# Solutions to Problems - Chapter 4 Fixed Interest Rate Mortgage Loans 

## Problem 4-1

A borrower makes a fully amortizing CPM mortgage loan.

| Principal | $=$ | $\$ 125,000$ |
| :--- | :--- | :--- |
| Interest | $=$ | $11.00 \%$ |
| Term | $=$ | 10 years |

CPM Payment:
The monthly payment for a CPM is found using the following formula:

| Monthly payment | $=$ | PMT (n,i,PV, FV) |
| :--- | :--- | :--- |
| Monthly payment | $=$ | PMT (10 yrs, 11\%,\$125,000, \$0) |
| Payment | $=$ | $\$ 1,721.88$ |

If the loan maturity is increased to 30 years the payment would be:

| Monthly payment | $=$ | PMT (n,i,PV, FV) |
| :--- | :--- | :--- |
| Monthly payment | $=$ | PMT (30 yrs, 11\%,\$125,000, \$0) |
| Payment | $=$ | $\$ 1,190.40$ |

## Problem 4-2

(a) Monthly payment (PMT (n,i,PV, FV) $=\$ 515.44$

Solution:

$$
\begin{array}{lll}
\mathrm{n} & = & 25 \times 12 \text { or } 300 \\
\mathrm{i} & = & 6 \% / 12 \text { or } .50 \\
\mathrm{PV} & = & \$ 80,000 \\
\mathrm{FV} & = & 0
\end{array}
$$

Solve for payment:

$$
\text { PMT }=-\$ 515.44
$$

(b) Month 1:
interest payment:
$\$ 80,000 \times(6 \% / 12)=\$ 400$
principal payment:
$\$ 515.44-\$ 400=\$ 115.44$
(c) Entire 25 Year Period:
total payments:
$\$ 515.44 \times 300=\$ 154,632$
total principal payment: $\$ 80,000$
total interest payments:
$\$ 154,632-\$ 80,000=\$ 74,632$
(d) Outstanding loan balance if repaid at end of ten years $=\$ 66,191.38$ Solution:

| n | $=$ | 120 (pay off period) |
| :--- | :--- | :--- |
| i | $=$ | $6 \% / 12$ or 0.50 |
| PMT | $=$ | $\$ 515.44$ |
| PV | $=$ | $\$ 80,000$ |
| FV: |  |  |
| FV | $=$ | $\$ 61,081.77$ |

(e) Trough ten years:
total payments:
$\$ 515.44 \times 120=\$ 61,852.80$
total principal payment (principal reduction):
$\$ 80,000-61,081.77^{*}=\$ 18,918.23$
*PV of loan at the end of year 10 total interest payment:
$\$ 61,852.80-\$ 18,918.23=\$ 42,934.57$
(f) Step 1, Solve for loan balance at the end of month 49:

| n | $=$ | 49 |
| :--- | :--- | :--- |
| i | $=$ | $6 \% / 12$ or 0.50 |
| PMT | $=$ | $\$ 515.44$ |
| PV | $=$ | $-\$ 80,000$ |

Solve for loan balance:
$\mathrm{PV}=\quad \$ 73,608.28$
Step 2, Solve for the interest payment at month 50: interest payment:
$\$ 73,608.28 \times(.06 / 12)=\$ 368.04$
principal payment:
$\$ 515.44-\$ 368.04=\$ 147.40$
Problem 4-3
(a) Monthly payment PMT (n,i,PV, FV) $=\$ 599.55$

Solution:

| n | $=$ | $30 \times 12$ or 360 |
| :--- | :--- | :--- |
| i | $=$ | $6 \% / 12$ or 0.50 |
| PV | $=$ | $-\$ 100,000$ |
| FV | $=$ | 0 |

Solve for payment:
PMT $=\$ 599.55$
(b) Quarterly Payment PMT (n,i,PV, FV) $=\$ 1,801.85$

Solution:

| n | $=$ | $30 \times 4$ or 120 |
| :--- | :--- | :--- |
| i | $=$ | $6 \% / 4$ or 1.50 |
| PV | $=$ | $-\$ 100,000$ |
| FV | $=$ | 0 |

Solve for payment:
PMT $=\quad \$ 1,801.85$
(c) Annual Payment PMT (n,i,PV, FV) $=\$ 7,264.89$

Solution:

| n | $=$ | 30 |
| :--- | :--- | :--- |
| i | $=$ | $6 \%$ |
| PV | $=$ | $-\$ 100,000$ |
| FV | $=$ | 0 |

