,101.09 i = 12%	FV <u>Solve</u> PV	= 0 = \$91,744.33		
	A.2	PMTs = 1,050.54 i = 12% n = 180	FV <u>Solve</u> PV	= 50,000 = \$95,872.16		
	A.3	PMTs = 1,000.00 i = 12% n = 180	FV <u>Solve</u> PV	= 100,000 = 100,000		
	A.4	PMTs = \$949.46 i = 12% n = 180	FV <u>Solve</u> PV	= 150,000 = 104,127		
C.	Interest at the end of month 61 for $A.1 - A.4$					
	A.1 A.2 A.3 A.4	\$91,744.33 * .01 \$95,872.16 * .01 \$100,000.00 * .01 \$104,127.84 * .01	= \$ 917.44 = \$ 958.72 = \$1,000.00 = \$1,041.28			
D.	APR* for loans in A.1 – A.4					
A.1 $PV = -97,000, PMT = 1,101.09, FV = 0, n = 240$ Solve A.2 $PV = -97,000, PMT = 1,050.54, FV = 50,000, n = 240$ A.3 $PV = -97,000, PMT = 1,000.00, FV = 100,000, n = 240$ A.4 $PV = -97,000, PMT = 0.40, 46, FV = 150,000, n = 240$			000, n = 240 <u>Solve</u> i = 12.44 ,000, n = 240 <u>Solve</u> i = 12.41			

PV = -97,000, PMT = 949.46, FV = 150,000, n = 240 Solve i = 12.375

*Solution shown based on calculation – final answers may be rounded to nearest 1/4%

E. Effective yield if loan prepaid EOY₅. Balances must be calculated at EOY₅ for each loan (not shown).

A.1	PV = -97,000, PMT = 1,101.09, FV = 91,744.33 n = 60 <u>Solve</u> i = 12.84
A.2	PV = -97,000, PMT = 1,050.54, FV = 95,872.16 n = 60 <u>Solve</u> i = 12.83
A.3	PV = -97,000, PMT = 1,000.00, FV = 100,000.00 n = 60 <u>Solve</u> i = 12.82
A.4	PV = -97,000, PMT = 949.46, FV = 104,127.00 n = 60 Solve i = 12.80

F. "Interest only" monthly payments in A.1 = $100,000 * (12\% \div 12)$ or 1,000 per month for 36 mos. What must payments be from yr. 4-17 to fully amortize the loan at the end of 240 mos.?

 $\frac{Part 1}{PV} = -100,000$ i = 12% n = 36 PMT = \$1,000 <u>Solve</u> FV = \$100,000

 $\begin{array}{l} \underline{Part \ 2}:\\ PV = -100,000\\ i = 12\% \ \div \ 12\\ n = 204\\ FV = 0\\ \underline{Solve} \ PMT = \$1,151.22 \end{array}$

G. (1) Total PMTs = (949.46 * 240) + 150,000 = \$377,870 Principal = 100,000 Interest = 277,870

(2) $n = 204$	FV	= 150,000
PMTs = 949.46	i	= 12%
	Solve	PV = 102,177 balance

(3) 12% because there are no points

(4) 4 points charged, loan payoff 36 months, what is effective interest rate?

PV = -96,000	PMT	= 949.46
n = 36	Solve i	= 1.13% * 12 = 13.62%
FV = 102,177		