CHAPTER 9

BREAK-EVEN POINT AND COST-VOLUME-PROFIT ANALYSIS

11. a. Break-even in units = $90,000 ÷ ($70 – $40) = 3,000 units

b. In dollars break-even = 3,000 × $70 = $210,000

12. a. Break-even point in rings = $345,000 ÷ ($600 – $300) = 1,150

b. Break-even point in sales dollars = 1,150 × $600 = $690,000

c. Break-even point $345,000 ÷ ($600 – $306) = 1,174 rings (rounded)

d. Break-even point would be $339,000 ÷ ($600 – $300) = 1,130 rings

14. a. Break-even in units is $260,000 ÷ ($1,800 – $1,000) = 325 garden sheds.

b. To earn a pre-tax profit of $200,000 = ($260,000 + $200,000) ÷ $800 = 575 garden sheds

c. To earn a pre-tax profit of $280,000 = ($260,000 + $280,000) ÷ $800 = 675 garden sheds

15. a. Contribution margin per unit = Sales less variable costs
$180 – ($30 + $25 + $17) = $108

b. Contribution margin ratio = Contribution margin ÷ Sales
$108 ÷ $180 = 60%

c. Break-even in units is fixed costs ÷ Contribution margin per unit
$62,640 ÷ $108 = 580 units

d. Break-even in dollars is fixed costs ÷ Contribution margin ratio
$62,640 ÷ 0.60 = $104,400

e. To earn $51,840 in pre-tax profit, Austin Automotive must sell:
($62,640 + $51,840) ÷ $108 = 1,060 units

16. a. Contribution margin per unit = Sales less variable costs
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f. Break-even in dollars is fixed costs ÷ Contribution margin ratio
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g. To earn $51,840 in pre-tax profit, Austin Automotive must sell:
($62,640 + $51,840) ÷ $108 = 1,060 units

17. a. Convert after-tax to pre-tax profit: $182,000 ÷ (1– 0.35) = $280,000
The number of garden sheds that must be sold to generate $280,000 =
($260,000 + $280,000) ÷ $800 = 675 garden sheds.

b. Let R = revenue; then 0.08R = After-tax income desired
Before-tax income = 0.08R ÷ (1 – 0.35) = 0.123R
Revenue – Variable costs – Fixed costs = Income before tax
Let X = Units sold
SP(X) – VC(X) – FC = Income before tax
$1,800X – $1,000X – $260,000 = 0.123($1,800)X
$800X – $260,000 = $221.4X
$578.6X = $260,000
X = 450 units (rounded) sold to earn 8 percent of revenue after tax
Amount of revenue = 450 × $1,800 = $810,000

Check: $810,000 × 0.08 = $64,800 after-tax income needed (round to $65,000) $64,800 ÷ 0.65 = $99,692 before-tax income (round to $100,000) $1,800(450) – $1,000(450) – $260,000 = $100,000 (before-tax income)
$100,000 – 0.35($100,000) = $100,000 – $35,000 = $65,000
$65,000 ÷ $810,000 = 8%

18. a. Convert the after-tax income to pre-tax desired income:
$135,800 ÷ (1 – 0.30) = $194,000
The number of units required to earn an after-tax profit of $135,800:
($62,640 + $194,000) ÷ $108 = 2,376.3 or 2,376 units

b. Convert the after-tax to pre-tax profit:
$7.20 ÷ $180 = 0.04, or 4%; 0.04 ÷ (1 – 0.30) = 5.7% of sales
A pre-tax return on sales of 5.7 percent is required to generate an after-tax profit of $7.20 per unit
Let R = the Level of revenue that generates a pre-tax return of 5.7%:
Variable costs = ($30 + $25 + 17) ÷ $180 = 0.4, or 0.4R
R – $62,640 – 0.4R = 0.057R
0.543R = $62,640
R = $115,359
$115,359 ÷ $180 = 640.88 or 641 units (rounded)

19. Let Y = Level of sales generating income equal to 30% of sales, then:
Y – 0.60Y – ($25,000 per month × 12 months) = 0.30Y
0.10Y = $300,000
Y = $3,000,000

Since existing sales are $2,250,000, sales would need to increase by $3,000,000 – $2,250,000 = $750,000.

20. a. First, convert the desired after-tax income to a pre-tax desired income:
$600,000 ÷ (1 – 0.40) = $1,000,000
Note that total variable costs per unit = $3,000, and total fixed costs = $370,000.

