Chapter Fourteen: Simulation

PROBLEM SUMMARY

1. Rescue squad emergency calls
2. Car arrivals at a service station
3. Machine breakdowns
4. Inventory analysis
5. Decision analysis
6. Machine repair time (14–3)
7. Product demand and order receipt
8. Bank drive-in window arrivals and service
9. Loading dock arrivals and service
10. Product demand and order receipt
11. Football game
12. Student advising, arrival and service
13. Markov process
14. Inventory analysis
15. Rental car agency
16. CPM/PERT network analysis
17. Store robbery and getaway
18. Stock price movement
19. Hospital emergency room staffing
20. Break-even analysis
21. Rating dates
22. Model analysis (14-21)
23. Production capacity
24. Baseball game
25. Crystal Ball, maintenance cost
26. Crystal Ball, B-E analysis (14-20)
27. Crystal Ball, rating dates (14-21)
28. Crystal Ball, production capacity (14-23)
29. Crystal Ball, investment selection
30. Crystal Ball, inventory management
31. Crystal Ball, hotel room rates
32. Crystal Ball, CPM/PERT network

PROBLEM SOLUTIONS

1.

Time Between Calls (hr)	Cumulative Probability	Random Numbers
1	.05	01–05
2	.15	06–15
3	.45	16–45
4	.75	46–75
5	.95	76–95
6	1.00	96–99, 00

a)

r	Time Between Calls	Cumulative Clock
65	4	4
71	4	8
20	3	11
15	2	13
48	4	17
89	5	22
18	3	25
83	5	30
08	2	32
90	5	37
05	1	38
89	5	43
18	3	46
08	2	48
26	3	51
47	4	55
94	5	60
06	2	62
72	4	66
40	3	69
62	4	73

b) $\mu = \frac{73}{21} = 3.48$ hr between calls; EV $= 1(.05) + 2(.10) + 3(.30) + 4(.30) + 5(.20) + 6(.05) = 3.65$. The results are different because there were not enough simulations to enable the simulated average to approach the analytical result.

c) 21 calls; no, this is not the average number of calls per 3 days. In order to determine this average, this simulation would have to be repeated a number of times in order to get enough observations of calls per 3-day period to compute an average.

2.0

Time Between Arrivals (min)	Cumulative Probability	Random Numbers
1	.15	01–15
2	.45	16–45
3	.85	46–85
4	1.00	86–99, 00

a)

Arrival	r	Time Between Arrivals (min)
1	39	2
2	73	3
3	72	3
4	75	3
5	37	2
6	02	1
7	87	4
8	98	4
9	10	1
10	47	3
11	93	4
12	21	2
13	95	4
14	97	4
15	69	3
16	41	2
17	91	4
18	80	3
19	67	3
20	59	3
		58

$\mu = 58/20 = 2.9$ min between arrivals

b)

r	Time Between Arrivals	Cumulative Clock
65	3	3
71	3	6
20	2	8
17	2	10
48	3	13
89	4	17
18	2	19
83	3	22
08	1	23
90	4	27
05	1	28
89	4	32
18	2	34
08	1	35
26	2	37
47	3	40
94	4	44
06	1	45
72	3	48
40	2	50
62	3	53
47	3	56
68	3	59
60	3	62

$\mu = 62/24 = 2.58$ min between arrivals

c) The results are different because of the few simulations combined with the different random number streams. If there were enough simulations, the different random numbers would have little or no effect.

3.

Machine Breakdowns/ Week	Cumulative Probability	Random Numbers
0	.10	01–10
1	.20	11–20
2	.40	21–40
3	.65	41–65
4	.95	66–95
5	1.00	96–99, 00

a)

Week	*r*	Breakdowns
1	20	1
2	31	2
3	98	5
4	24	2
5	01	0
6	56	3
7	48	3
8	00	5
9	58	3
10	27	2
11	74	4
12	76	4
13	79	4
14	77	4
15	48	3
16	81	4
17	92	4
18	48	3
19	64	3
20	06	0
		59

b) $\mu = 59/20 = 2.95$ breakdowns per week

4.

Snowfall Level (in)	Cumulative Probability	RN	Financial Return
>40	0.40	01-40	$120,000
20-40	0.60	41-60	40,000
<20	1.00	61-99,00	-40,000

Use fifth column of random numbers in Table 16.3

Random Number	Return
45	40,000
90	-40,000
84	-40,000
17	120,000
74	-40,000
94	-40,000
07	120,000
15	120,000
04	120,000
31	40,000
07	120,000
99	-40,000
97	-40,000
73	-40,000
13	120,000
03	120,000
62	-40,000
47	40,000
99	-40,000
75	-40,000
	560,000

$$\text{Average return} = \frac{560{,}000}{20} = \$28{,}000$$

The expected value is $40,000 so the simulated average for 20 weeks is significantly lower indicating the need for more trials.