Solve for payment:
PMT $=\quad \$ 7,264.89$
(d) Weekly Payment (n,i,PV, FV) $=\$ 138.26$

Solution:

| n | $=$ | $52 \times 30$ or 1,560 |
| :--- | :--- | :--- |
| i | $=$ | $6 \% / 52$ or 0.12 |
| PV | $=$ | $-\$ 100,000$ |
| FV | $=$ | 0 |

Solve for payment:
PMT = $\$ 138.26$

## Problem 4-4

Monthly:
total principal payment: \$100,000
total interest:
$(\$ 599.55 \times 360)-\$ 100,000=\$ 115,838$
Quarterly:
total principal payment: \$100,000
total interest:
$(\$ 1,801.85 \times 120)-\$ 100,000=\$ 116,222$
Annually:
total principal payment: \$100,000
total interest:
$(\$ 7,264.89 \times 30)-\$ 100,000=\$ 117,946.70$
Weekly:
total principal payment: \$100,000
total interest:
$(\$ 138.26 \times 1560)-\$ 100,000=\$ 115,685.60$
The greatest amount of interest payable is with the Annual Payment Plan because you are making payments less frequently. Therefore, the balance is reduced slower and interest is paid on a larger loan balance each period.

## Problem 4-5

(a) Monthly Payment PMT (n,i,PV,FV):

Solution:

| n | $=$ | $20 \times 12$ or 240 |
| :--- | :--- | :--- |
| i | $=$ | $6 \% / 12$ or 0.50 |
| PV | $=$ | $-\$ 100,000$ |
| FV | $=$ | 0 |

Solve for payment:
PMT $=\quad \$ 716.43$
(b) Entire Period:

Monthly Payment PMT (n,i,PV,FV):
total payment:
$\$ 716.43 \times 240=\$ 171,943.45$
total principal payment: $\$ 100,000$
total interest:
$\$ 171,943.45-100,000=\$ 71,943.45$
(c) Outstanding loan balance if repaid at end of year eight $=\$ 73,415.98$

Solution:

| n | $=$ | 96 |
| :--- | :--- | :--- |
| i | $=$ | $6 \% / 12$ or 0.50 |
| PMT | $=$ | $-\$ 716.43$ |
| PV | $=$ | $\$ 100,000$ |

Solve for mortgage balance:

$$
F V \quad=\quad \$ 73,416.22
$$

Total interest collected:
total payment + mortgage balance - principal $\$ 716.43$ x ( $8 \times 12$ ) $+\$ 73,416.22-100,000$
total interest collected $=\$ 42,193.50$
(d) Step 1, Solve for the loan balance at the end of year 8:

| n | $=$ | 96 |
| :--- | :--- | :--- |
| i | $=$ | $6 \% / 12$ |
| PMT | $=$ | $-\$ 716.43$ |
| PV 0.50 |  |  |
|  | $=\$ 100,000$ |  |

Solve for loan balance:
$\mathrm{FV}=\$ \$ 3,416.22$

After reducing the loan by $\$ 5,000$, the balance is:

$$
\$ 73,416.22-5,000=\$ 68,416.22
$$

(e) The new loan maturity will be 78 months after the loan is reduced at the end of year 8 .

| Solution: |  |  |
| :---: | :--- | :--- |
| i | $=$ | $6 \% / 12$ or 0.50 |
| PMT | $=$ | $-\$ 716.43$ |
| PV | $=$ | $\$ 68,416.22$ |
| FV | $=$ | 0 |

Solve for maturity:
$\mathrm{n} \quad=\quad 78.26$ (months)
(f) The new payment would be $\$ 667.64$

Solution:
i $=6 \% / 12$ or 0.50
$\mathrm{n} \quad=\quad 12 \times 12$ or 144
$\mathrm{PV}=\$ \quad \$ 68,416.22$
$\mathrm{FV}=0$
Solve for payment:
PMT $=-\$ 667.64$
Problem 4-6
Step 1, Solve for the original monthly payment:

| i | $=$ | $6 \% / 12$ or 0.50 |
| :--- | :--- | :--- |
| n | $=$ | $30 \times 12$ or 360 |
| PV | $=$ | $-\$ 75,000$ |
| FV | $=$ | 0 |

Solve for payment:
PMT $=\$ 449.66$

Step 2, Solve for current balance:

| i | $=$ | $6 \% / 12$ or 0.50 |
| :--- | :--- | :--- |
| n | $=$ | $20 \times 12$ or 240 |
| PV | $=$ | $-\$ 75,000$ |
| PMT | $=\$ 658.18$ |  |