Next, let P represent the number of golf carts that must be sold to generate $1,000,000 in pre-tax income:

$5,000P – $3,000P – $370,000 = $1,000,000
$2,000P = $1,370,000
= 685 golf carts

b. Find after-tax equivalent of 20%: 20% ÷ (1 – 0.40) = 33.33%
Variable costs as a percentage of sales: $3,000 ÷ $5,000 = 60%
Let R = Level of revenue that generates a pre-tax return of 33.33%:

R – 0.6R – $370,000 = 0.3333R
0.0667R = $370,000
R = $5,547,226

Proof: Sales	$ 5,547,226
Variable costs (60%)	(3,328,336)
Contribution margin	$ 2,218,890
Fixed costs	(370,000)
Income before tax	$ 1,848,890
Income tax (40%)	(739,556)
Net income	$ 1,109,334

$1,109,334 ÷ $5,547,226 = 20%

22. a. $1,450 ÷ $0.50 = 2,900 passengers per day

i. Break-even: $2,000 ÷ 2,900 = $0.69 (rounded) per passenger
Earn $250: ($2,000 + $250) ÷ 2,900 = $0.78 (rounded)

ii. Total variable cost = $2,000 – ($2,000 × 0.80) = $400
Variable cost per passenger = $400 ÷ 2,900 = $0.14 (rounded)
Profit if fare is $0.60 = (2,900 × 0.90 × $0.60) – (2,900 × 0.9 × $0.14) – $1,600 = $(399.40)
Current loss = $1,450 – $2,000 = $(550)
County will be better off by $(399.40) – ($550) = $150.60.

iii. At a fare of $0.70:
(2,900 × $0.70 × 0.95) – (2,900 × $0.14 × 0.95) – $1,600 = $(57.20)
The county would incur a slight loss at a fare of $0.70.

At a fare of $0.90:
(2,900 × $0.90 × 0.90) – (2,900 × $0.14 × 0.90) – $1,600 = $383.60
The company would first make a profit when the fare is set at $0.90.

iv. Increasing volume will help improve profitability only if the volume change increases total contribution margin. Because an increase in volume can often

be achieved only with a decrease in price, the change in contribution margin may be negative rather than positive.

23. a. Current sales volume for both companies = \$2,000,000 ÷ \$40 = 50,000
New selling price \$40 – (0.3 × \$40) = \$28; Variable costs = \$1,400,000 ÷ 50,000 = \$28
Ainsley: (50,000 × 1.60 × \$28) – (50,000 × 1.60 × \$28) – \$0 = \$0
Bard: (50,000 × 1.60 × \$28) – (50,000 × 1.60 × \$0) – \$1,400,000 = \$840,000
This strategy is best used by Bard.

b. New selling price: \$40 × 1.3 = \$52
Ainsley: (50,000 × 0.85 × \$52) – (50,000 × 0.85 × \$28) – \$0 = \$1,020,000
Bard: (50,000 × 0.85 × \$52) – (50,000 × 0.85 × \$0) – \$1,400,000 = \$810,000
This strategy is best used by Ainsley.

c. Ainsley: (65,000 × \$40) – (65,000 × \$28) – \$200,000 = \$580,000
Bard: (65,000 × \$40) – (65,000 × \$0) – \$1,600,000 = \$1,000,000
This strategy is best used by Bard.

24. a. CM per unit of sales mix = (\$3 × 8) + (1 × \$6) = \$30
Break-even = \$180,000 ÷ \$30 = 6,000 units of sales mix, or 18,000 wallets and 6,000 money clips
Total revenue = (18,000 × \$30) + (6,000 × \$15) = \$630,000

b. Sales mix units = (\$180,000 + \$150,000) ÷ \$30 = 11,000 = 33,000 wallets and 11,000 money clips
Total revenue = (33,000 × \$30) + (11,000 × \$15) = \$1,155,000

c. Equivalent pre-tax profit = \$150,000 ÷ (1 – 0.40) = \$250,000
Sales mix units = (\$180,000 + \$250,000) ÷ \$30 = 14,333.33 = 43,000 wallets and 14,333 money clips
Total revenue = (43,000 × \$30) + (14,333 × \$15) = \$1,504,995

d. Units of sales mix = \$1,155,000 ÷ [(5 × \$30) + (2 × \$15)] = 6,417 (rounded) = 32,085 wallets and 12,834 money clips
Income = (32,085 × \$8) + (12,834 × \$6) – \$180,000 = \$153,684

The sales mix shifted such that the ratio of wallets to money clips declined, and the break-even point was reduced because money clips have a higher contribution margin ratio than money clips. Hence, at a sales level of \$1,155,000, more contribution margin is generated at the actual sales mix than at the planned sales mix.