5.

Weather Conditions	Random Numbers
Rain	01–30
Overcast	31–45
Sunshine	46–99, 00

Sixth column in Table 15.3.

	Profit	
r	Sun Visors	Umbrellas
19	−500	2,000
65	1,500	−900
51	1,500	−900
17	−500	2,000
63	1,500	−900
85	1,500	−900
37	−200	0
89	1,500	−900
76	1,500	−900
71	1,500	−900
34	−200	0
11	−500	2,000
27	−500	2,000
10	−500	2,000
59	1,500	−900
87	1,500	−900
08	−500	2,000
08	−500	2,000
89	1,500	−900
42	−200	0

Sun visors: $\mu = 10{,}900/20 = \$545$; umbrellas: $\mu = 5{,}000/20 = \$250$. The best decision according to the simulation (for only 20 weeks) would be sun visors.

6. a) The first three columns are from Problem 3. Select as many r_2's as there are breakdowns from a different random number stream.

Week	r_1	Breakdowns	r_2	Repair Time
1	20	1	58	2
2	31	2	47, 23	2 + 1 = 3
3	98	5	69, 35, 21, 41, 14	2 + 2 + 1 + 2 + 1 = 8
4	24	2	59, 28	2 + 1 = 3
5	01	0	—	0
6	56	3	13, 09, 20	1 + 1 + 1 = 3
7	48	3	73, 77, 29	2 + 2 + 1 = 5
8	00	5	72, 89, 81, 20, 85	2 + 3 + 3 + 1 + 3 = 12
9	58	3	59, 72, 88	2 + 2 + 3 = 7
10	27	2	11, 89	1 + 3 = 4
11	74	4	87, 59, 66, 53	3 + 2 + 2 + 2 = 9
12	76	4	45, 56, 32, 44	2 + 2 + 2 + 2 = 8
13	79	4	08, 82, 55, 27	1 + 3 + 2 + 1 = 7
14	77	4	49, 24, 83, 05	2 + 1 + 3 + 1 = 7
15	48	3	81, 07, 78	3 + 1 + 2 = 6
16	81	4	92, 36, 53, 04	3 + 2 + 2 + 1 = 8
17	92	4	95, 79, 61, 44	3 + 2 + 2 + 2 = 9
18	48	3	37, 45, 18	2 + 2 + 1 = 5
19	64	3	65, 37, 30	2 + 2 + 1 = 5
20	06	0	—	0

Total repair time = 111 hr; $\mu = 111/20 = 5.55$ hr/week.

b) It could bias the results. If a high random number is selected—for example, 98—this results in a high number of breakdowns (i.e., 5). If the same random number is used, it will result in a high repair time (i.e., 3 hr). Thus, a relationship will result wherein high number of breakdowns equals high repair times, and vice versa. The effect in this model will not be too bad since *several* repair time random numbers are selected for each breakdown.

c) Average weekly breakdown cost: 111 hours × \$50 = \$5,550; $\mu = 5{,}550/20 = \$277.50$ per week.

d)

Breakdowns per week	Cumulative Probability	Random Numbers
0	.20	01–20
1	.50	21–50
2	.70	51–70
3	.85	71–85
4	.95	86–95
5	1.00	96–99, 00

Week	r_1	Breakdowns	r_2	Repair Time
1	20	0	—	0
2	31	1	58	2
3	98	5	47, 23, 69, 35, 21	2 + 1 + 2 + 2 + 1 = 8
4	24	1	41	2
5	01	0	—	0
6	56	2	14, 59	1 + 2 = 3
7	48	1	28	1
8	00	5	13, 09, 20, 73, 77	1 + 1 + 1 + 2 + 2 = 7
9	58	2	29, 72	1 + 2 = 3
10	27	1	89	3
11	74	3	81, 20, 85	3 + 1 + 3 = 7
12	76	3	59, 72, 88	2 + 2 + 3 = 7

(continued)

Week	r_1	Breakdowns	r_2	Repair Time
13	79	3	11, 89, 87	1 + 3 + 3 = 7
14	77	3	59, 66, 53	2 + 2 + 2 = 6
15	48	1	45	2
16	81	3	56, 32, 44	2 + 2 + 2 = 6
17	92	4	08, 82, 55, 27	1 + 3 + 2 + 1 = 7
18	48	1	49	2
19	64	2	24, 83	1 + 3 = 4
20	06	0	—	0

Total repair time = 77 hr; average weekly breakdown cost: 77 × \$50 = \$3,850; μ = 3,850/20 = \$192.50; reduction in average weekly repair cost = 277.50 − 192.50 = \$85. Since the maintenance program costs \$150 and only \$85 would be saved, it should not be put into effect. However, the results should be applied with some hesitancy, since they were derived for only one actual simulation. This whole simulation process should be repeated a number of times. *Note:* In part d the same random number streams were used as in part a. This was done to replicate as much as possible the conditions of the first simulation since the results were to be compared. However, if many simulations were conducted, this would not have been necessary.