Solve for mortgage balance:
FV $=\$ 68,203.24$
(a) New Monthly Payment $=\$ 561.67$

Solution:

| i | $=$ | $10 \% / 12$ or 0.83 |
| :--- | :--- | :--- |
| n | $=$ | $12 \times 20$ or 240 |
| PV | $=$ | $\$ 58,203.24^{*}$ |
| FV | $=$ | 0 |

Solve for payment:

```
        PMT = $561.67
```

(b) New Loan Maturity = 161 months

Solution:

| i | $=$ | $10 \% / 12$ or 0.83 |
| :--- | :--- | :--- |
| PMT | $=$ | $-\$ 658.18$ |
| PV | $=$ | $\$ 58,203.24^{*}$ |
| FV | $=$ | 0 |

Solve for maturity:
n =
161
*\$68,203.24-10,000

## Problem 4-7

The loan will be repaid in 145 months.
Solution: n (PMT,i,PV,FV)
i $\quad=\quad 6.5 \% / 12$ or 0.54
PMT $=\$ 1,000$
$\mathrm{PV}=\$ 100,000$
FV $=0$
Solve for maturity:
$\mathrm{n} \quad=\quad 144.42$

Problem 4-8
The interest rate on the loan is $12.96 \%$.
Solution:

| n | $=$ | $25 \times 12$ or 300 |
| :--- | :--- | :--- |
| PMT | $=$ | $-\$ 900$ |
| PV | $=$ | $\$ 80,000$ |
| FV | $=$ | 0 |

Solve for the annual interest rate:

$$
\mathrm{i} \quad=\quad 1.08(\mathrm{x} 12) \text { or } 12.96 \%
$$

## Problem 4-9

(a) Monthly Payments $=\$ 656.70$

Solution:

$$
\begin{array}{lll}
\mathrm{n} & = & 10 \times 12 \text { or } 120 \\
\mathrm{i} & = & 7 \% / 12 \text { or } 0.58
\end{array}
$$

$$
\begin{array}{lll}
\mathrm{PV} & = & -\$ 60,000 \\
\mathrm{FV} & = & \$ 20,000
\end{array}
$$

Solve for monthly payment:

```
PMT = $581.10
```

(b) Loan balance at the end of year five $=\$ 43,454.81$

Solution:

$$
\begin{array}{lll}
\mathrm{n} & = & 5 \times 12 \text { or } 60 \\
\mathrm{i} & = & 7 \% / 12 \text { or } 0.58 \\
\text { PMT } & = & \$ 581.10 \\
\text { FV } & = & \$ 20,000
\end{array}
$$

Solve for the loan balance:

$$
P V \quad=\quad-\$ 43,454.81
$$

## Problem 4-10

(a) Monthly Payments = \$666.67

Solution:

| n | $=$ | $10 \times 12$ or 120 |
| :--- | :--- | :--- |
| i | $=$ | $10 \% / 12$ or 0.83333 |
| PV | $=$ | $-\$ 80,000$ |
| FV | $=$ | $\$ 80,000$ |

Solve for monthly payments:

$$
\text { PMT }=\quad \$ 666.67
$$

(b) Loan balance $=\$ 80,000$

Solution:

| n | $=$ | $12 \times 5$ or 60 |
| :--- | :--- | :--- |
| i | $=$ | $10 \% / 12$ or 0.83333 |
| PV | $=$ | $-\$ 80,000$ |
| PMT | $=$ | $\$ 666.67$ |

Solve for loan balance:

```
FV = $80,000
```

The solution does not have to be calculated because the loan balance will be the same as initial loan amount. This is because it is an interest only loan and there is no loan amortization or reduction of principal.
(c) Yield to the lender $\mathrm{i}(\mathrm{n}, \mathrm{PV}, \mathrm{PMT}, \mathrm{FV})=10 \%$

Solution:

| n | $=$ | $12 \times 5$ or 60 |
| :--- | :--- | :--- |
| PMT | $=$ | $\$ 666.67$ |
| PV | $=$ | $-\$ 80,000$ |
| FV | $=$ | $\$ 80,000$ |

Solve for the annual yield:
$\mathrm{i}=0.83333(\mathrm{x} 12)$ or $10 \%$
(d) Yield to the lender i(n,PV,PMT,FV) $=10 \%$

Solution:

| n | $=$ | $12 \times 10$ or 120 |
| :--- | :--- | :--- |
| PMT | $=$ | $\$ 666.67$ |
| PV | $=$ | $-\$ 80,000$ |
| FV | $=$ | $\$ 80,000$ |