25. a. Fixed costs ÷ Contribution margin = Break-even point in units
\$1,080,000,000 ÷ [(3 × \$300) + (5 × \$700) + (2 × \$1,000)] = \$1,080,000,000 ÷ \$6,400 = 168,750 bags

Mod = 3 × 168,750 = 506,250 units × \$2,200 =	\$1,113,750,000
Rad = 5 × 168,750 = 843,750 units × \$3,700 =	3,121,875,000
X-treme = 2 × 168,750 = 337,500 units × \$6,000 =	2,025,000,000
Revenue to break-even	\$6,260,625,000

b. Convert after-tax to pre-tax income. $1,000,000,000 ÷ (1 – 0.5) = $2,000,000,000
($2,000,000,000 + $1,080,000,000) ÷ $6,400 = 481,250 bags

Mod = 3 × 481,250 = 1,443,750 units × $2,200 =	$ 3,176,250,000
Rad = 5 × 481,250 = 2,406,250 units × $3,700 =	8,903,125,000
X-treme = 2 × 481,250 = 962,500 units × $6,000 =	5,775,000,000
Total revenue needed	$17,854,375,000

c. This change will increase the number of units required to break even because fewer units of Rad and X-treme, which have the greatest contribution margin, are being sold and more units of Mod, which has the lowest contribution margin, are being sold.

Scooter	Contribution Margin		
Mod	5 × $300	=	$1,500
Rad	4 × $700	=	2,800
X-treme	1 × $1,000	=	1,000
Total			$5,300

Now the contribution margin is $5,300 per bag, which is less than the contribution margin per bag of $6,400 in (a) above.

d. If Green Rider sells more of its scooters with the greatest contribution margin (X-treme) and fewer of the scooters with the lowest contribution margin (Mod), then fewer scooters would be needed to be sold to break even.

26. a. Break-even is $264,000 ÷ ($9.60 – $7.60) = 132,000 bushels
132,000 bushels × $9.60 = $1,267,200
Bushels per acre = 132,000 ÷ 1,200 = 110 bushels per acre

b. Bushels sold – Break-even bushels = Margin of safety
174,000 – 132,000 = 42,000 bushels
(174,000 × $9.60) – $1,267,200 = $403,200
$403,200 ÷ $1,670,400 = 24.1%

31. a. Each "bag" contains one unit of liquid and two units of spray. Thus, each bag generates contribution margin of: (1 × $10) + (2 × $5) = $20.

The break-even point would be: $100,000 ÷ $20 = 5,000 bags. Since each bag contains two units of spray, at the break-even point 5,000 × 2 or 10,000 units of spray must be sold.

i. At the break-even point, Total CM = Total FC; and the CM per unit would be $1,600 ÷ 4,000 = $0.40. If one unit is sold beyond the break-even point, net income would rise by $0.40.

ii. $10X – 0.40($10X) – $216,000 = 0.25($10X)
$3.50X = $216,000
X = 61,715 units (rounded)

iii. In units: 3,200 – 2,800 = 400 units
In dollars: 400 units × $65 per unit = $26,000

Percentage: $26,000 ÷ ($65 × 3,200) = 12.5%

38. a. Total variable cost = $28 + $12 + $8 = $48
Contribution margin per unit = $70 – $48 = $22 per unit
Contribution margin ratio = $22 ÷ $70 = 31.4% (rounded)
Total fixed costs = $10,000 + $24,000 = $34,000
Break-even point in units = $34,000 ÷ $22 per unit = 1,545 units (rounded)
Break-even point in dollars = $34,000 ÷ 0.314 = $108,280 (rounded)

b. ($40,000 + $34,000) ÷ 0.314 = $235,669 (rounded)
($235,669 ÷ $70) = 3,367 units (rounded)

c. Convert after-tax earnings to pre-tax earnings: $40,000 ÷ (1 – 0.40) = $66,667
Required sales = ($66,667 + $34,000) ÷ 0.314 = $320,596 (rounded)
$320,596 ÷ $70 = 4,580 units (rounded)

d. Convert the after-tax rate of earnings to a pre-tax rate of earnings:
[20% ÷ (1 – 0.40)] = 33.33%

Because the CM% is only 31.4%, no level of sales would generate net income equal to, on a pre-tax basis, 33.33% of sales.

e.