7. a)

Demand/Month	Random Numbers
0	01–04
1	05–12
2	15–40
3	41–80
4	81–96
5	97–98
6	99, 00

Lead Time	Random Numbers
1	01–60
2	61–90
3	91–99, 00

Month	r_1	r_2	Demand	Order Placed	Order Received	Balance	Carrying Cost	Order Cost	Stockout Cost	Total
0						5	—			
1	39	73	2	5		3	120	100	—	220
2	72	75	3	5		0	0	100	—	100
3	37	02	2	5	5	3	120	100	—	220
4	87	—	4	—	5 + 5	9	360	—	—	360
5	98	—	5		—	4	160	—	—	160
6	10	47	1	5	—	3	120	100	—	220
7	93	—	4	—	5	4	160	—	—	160
8	21	95	2	5	—	2	80	100	—	180
9	97	69	5	5	—	0	0	100	400	500
10	41	91	3	5	—	0	0	100	400	500
11	80		3		5 + 5	7	280	—	—	280
12	67		3		—	4	160	—	—	160
13	59		3		5	6	240	—	—	240
14	63	78	3	5	—	3	120	100	—	220
15	87	47	4	5	—	0	0	100	400	500
16	56		3		5 + 5	7	280	—	—	280
17	22		2		—	5	200	—	—	200
18	19	16	2	5	—	3	120	—	—	120
19	78		3		5	5	200	—	—	200
20	03		0	—	—	5	200	—	—	200
								μ = \$251		5,020

8. One-teller system:

Customer	r_1	Arrival Interval	Arrival Clock	Enter Facility Clock	Waiting Time	Length of Queue at Entry	r_2	Service Time	Departure Time	Time in System
1	—	—	—	—	—	—	39	3	3	3
2	73	2	2	3	1	0	72	5	8	6
3	75	2	4	8	4	0	37	3	11	7
4	02	1	5	11	6	1	87	5	16	11
5	98	4	9	16	7	1	10	3	19	10
6	47	2	11	19	8	1	93	6	25	14
7	21	2	13	25	12	2	95	6	31	18
8	97	4	17	31	14	2	69	4	35	18
9	41	2	19	35	16	2	91	6	41	22
10	81	3	22	41	19	3	67	4	45	23
11	59	2	24	45	21	4	63	4	49	25
12	78	2	26	49	23	4	87	5	54	28
13	47	2	28	54	26	5	56	4	58	30
14	22	2	30	58	28	6	19	3	61	31
					$\mu = 13.1$	$\mu = 2.2$				$\mu = 17.6$

Notice that statistics are misleading since the values keep increasing. In other words, a steady state has not been reached; thus, it is difficult to draw inferences.

Two-teller system:

Customer	r_1	Arrival Interval	Arrival Clock	Enter Facility Clock	Waiting Time	Length of Queue	r_2	Service Time	Departure Time	Time in System
1	—	—	—	—	—	—	39	3	3	3
3	75	2	4	4	0	0	37	3	7	3
5	10	1	6	7	1	0	47	3	10	4
6	93	4	10	10	0	0	21	3	13	3
8	41	2	16	16	0	0	91	6	22	6
11	87	3	24	24	0	0	47	3	27	3
12	56	2	26	27	1	0	19	3	30	4
15	15	1	29	30	1	0	58	4	34	4
					$\mu = .4$	0				$\mu = 3.8$

Customer	r_1	Arrival Interval	Arrival Clock	Enter Facility Clock	Waiting Time	Length of Queue	r_2	Service Time	Departure Time	Time in System
2	73	2	2	2	0	0	72	5	7	5
4	02	1	5	7	2	0	98	6	13	8
7	95	4	14	14	0	0	69	4	18	4
9	80	3	19	19	0	0	67	4	23	4
10	59	2	21	23	4	0	78	5	28	7
13	16	1	27	28	1	0	03	2	30	2
14	04	1	28	30	2	0	23	3	33	5
16	93	4	33	33	0	0	78	5	38	5
					$\mu = 1.1$	0				$\mu = 5$

Ties for queue length:

Line	*Random Numbers*
1	00–49
2	50–99

Customer	r_3
4	87
7	97
10	63
12	22
13	78
28	61

It appears that the two-teller system is most appropriate; however, some trade-off analysis between the cost of the extra facility and the cost of customers waiting and possibly leaving is necessary.