Solve for the annual yield:
$\mathrm{i}=0.83333(\mathrm{x} 12)$ or $10 \%$

## Problem 4-11

Monthly Payments PMT (n,i,PV,FV) $=\$ 877.14$
Solution:

| n | $=$ | $10 \times 12$ or 120 |
| :--- | :--- | :--- |
| i | $=$ | $6 \% / 12$ or 0.50 |
| PV | $=$ | $\$ 90,000$ |
| FV | $=$ | $-\$ 20,000$ |

Solve for monthly payments:

```
PMT = $877.14
```

Yield to the lender $\mathrm{i}(\mathrm{n}, \mathrm{PV}, \mathrm{PMT}, \mathrm{FV})=6.39 \%$
Solution:

| n | $=$ | $12 \times 10$ or 120 |
| :--- | :--- | :--- |
| PMT | $=$ | $\$ 877.14$ |
| PV | $=$ | $-\$ 88,200^{*}$ |
| FV | $=$ | $\$ 20,000$ |

Solve for the annual yield:
$\mathrm{i}=6.39 \%$
*-\$90,000 x (100-2)\% = $-\$ 88,200$ (amount disbursed)

Step 1, Solve the loan balance if repaid in four years:
Solution:

$$
\begin{array}{lll}
\mathrm{n} & = & 4 \times 12 \text { or } 48 \\
\mathrm{i} & = & 6 \% / 12 \text { or } 0.50 \\
\text { PV } & = & -\$ 90,000 \\
\text { PMT } & = & \$ 877.14
\end{array}
$$

Solve for the loan balance:

$$
F V \quad=\quad \$ 66,892.65
$$

Step 2, Solve for the yield:
Solution:

| n | $=$ | $12 \times 4$ or 48 |
| :--- | :--- | :--- |
| PMT | $=$ | $\$ 877.14$ |
| PV | $=$ | $-\$ 88,200^{*}$ |
| FV | $=$ | $\$ 66,892.65$ |

Solve for the annual yield:

| i | $=$ | $\mathrm{i}(\mathrm{n}, \mathrm{PV}, \mathrm{PMT}, \mathrm{FV})$ |
| :--- | :--- | :--- |
| i | $=$ | $6.64 \%$ |

*-\$90,000 x (100-2) \% = $-\$ 88,200$

## Problem 4-12

(a) At the end of year ten $\$ 94,622.86$ will be due:

Solution:

$$
\begin{array}{lll}
\mathrm{n} & = & 12 \times 10 \text { or } 120 \\
\mathrm{i} & = & 8 \% / 12 \text { or } 0.67 \\
\mathrm{PV} & = & -\$ 50,000 \\
\text { PMT } & =0
\end{array}
$$

Solve for loan balance:
$\mathrm{FV} \quad=\quad \$ 110,982.01$
(b) Step 1, the loan yield remains 8\%, this can be "proved" by solving for loan balance at end of year eight.

Solution:

| n | $=$ | $8 \times 12$ or 96 |
| :--- | :--- | :--- |
| i | $=$ | $8 \% / 12$ or 0.67 |
| PV | $=$ | $-\$ 50,000$ |
| PMT | $=$ | 0 |

Solve for loan balance:

```
FV = $94,622.86
```

Step 2, Solve for the yield: Solution:

| n | $=$ | $8 \times 12$ or 96 |
| :--- | :--- | :--- |
| PMT | $=$ | 0 |
| PV | $=$ | $-\$ 50,000$ |
| FV | $=$ | $\$ 94,622.86$ |

Solve for the annual yield:
$\mathrm{i} \quad=\quad .67(\mathrm{x} 12)$ or $8 \%$
Note: because there were no points, the yield must be the same as the initial interest rate of $8 \%$ so no calculations were really necessary.
(c) Yield to lender with one point charged $=8.13 \%$

Solution:

| $n$ | $=$ | $8 \times 12$ or 96 |
| :--- | :--- | :--- |
| PMT | $=$ | 0 |
| PV | $=$ | $-\$ 49,500^{*}$ |
| FV | $=$ | $\$ 94,622.86$ |

Solve for the annual yield:

$$
\mathrm{i}=\quad=\quad 68(\mathrm{x} 12) \text { or } 8.13 \% \text { (annual rate, compounded monthly) }
$$