Variable cost savings (5,000 × $6.00)	$ 30,000
Additional fixed costs	(40,000)
Decrease in profit	$(10,000)

The company should not buy the new sewing machine.

f. Existing CM per unit = $22
CM under proposal = ($70 × 0.90) – $48 = $15

Total CM under proposal (3,000 × 1.30 × $15)	$ 58,500
Existing CM (3,000 × $22)	(66,000)
Change in CM	$ (7,500)
Change in fixed costs	(10,000)
Change in net earnings before taxes	$ (17,500)

No, these two changes should not be made because they would lower pre-tax profits by $17,500 relative to existing levels.

41. a.

Total sales price per bag:		
Commercial ($5,600 × 1)	$5,600	
Residential ($1,800 × 3)	5,400	$11,000
Total variable costs per bag:		
Commercial ($3,800 × 1)	$3,800	
Residential ($1,000 × 3)	3,000	(6,800)
Total contribution margin		$ 4,200

Break-even point in units = $8,400,000 ÷ $4,200 = 2,000 bags
Commercial: 2,000 × 1 = 2,000 mowers
Residential: 2,000 × 3 = 6,000 mowers

b. ($8,400,000 + $1,260,000) ÷ $4,200 = 2,300 bags

Commercial: 2,300 × 1 = 2,300 mowers
Residential: 2,300 × 3 = 6,900 mowers

c. Pre-tax equivalent of $1,008,000 after-tax = $1,008,000 ÷ (1 – 0.40) = $1,680,000
($8,400,000 + $1,680,000) ÷ $4,200 = 2,400 bags
Commercial: 2,400 × 1 = 2,400 mowers
Residential: 2,400 × 3 = 7,200 mowers

d. Let X = number of bags that must be sold to produce pre-tax earnings equaling 12 percent of sales revenue, then:
$4,200X – $8,400,000 = 0.12($11,000X)
X = 2,917 bags (rounded)
Commercial: 2,917 × 1 = 2,917 mowers
Residential: 2,917 × 3 = 8,751 mowers

e. Convert the after-tax return to a pre-tax rate of return:
0.08 ÷ (1 – 0.40) = 13% (rounded)
$4,200X – $8,400,000 = 0.13($11,000X)
X = 3,032 bags (rounded)
Commercial: 3,032 × 1 = 3,032 mowers
Residential: 3,032 × 3 = 9,096 mowers

42. a.

	Ducks	Ducklings
Sales	$ 24.00	$12.00
Variable costs	(12.00)	(8.00)
Contribution margin	$ 12.00	$ 4.00
Mix	× 1	× 5
Total contribution margin	$ 12.00	$20.00

The average contribution margin ratio is $32 ÷ $84 = 38.1% (rounded)

b. Break-even point = $288,000 ÷ $32 = 9,000 bags per year or 750 bags a month
Ducks: 750 × 1 = 750 per month
Ducklings: 750 × 5 = 3,750 per month

c. Target profit is $96,000 × 12 = $1,152,000
($288,000 + $1,152,000) ÷ $32 = 45,000 bags per year or 3,750 bags a month.
Ducks: 3,750 × 1 = 3,750 per month
Ducklings: 3,750 × 5 = 18,750 per month

d.

	Ducks	Ducklings
Sales	$ 24.00	$12.00
Variable costs	(12.00)	(8.00)
Contribution margin	$ 12.00	$ 4.00
Mix	× 1	× 9
Total contribution margin	$ 12.00	$36.00

Target profit after tax is $31,680.
Pre-tax profit is $31,680 ÷ (1 – 0.40) = $52,800 monthly or $633,600 per year.
Break-even = ($633,600 + $288,000) ÷ $48 = 19,200 bags per year, or 1,600 per month

	Units	Revenue

Ducks (19,200 × $24)	19,200	$ 460,800
Ducklings (19,200 × 9 × $12)	172,800	2,073,600
Total		$2,534,400

e. [$288,000 + ($8,500 × 12)] ÷ [$12 + ($8 × 5)]
($288,000 + $102,000) ÷ $52 = 7,500

Yes, the company would want to make the change because the break-even point is reduced from 9,000 mix units to 7,500 mix units.