9.

Days Between Arrivals	Cumulative Probability	Random Numbers
1	.05	1–5
2	.15	6–15
3	.35	16–35
4	.65	36–65
5	.85	66–85
6	.95	86–95
7	1.00	96–99, 00

Days to Fill and Prepare	Cumulative Probability	Random Numbers
3	.10	1–10
4	.30	11–30
5	.70	31–70
6	1.00	71–99, 00

a)

Observation	Random Number	Time Between Arrivals
No ship–1st ship	43	4
1st ship–2nd ship	96	7
2nd ship–3rd ship	20	3
3rd ship–4th ship	86	6
4th ship–5th ship	92	6
5th ship–6th ship	22	3
6th ship–7th ship	86	6
7th ship–8th ship	91	6
8th ship–9th ship	46	4
9th ship–10th ship	29	3
10th ship–11th ship	49	4
11th ship–12th ship	79	5
12th ship–13th ship	22	3
13th ship–14th ship	66	5
14th ship–15th ship	08	2
15th ship–16th ship	62	4
16th ship–17th ship	09	2
17th ship–18th ship	81	5
18th ship–19th ship	58	4
19th ship–20th ship	39	4
		86

Ship No.	Arrival Day	Time to Arrival of Next Ship	Day Loading Begins	Loading Time	Departure Day	Waiting Time	No. of Ships Waiting
1	4	7	4	5	9	0	0
2	11	3	11	4	15	0	1
3	14	6	15	6	21	1	1
4	20	6	21	3	24	1	0
5	26	3	26	4	30	0	1
6	29	6	30	6	36	1	1
7	35	6	36	6	42	1	1
8	41	4	42	5	47	1	1
9	45	3	47	5	52	2	1
10	48	4	52	6	58	4	2
11	52	5	58	6	64	6	2
12	57	3	64	4	68	7	3

(continued)

Ship No.	Arrival Day	Time to Arrival of Next Ship	Day Loading Begins	Loading Time	Departure Day	Waiting Time	No. of Ships Waiting
13	60	5	68	5	73	8	3
14	65	2	73	4	77	8	3
15	67	4	77	6	83	10	4
16	71	2	83	6	89	12	4
17	73	5	89	3	92	16	3
18	78	4	92	6	98	14	2
19	82	4	98	5	103	16	1
20	86		103	5	108	17	0
						125	

Average time between arrivals = 86/20 = 4.3 days; average waiting time to load = 125/20 = 6.25 days; average number of tankers waiting to unload = waiting time/simulation period = 125/108 = 1.16 tankers (*Note:* You must use waiting time instead of tankers waiting per arrival.)

b) It should first be pointed out that it would be an error to make inferences about the true system based upon summary statistics (i.e., averages) computed from the simulation of this problem. Why? Because the simulated system does not reach *steady state*. That is, note that ship waiting times are continuously increasing, from zero up to 15 days, over the 20 ship arrivals. This is easily explained by the fact that the mean arrival rate exceeds the service rate; thus, no steady state can be reached.

The relevant output data for analysis of this problem is ship waiting time for each ship, as opposed to any summary statistics. The management should conclude that additional manpower or equipment must be added so that ship servicing time could be reduced to less than 4 days (mean of arrival distribution). If ship service times were reduced to 3.5 days, the simulated system would then reach steady state, and summary statistics would be relevant for analysis of the problem.

Another problem that should be considered when computing summary statistics is the time it takes the simulated system to reach steady state and the output generated as the sample is concluded (ending conditions). In both cases, the output may bias the computations of summary statistics. That is, note the reduction in number of ships waiting for the last three observations (ship arrivals 18, 19, and 20), which is simply due to the ending of the simulation. The conclusion reached is that a portion of both the beginning and the end of the simulation output data should be discarded when computing summary statistics.

10.