*-\$50,000 x (100-1)\% = $-\$ 49,500$

## Problem 4-13

(a)

| Property value | $=$ | $\$ 105,000$ |
| :--- | :--- | :---: |
| Principal | $=$ | $\$ 84,000$ |
| Interest rate | $=$ | $8.00 \%$ |
| Maturity | $=$ | 30 years |
| Loan origination fee $=$ | $\$ 3,500$ |  |

Lender will disburse $\$ 84,000.00$ less the loan origination fee of $\$ 3,500.00$ or $\$ 80,500.00$
(b) Monthly payments are based on the loan amount of $\$ 84,000$ and would be PMT (n,i,PV,FV):

| Monthly Payment |  |  |
| :---: | :---: | :--- |
| n | $=$ | 360 |
| i | $=$ | $8 \% \div 12$ |
| FV | $=$ | 0 |
| PV | $=$ | $-\$ 84,000$ |

Monthly Payment $=\quad \$ 616.36$

The effective interest rate would be:

| Effective Interest rate |  | $=$ | $i(n, P V, P M T, F V)$ |
| :---: | :--- | :--- | :--- |
| n | $=$ | 360 |  |
| PMT $=$ | 616.36 | 0 |  |
| FV $=$ | $\$ 80,500$ |  |  |
| PV $=$ | $=$ | $.7045 * 12=8.45 \%$ |  |

(c) Assuming the loan payoff occurs after 5 years, determine the mortgage balance:

Mortgage balance $=\mathrm{PV}$ of 300 monthly payments of $\$ 616.36$ discounted at $8.00 \%$

|  |  | PV | = | PV (n,i,PMT,FV) |
| :---: | :---: | :---: | :---: | :---: |
| n | $=$ | 60 |  |  |
| PMT | = | 616.36 |  |  |
| FV | = | 0 |  |  |
| i | = | $8 \div 12$ |  |  |

The effective interest rate would be:

```
n = 60
PMT = $616.36
PV = -$80,500
FV = $79,858.39
i = i(n,PV,PMT,FV)
i}=\quad.755*12=9.06
```

The effective interest rate in this part is different from the APR because the loan origination fee is amortized over a much shorter period (5 years instead of 30 years).
(d) With a prepayment penalty of $2 \%$ on the outstanding loan balance of $\$ 79,858.39$, the penalty would be $\$ 1,597.17$.

The effective interest cost would be:

| n | $=$ | 60 |
| :--- | :--- | :--- |
| PMT | $=$ | $\$ 616.36$ |
| PV | $=$ | $-\$ 80,500$ |
| FV | $=$ | $\$ 81,455.56(\$ 79,858.39+\$ 1,597.17)$ |
| i | $=$ | i(n,PV,PMT,FV) |
| i | $=$ | $9.37 \%$ |

This rate is different from the APR because penalty points are not used in the calculation of the APR.
Note: Penalty equals 79858.39 *. 02 = \$1597.17

## Problem 4-14

Points required to achieve a yield to $10 \%$ for the 25 year loan.

Monthly payments PMT (n,i,PV,FV):

| n | $=$ | 300 |
| :--- | :--- | :--- |
| i | $=$ | $8 \% \div 12$ |
| PV | $=$ | $\$ 95,000$ |
| FV | $=$ | $\$ 0$ |

Solve for monthly payments:
PMT $=\$ 733.23$

PV (n,i,PMT,FV) of 300 payments of $\$ 733.23$ discounted at $10 \%=\$ 80,689.93$
Subtracting \$80,689.93from $\$ 95,000.00$, we get $\$ 14,310.07$
The loan origination fee should be $\$ 14,310.07$ if the loan is to be repaid after 25 years and the lender requires a $10 \%$ yield.
If the loan is expected to be repaid after 10 years, the loan balance at the end of 10 years must be determined:

| n | $=$ | 180 |
| :--- | :--- | :--- |
| i | $=$ | $8 \%$ |
| PMT | $=$ | $\$ 733.23$ |
| PV | $=$ | $\$ 95,000$ |

Solve for FV:
$F V=\$ 83,423.67$
Loan balance after 10 years $=\$ 83,423.67$

Discounting $\$ 733.23$ monthly for 120 months and $\$ 83,423.67$ at the end of the $120^{\text {th }}$ month by the desired yield of $10 \%$ gives:
Present value $=\$ 86,301.65$
Subtracting $\$ 86,301.65$ from $\$ 95,000.00$, we get $\$ 8,698.35$.
The loan origination fee should be $\$ 8,698.35$ if the loan is to be repaid after 10 years, and the lender requires a yield of $10 \%$.

## Problem 4-15

(a) In order to find which loan is the better choice after 20 years, the effective interest rate for each loan must be calculated.

|  | Loan A | $\underline{\text { Loan B }}$ |
| :--- | ---: | ---: |
| Principal | $\$ 75,000$ | $\$ 75,000$ |
| Nominal interest rate | $6.00 \%$ | $7.00 \%$ |
| Term (years) | 30 | 30 |
| Points | 6 | 2 |
| Payment | $\$ 449.66$ | $\$ 498.98$ |
| Loan Balance after 20 years | $\$ 40,502.43$ | $\$ 42,975.33$ |
| Loan Balance after 5 years | $\$ 69,790.32$ | $\$ 70,599.14$ |

Loan A

| $n$ | $=$ | 240 |
| :--- | :--- | :--- |
| PMT | $=$ | $\$ 449.66$ |
| PV | $=$ | $-\$ 70,500$ |
| FV | $=$ | $\$ 40,502.43$ |
| i | $=$ | i(n,PV,PMT,FV) |
| i | $=$ | $.5525 \% * 12=6.63 \%$ |

## Loan B

```
n = 240
PMT = $498.98
PV = -$73,500
FV = $42,975.33
i = i(n,PV,PMT,FV)
i = . 6008% * 12 = 7.21%
```

Loan A is the better alternative if the loan is repaid after 20 years.
(b) This part is solved the same as (a) except using the assumption that the loan is repaid after 5 years.

## Loan A

```
n = 60
PMT = $449.66
PV = -$70,500
FV = $69,790.32
i = i(n,PV,PMT,FV)
i = . 623917% * 12 = 7.49%
```

Loan B

Note: Balance at the end of 60 months $=\$ 69,790.32$

Note: Balance at the end of 60 months $=\$ 70,599.14$

```
n = 60
PMT = $498.98
PV = -$73,500
FV = $70,599.14
i = i(n,PV,PMT,FV)
i}=\quad.624417*12=7.49
```

The borrower would be indifferent between the two loans if the repayment period is 5 years.

## Problem 4-16

(a) Monthly Payments $=\$ 1,382.50$ to be made to the borrower

Solution:

| n | $=$ | $10 \times 12$ or 120 |
| :--- | :--- | :--- |
| i | $=$ | $11 \% / 12$ or 0.92 |
| PV | $=$ | 0 |
| FV | $=$ | $-\$ 300,000$ |

Solve for monthly payments:

```
PMT = $1,382.50
```

(b) The borrower will have received monthly payments of $\$ 1,382.50$ during months 1 to 36

Solve for loan balance at the end of month 36
Solution:

| n | $=$ | 36 |
| :--- | :--- | :--- |
| i | $=$ | $11 \% / 12$ or 0.92 |
| PV | $=$ | 0 |
| PMT | $=$ | $\$ 1,382.50$ |

Solve for loan balance*:
FV $=\quad-\$ 58,649.97$
*Note that this is equivalent to finding the Future Value of a $\$ 1382.50$ monthly ordinary annuity at an annual rate of $11 \%$, compounded monthly.
(c) The borrower will receive $\$ 2,000$ per month for 50 months and then will receive monthly payments of $\$ 626.22$ during months 51 to 120 . This is calculated as follows:
Step 1, Solve for loan balance at the end of month 50
Solution:

| n | $=$ | 50 |
| :--- | :--- | :--- |
| i | $=$ | $11 \% / 12$ or 0.92 |
| PV | $=$ | 0 |
| PMT | $=$ | $\$ 2,000$ |

Solve for loan balance at the end of month 50 :

$$
\mathrm{FV} \quad=\quad-\$ 126,139.10
$$

Step 2, Solve for payments during months 51 to 120
Solution:

$$
\begin{array}{lll}
\mathrm{n} & = & 120-50 \text { or } 70 \\
\mathrm{i} & = & 11 \% / 12 \text { or } 0.92 \\
\mathrm{PV} & = & \$ 126,139.10 \\
\mathrm{FV} & = & -\$ 300,000
\end{array}
$$

Solve for monthly payments beginning in month 51 through 120 or for the next 70 months:

```
PMT = $626.22
```


## Problem 4-17

Find the balance at the end of 5 years for a fully amortizing $\$ 200,000,10 \%$ mortgage with a 25 year amortization schedule:

| PV | $=-200,000$ |  | FV |
| :--- | :--- | :--- | :--- |
| i | $=10 \% \div 12$ |  | $=0$ |
| n | $=300$ | Solve PMT | $=\$ 1,817.40$ |