Months to Receive an Order	Cumulative Probability	Random Numbers
1	.50	01–50
2	.80	51–80
3	1.00	81–99, 00

Demand per Month	Cumulative Probability	Random Numbers
1	.10	01–10
2	.40	11–40
3	.80	41–80
4	1.00	81–99, 00

Reorder Number	Random Number	Months Lead Time	Random Numbers	Demand per Month
1	21	1	70	3
2	41	1	38	2
3	14	1	11	2
4	59	2	70, 52	3, 3 = 6
5	28	1	88	4
6	68	2	25, 17	2, 2 = 4
7	13	1	18	2
8	09	1	00	1
9	20	1	70	3
10	73	2	04, 11	1, 2 = 3
11	77	2	76, 29	3, 2 = 5

(continued)

Reorder Number	Random Number	Months Lead Time	Random Numbers	Demand per Month
12	29	1	01	1
13	72	2	11, 42	2, 3 = 5
14	89	3	70, 17, 48	3, 2, 3 = 8
15	81	3	87, 20, 16	4, 2, 2 = 8
16	20	1	56	3
17	85	3	35, 94, 91	2, 4, 4 = 10
18	59	2	65, 20	3, 2 = 5
19	72	2	97, 46	4, 3 = 7
20	88	3	10, 98, 32	2, 4, 2 = 8
21	11	1	57	3
22	89	3	92, 55, 35	4, 3, 2 = 9
23	87	3	57, 99, 20	3, 4, 2 = 9
24	59	2	52, 45	3, 3 = 6
25	66	2	01, 73	1, 3 = 4
26	53	2	58, 24	3, 2 = 5
27	45	1	72	3
28	56	2	84, 35	4, 2 = 6
29	22	1	41	3
30	49	1	32	2
				140

Total demand = 140; average demand during lead time = 140/30 = 4.67; thus, should reorder at 5-car level.

11. State University:

Play	Cumulative Probability	Random Numbers (r_1)
Sweep	.10	01–10
Pass	.30	11–30
Draw	.50	31–50
Off tackle	1.00	51–99, 00

Tech:

Defense	Cumulative Probability	Random Numbers (r_2)
Wide tackle	.30	01–30
Oklahoma	.80	31–80
Blitz	1.00	81–99, 00

Play Number	r_1	Play	r_2	Defense	Yardage
1	39	Draw	65	Oklahoma	1
2	73	Off tackle	71	Oklahoma	3
3	72	Off tackle	20	Wide tackle	7
4	75	Off tackle	17	Wide tackle	7
5	37	Draw	48	Oklahoma	1
6	02	Sweep	89	Blitz	12
7	87	Off tackle	18	Wide tackle	7
8	98	Off tackle	83	Blitz	−3
9	10	Sweep	08	Wide tackle	−3
10	47	Draw	90	Blitz	20
11	93	Off tackle	05	Wide tackle	7
12	21	Pass	89	Blitz	−10
13	95	Off tackle	18	Wide tackle	7
14	97	Off tackle	08	Wide tackle	7
15	69	Off tackle	26	Wide tackle	7
16	41	Draw	47	Oklahoma	1
17	91	Off tackle	94	Blitz	−3
18	80	Off tackle	06	Wide tackle	7
19	67	Off tackle	72	Oklahoma	3
20	59	Off tackle	40	Oklahoma	3
21	63	Off tackle	62	Oklahoma	3
22	78	Off tackle	47	Oklahoma	3
23	87	Off tackle	68	Oklahoma	3
24	47	Draw	60	Oklahoma	1
25	56	Off tackle	88	Blitz	−3
26	22	Pass	17	Wide tackle	12
27	19	Pass	36	Oklahoma	4
28	16	Pass	77	Oklahoma	4
29	78	Off tackle	43	Oklahoma	3
30	03	Sweep	28	Wide tackle	−3
31	04	Sweep	31	Oklahoma	5
32	61	Off tackle	06	Wide tackle	7
33	23	Pass	68	Oklahoma	4
34	15	Pass	39	Oklahoma	4
35	58	Off tackle	71	Oklahoma	3
36	93	Off tackle	22	Wide tackle	7
37	78	Off tackle	76	Oklahoma	3
38	61	Off tackle	81	Blitz	−3
39	42	Draw	88	Blitz	20
40	77	Off tackle	94	Blitz	−3

Total yardage = 155 yd; the sportswriter will predict that Tech will win.

12.

Time Between Arrivals	Cumulative Probability	Random Numbers (r_1)
4	.20	1–20
5	.50	21–50
6	.90	51–90
7	1.00	91–99, 00

Schedule Approval	Cumulative Probability	Random Numbers (r_2)
6	.30	1–30
7	.80	31–80
8	1.00	81–99, 00

r_1	Time Between Arrivals	Arrival Clock	r_2	Approval Time	Departure Clock	Waiting Line	Waiting Time
19	4	4	65	7	11	0	0
15	6	10	17	6	17	1	1
63	6	16	85	8	25	1	1
37	5	21	89	8	33	1	4
76	6	27	71	8	41	1	6
34	5	32	11	6	47	2	9
27	5	37	10	6	53	2	10
59	6	43	87	8	61	2	10
08	4	47	08	6	67	2	14
89	6	53	42	7	74	2	14
79	6	59	79	7	81	3	15
97	7	66	26	6	87	3	15
06	4	70	87	8	95	3	17
39	5	75	28	6	101	3	20
97	7	82	69	7	108	3	19
33	5	87	87	8	116	3	21
99	7	94	93	8	124	4	22
							198

Total waiting time = 198 min; 17 arrivals; average waiting time = 198/17 = 11.65 min; average waiting line = 36/17 = 2.12. The queue size and waiting time seem to be increasing and not approaching a steady state. Computing the expected time between arrivals and approval times gives E (time between arrivals) = 4(.2) + 5(.3) + 6(.4) + 7(.1) = 5.4 min; E (approval time) = 6(.3) + 7(.5) + 8(.2) = 6.9 min. Therefore, students are arriving at a faster rate than their schedules are being approved. The queue will never reach a steady state but will increase infinitely.

13. *Tribune:*

Paper	Random Numbers
Tribune	01–65
Daily News	66–99, 00

Daily News:

Paper	Random Numbers
Tribune	01–45
Daily News	46–99, 00

Fifth column in Table 15.3; start with *Tribune.*

r	*Daily News*	*Tribune*
45		1
90	1	
84	1	
17		1
74	1	
94	1	
07		1
15		1
04		1
31		1
67	1	
99	1	
97	1	
73	1	
13		1
03		1
62		1
47		1
99	1	
75	1	

Simulation results: [*Tribune*   *Daily News*] = [.50   .50]; the results of Markov analysis: [*Tribune*   *Daily News*] = [.563   .437]. The results differ because the simulation had too few iterations (runs) to approach the actual steady-state results determined by Markov analysis.

14. Order size = 16 cases. Order size ≥ demand: profit = $DP - QC - (Q/2)C_c = 16D - 164$. Demand > order size: profit = $QP - QC - (Q/2)C_c - C_s(D - Q) = 108 - D$.

Demand	Random Numbers
15	01–20
16	21–45
17	46–85
18	86–99, 00

r	Demand	Profit
39	16	92
73	17	91
72	17	91
75	17	91
37	16	92
02	15	76
87	18	90
98	18	90
10	15	76
47	17	91
93	18	90
21	16	92
95	18	90
97	18	90
69	17	91
41	16	92
91	18	90
80	17	91
67	17	91
59	17	91
		1,788

$\mu = 1{,}788/20 = \$89.40$; the complete simulation would be achieved by performing the same simulation (above) for all four order sizes and selecting the order size with the maximum profit.

15.

Customers/Day	Random Numbers
0	01–20
1	21–40
2	41–90
3	91–99, 00

Duration	Random Numbers
1	01–10
2	11–40
3	41–80
4	81–90
5	91–99, 00

Day	Available Cars	r_1	Customers	r_2	Duration/Car	Car Day Available	Customers Not Served
1		62	2	19	2	3	
				66	3	4	
2	2	48	2	27	2	4	
				43	3	5	
3	1	96	3	20	2	5	2
4	2	86	2	92	5	9	
				22	2	6	
5	2	86	2	91	5	10	
				46	3	8	
6	1	29	1	49	3	9	
7	0	79	2	—	—	—	2
8	1	22	1	66	3	11	
9	2	08	0	—	—	—	
10	3	62	2	09	1	11	
				81	4	14	

Probability of not having a car available = customers not served/total customers = 4/17 = .235; since almost 24% of customers are not served, expansion would probably be warranted. A simulation model of this system necessary to make a decision would need to perform this simulation for a number of different fleet sizes (in addition to the four cars used in this experiment). The simulation should also include the daily cost of the car to the rental agency and the daily rental price plus some estimate of lost current and potential sales when a customer is turned away. The fleet size selected would be the one that maximized average daily profit.

16. Activity 1–2:

Day	Random Numbers
5	01–60
9	61–99, 00

Activity 3–5:

Day	Random Numbers
3	01–20
5	21–70
7	71–99, 00

Activity 1–3:

Day	Random Numbers
6	01–40
10	41–99, 00

Activity 4–5:

Day	Random Numbers
3	01–50
5	51–99, 00

Activity 2–4:

Day	Random Numbers
2	01–20
6	21–90
8	91–99, 00

Activity 5–6:

Day	Random Numbers
1	01–40
2	41–99, 00

Activity 3–4:

Day	Random Numbers
1	01–30
3	31–60
6	61–99, 00

Paths through the network: A = 1–3–5–6; B = 1–3–4–5–6; C = 1–2–4–5–6.