Solve for balance at end of 5 years:

$$
\begin{array}{llll}
\mathrm{i} & =10 \% & \text { PMT } & =\$ 1,817.40 \\
\mathrm{n} & =240 & \text { FV } & =0 \\
& & \text { Solve PV } & =-188,327.38
\end{array}
$$

## Problem 4-18

CAM loan:
(a) Calculate constant monthly amortization:
$\$ 125,000 \div 240$ months $=\$ 520.83$ per month

Calculate Monthly Interest:

|  | Beg. <br> Month | Balance | Rate | Interest | Amortization | Total Payment |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | End Balance |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 125,000 | $* 11 \% / 12$ | $1,145.83$ | 520.83 | $1,666.66$ |
| 2 | $124,479.17$ | $* 11 \% / 12$ | $1,141.05$ | 520.83 | $1,661.88$ |
| 3 | $123,958.34$ | $* 11 \% / 12$ | $1,136.28$ | 520.83 | $1,657.11$ |
| 4 | $123,437.51$ | $* 11 \% / 12$ | $1,131.51$ | 520.83 | $1,652.34$ |
| 5 | $122,916.68$ | $* 11 \% / 12$ | $1,126.74$ | 520.83 | $1,647.57$ |
| 6 | $122,395.85$ | $* 11 \% / 12$ | $1,121.96$ | 520.83 | $1,642.79$ |

(b) For a constant payment loan (CPM) we have:

| PV | $=$ | $-\$ 125,000$ |
| :--- | :--- | :--- |
| n | $=$ | 240 |
| i | $=$ | $11 \% \div 12$ |
| FV | $=$ | 0 |

Solve PMT = \$1,290.24
(c) In the absence of point and origination fees, the effective interest rates on both loans will be an annual rate of $11 \%$, compounded monthly. This is true regardless of when either of the loans are repaid. Monthly payments are different, however i is the same for both loans.

## Problem 4-19

(a) Determine monthly payments based on interest being accrued daily.

Solve for interest due at the end of month one:

| PV | $=$ | $\$ 50,000$ |
| :--- | :--- | :--- |
| i | $=$ | $6 \% \div 365$ |
| n | $=$ | 30 |

Solve for FV
FV $=\quad \$ 50,247.16$

Because this is an "interest only" loan, payments of $\$ 247.16$ will be due at the end of each month for 360 months.
(b) The loan balance will be $\$ 50,000$ at the end of each month for the life of the loan. At the end of 30 years it also will be $\$ 50,000$.
(c) The equivalent annual rate will be:

| FV | $=$ | $\$ 50,000$ |
| :--- | :--- | :--- |
| n | $=$ | 360 |
| PV | $=$ | $-\$ 50,000$ |
| PMT | $=$ | 247.16 |

Solve for $\mathrm{i}=.4943 * 12=5.93 \%$ (annual rate, compounded monthly)
Or $\frac{\$ 50,247.16-\$ 50,000}{\$ 50,000}=.4943 * 12=5.93 \%$

Interpretation: A loan could be made at an annual interest rate of $5.93 \%$, compounded monthly, which would be equivalent to a loan made at an annual rate of 6\%, compounded daily.

## Problem 4- 20 Comprehensive Review Problem

Loan $=100,000,12 \%$ interest, 20 years
A. Monthly payments if
(1) Fully amortizing:

| PV $=-100,000$ | $n$ | $=240$ |
| :--- | :--- | :--- |
| $i=12 \%$ |  | Solve PMTs |

(2) Partial amortizing:

| PV $=-100,000$ | $n$ | $=240$ |
| :--- | :--- | :--- |
| $i=12 \%$ | $\underline{\text { Solve PMTs }}$ | $=\$ 1,050.54$ |

$\mathrm{FV}=\$ 50,000$
(3) Interest only

| PV $=100,000$ |  | $n$ |
| :--- | :--- | :--- |
| $\mathrm{i}=12 \%$ |  | $=240$ |
| FV | $=100,000$ |  |

(4) Negative amortization:

| PV $=-100,000$ | $n$ | $=240$ |
| :--- | :--- | :--- |
| $\mathrm{i}=12 \%$ | $\underline{\text { Solve PMTs }}$ | $=\$ 949.46$ |