	1–2		1–3		2–4		3–4		3–5		4–5		5–6				
Run	r_1	x_{12}	r_2	x_{13}	r_3	x_{24}	r_4	x_{34}	$\bar{x}_5$	x_{35}	r_6	x_{45}	r_7	x_{56}	A	B	C
1	39	5	65	10	76	6	45	3	45	5	19	3	90	2	17	18*	16
2	69	9	64	10	61	6	20	1	26	5	36	3	31	1	16	15	19*
3	62	9	58	10	24	6	97	6	14	3	97	5	95	2	15	23*	22
4	06	5	70	10	99	8	00	6	73	7	71	5	23	1	18	21*	19
5	70	9	90	10	65	6	97	6	60	5	12	3	11	1	16	19*	19*
6	31	5	56	10	34	6	19	1	19	3	47	3	83	2	15	16*	16*
7	75	9	51	10	33	6	30	1	62	5	38	3	20	1	16	15	19*
8	46	5	72	10	18	2	47	3	33	5	84	5	51	2	17	20*	14
9	67	9	47	10	97	8	19	1	98	7	40	3	07	1	18	15	21*
10	17	5	66	10	23	6	05	1	09	3	51	5	80	2	15	18*	18*

Average project completion time (i.e., mean critical path time) = 19.4 weeks; % of time A is critical = 0%, % of time B is critical = 40%, % of time C is critical = 30%; % of time B and C are critical together = 30%. These results indicate that since activity times are probabilistic, a single path may not always be critical; as shown, B and C were both critical. Since the PERT technique always assumes only one path is critical all of the time, the PERT critical path time will be overly optimistic, i.e., it does not recognize that a path might be longer than the critical path.

17.

Direction	Probability	Cummulative Probability	RN Ranges
East ($x = +1$)	.25	.25	01–25
West ($x = -1$)	.25	.50	26–50
North ($y = +1$)	.25	.75	51–75
South ($y = -1$)	.25	1.00	76–99,00

Monte Carlo Simulation (using 16th row of random numbers from Table 15.3)

Considering the city as a grid with an *x* and *y* axis with the store at point (0,0) each random number selected indicates a movement of 1 unit (block) in either an *x* or *y* direction.

End of Block	Trial 1 *r*	Trial 1 *(x,y)*	Trial 2 *r*	Trial 2 *(x,y)*	Trial 3 *r*	Trial 3 *(x,y)*	Trial 4 *r*	Trial 4 *(x,y)*	Trial 5 *r*	Trial 5 *(x,y)*
1	58	(0,1)	68	(0,1)	20	(1,0)	53	(0,1)	24	(1,0)
2	47	(–1,1)	13	(1,1)	85	(1,–1)	45	(–1,1)	83	(1,–1)
3	23	(0,1)	09	(2,1)	59	(1,0)	56	(–1,2)	05	(2,–1)
4	69	(0,2)	20	(3,1)	72	(1,1)	22	(0,2)	81	(2,–2)
5	35	(–1,2)	73	(3,2)	88	(1,0)	49	(–1,2)	07	(3,–2)
6	21	(0,2)	77	(3,1)	11	(2,0)	08	(0,2)	78	(3,–3)
7	41	(–1,2)	29	(2,1)	89	(2,–1)	82	(0,1)	92	(3,–4)
8	14	(0,2)	72	(2,2)	87	(2,–2)	55	(0,2)	36	(2,–4)
9	59	(0,3)	89	(2,1)	59	(2,–1)	27	(–1,2)	53	(2,–3)
10	28	(–1,3)	81	(2,0)	66	(2,0)	49	(–2,2)	04	(3,–3)
Within 2 blocks?		no		yes		yes		no		no

In 2 of the 5 trials the robber is within 2 blocks of the store. As an example, at the end of 10 blocks in trial 1, the robber is 1 block west and 3 blocks north.

18.

Stock Price Movement	Probability	Cummulative Probability	RN Ranges
Increase (+)	.45	.45	01–45
Same (0)	.30	.75	46–75
Decrease (–)	.25	1.00	76–99,00

Stock Price	Probability Increase	Cummulative Probability	RN Ranges	Probability Decrease	Cummulative Probability	RN Ranges
1/8	.40	.40	01–40	.12	.12	01–12
1/4	.17	.57	41–57	.15	.27	13–27
3/8	.12	.69	58–69	.18	.45	28–45
1/2	.10	.79	70–79	.21	.66	46–66
5/8	.08	.87	80–87	.14	.80	67–80
3/4	.07	.94	88–94	.10	.90	81–90
7/8	.04	.98	95–98	.05	.95	91–95
1	.02	1.00	99,00	.05	1.00	96–99,00

Monte Carlo Simulation (using the third column of random numbers from Table 15.3)