$F V=150,000$
Solve PMTs $=\$ 949.46$
FV 150,000
B. Loan Balances for A.1. - A. 4 after 5 years

| A. 1 | PMTs $=1,101.09$ | FV | $=0$ |
| :---: | :---: | :---: | :---: |
|  | $=12 \%$ | Solve PV | $=\$ 91,744.33$ |
| A. 2 | PMTs $=1,050.54$ | FV | = 50,000 |
|  | = $12 \%$ | Solve PV | = \$95,872.16 |
|  | $\mathrm{n}=180$ |  |  |
| A. 3 | PMTs $=1,000.00$ | FV | $=100,000$ |
|  | = $12 \%$ | Solve PV | $=100,000$ |
|  | $\mathrm{n}=180$ |  |  |
| A. 4 | PMTs $=\$ 949.46$ | FV | $=150,000$ |
|  | = $12 \%$ | Solve PV | $=104,127$ |
|  | $\mathrm{n} \quad=180$ |  |  |

C. Interest at the end of month 61 for A. 1 - A. 4

| A. 1 | $\$ 91,744.33 * .01$ | $=\$ 917.44$ |
| :--- | :--- | :--- |
| A. 2 | $\$ 95,872.16 * .01$ | $=\$ 958.72$ |
| A. 3 | $\$ 100,000.00 * .01$ | $=\$ 1,000.00$ |
| A. 4 | $\$ 104,127.84 * .01$ | $=\$ 1,041.28$ |

D. $A P R$ * for loans in A. 1 - A. 4
A. $1 \quad \mathrm{PV}=-97,000, \mathrm{PMT}=1,101.09, \mathrm{FV}=0, \mathrm{n}=240 \underline{\text { Solve } \mathrm{i}}=12.50$
A. $2 \quad \mathrm{PV}=-97,000, \mathrm{PMT}=1,050.54, \mathrm{FV}=50,000, \mathrm{n}=240 \underline{\text { Solve } \mathrm{i}}=12.44$
A. $3 \quad \mathrm{PV}=-97,000, \mathrm{PMT}=1,000.00, F V=100,000, n=240$ Solve $i=12.41$
A. $4 \quad \mathrm{PV}=-97,000, \mathrm{PMT}=949.46, \mathrm{FV}=150,000, \mathrm{n}=240 \underline{\text { Solve }} \mathrm{i}=12.375$
*Solution shown based on calculation - final answers may be rounded to nearest $1 / 4 \%$
E. Effective yield if loan prepaid EOY $_{5}$. Balances must be calculated at EOY ${ }_{5}$ for each loan (not shown).
A. $1 \quad \mathrm{PV}=-97,000, \mathrm{PMT}=1,101.09, \mathrm{FV}=91,744.33 \mathrm{n}=60 \underline{\text { Solve } \mathrm{i}}=12.84$
A. $2 \quad \mathrm{PV}=-97,000, \mathrm{PMT}=1,050.54, \mathrm{FV}=95,872.16 \mathrm{n}=60$ Solve $\mathrm{i}=12.83$
A. $3 \quad \mathrm{PV}=-97,000, \mathrm{PMT}=1,000.00, \mathrm{FV}=100,000.00 \mathrm{n}=60$ Solve $\mathrm{i}=12.82$
A. $4 \quad \mathrm{PV}=-97,000, \mathrm{PMT}=949.46, \mathrm{FV}=104,127.00 \mathrm{n}=60$ Solve $\mathrm{i}=12.80$
F. "Interest only" monthly payments in A. $1=\$ 100,000 *(12 \% \div 12)$ or $\$ 1,000$ per month for 36 mos. What must payments be from yr. 4-17 to fully amortize the loan at the end of 240 mos.?

Part 1:
PV =-100,000
$\mathrm{i}=12 \%$
n = 36
PMT = \$1,000
Solve FV = \$100,000
Part 2:
PV = $-100,000$
$\mathrm{i}=12 \% \div 12$
$\mathrm{n}=204$
$\mathrm{FV}=0$
Solve $\mathrm{PMT}=\$ 1,151.22$
G. (1) Total PMTs $=(949.46 * 240)+150,000=\$ 377,870$

Principal $=100,000$
Interest $=277,870$
(2) $n=204 \quad$ FV $=150,000$

PMTs $=949.46 \quad i \quad=12 \%$
Solve PV = 102,177 balance
(3) $12 \%$ because there are no points
(4) 4 points charged, loan payoff 36 months, what is effective interest rate?

$$
\begin{array}{ll}
\text { PV }=-96,000 & \text { PMT }=949.46 \\
\mathrm{n}=36 & \underline{\text { Solve } \mathrm{i}}=1.13 \% * 12=13.62 \%
\end{array}
$$

$$
F V=102,177
$$