Day	r	Stock Price Movement	r	Price Change (+,–)	Stock Price
1	76	–	23	1/4	61 3/4
2	47	0			61 3/4
3	25	+	79	1/2	62 1/4
4	08	+	15	1/8	62 3/8
5	71	–	58	1/2	61 7/8
6	56	0			61 7/8
7	31	+	11	1/8	62
8	94	–	31	3/8	61 5/8
9	88	–	10	1/8	61 1/2
10	14	+	54	1/4	61 3/4
11	77	–	24	1/4	61 1/2
12	06	+	23	1/8	61 5/8
13	62	0			61 5/8
14	92	–	87	3/4	60 7/8
15	68	0			60 7/8
16	27	+	23	1/8	61
17	76	–	28	3/8	60 5/8
18	17	+	98	7/8	61 1/2
19	35	+	25	1/8	61 5/8
20	96	–	53	1/2	61 1/8
21	58	0			61 1/8
22	31	+	07	1/8	61 1/4
23	30	+	45	1/4	61 1/2
24	70	0			61 1/2
25	33	+	69	3/8	61 7/8
26	88	–	16	1/4	61 5/8
27	70	0			61 5/8
28	70	0			61 5/8
29	07	+	37	1/8	61 3/4
30	03	+	47	1/4	62

In order to expand the model for practical purposes the length of the simulation trial would be increased to one year. Then this simulation would need to be repeated for many trials, i.e., 1,000 trials.

19.

Time Between Arrivals	Cummulative Probability	RN Ranges
5	.06	01–06
10	.16	07–16
15	.39	17–39
20	.68	40–68
25	.86	69–86
30	1.00	87–99,00

Service	Cummulative Probability	RN Ranges
Doctor (D)	.50	01–50
Nurse (N)	.70	51–70
Both (B)	1.00	71–99,000

Doctor			Nurse			Both		
Time	Cummulative Probability	RN Ranges	Time	Cummulative Probability	RN Ranges	Time	Cummulative Probability	RN Ranges
10	.22	01–22	5	.08	01–08	15	.07	01–07
15	.53	23–53	10	.32	09–32	20	.23	08–23
20	.78	54–78	15	.83	33–83	25	.44	24–44
25	.90	79–90	20	1.00	84–99,000	20	.72	45–72
30	1.00	91–99,000				35	.89	73–89
						40	1.00	90–99,00

Arrival						Time of Service (mins.)			Departure Clock	
Patient	*r*	Clock	*r*	Service	*r*	Doctor	Nurse	Wait?	*D*	*N*
1	20	15	31	D	98	30			30	
2	24	30	01	D	56	20			50	
3	48	50	00	B	58	30	30		80	80
4	27	65	74	B	76	35	35	15	115	115
5	79	90	77	B	48	30	30	25	140	140
6	81	115	92	B	48	30	30	25	170	170
7	64	135	06	D	94	30		35	200	
8	34	150	53	N	88		20	20		190
9	65	170	68	N	79		15	20		205
10	22	185	30	D	53	15		15	215	
11	26	200	43	D	52	15		15	230	
12	15	210	85	B	87	35	35	20	265	265
13	52	230	46	D	47	15		35	300	
14	12	240	26	D	56	20		60	320	
15	40	260	33	D	31	15		60	335	
16	27	275	13	D	06	10		60	345	
17	76	300	55	N	13	10				310
18	51	320	57	N	31	10				330
19	38	335	83	B	79	35	35	10	380	380
20	40	355	71	B	94	40	40	25	420	420

$$P\,(\text{wait}) = \frac{15}{20} = .75$$

$$\text{Average waiting time} = \frac{440}{20}\ \text{mins.} = 22\ \text{mins.}$$

It would seem that this system is inadequate given that the probability of waiting is high and the average time is high. Also, observing the actual simulation three customers had to wait an hour and two others had to wait 35 minutes which seems excessive. Of course in order to make a fully informed decision this simulation experiment would need to be extended for more patients and then repeated several hundred times.

20.

Sales Volume	RN1 Range
300	1-12
400	13-30
500	31-50
600	51-73
700	74-91
800	91-99,00

Price	RN2 Range
$22	1-7
23	8-23
24	24-47
25	48-72
26	73-90
27	91-99,00

Variable Cost	RN3 Range
$ 8	1-17
9	18-49
10	50-78
11	79-92
12	93-99,00

c_f = $9,000

Month	**RN1**	**Sales Volume (V)**	**RN2**	**Price (p)**	**RN3**	**Variable Cost (C_v)**	$Z=VP\text{-}9{,}000\text{-}VCv$
1	58	600	47	$24	23	9	0
2	69	600	35	24	21	9	0
3	41	500	14	23	59	10	–2,500
4	28	400	68	25	13	8	–2,500
5	09	300	20	23	73	10	–5,100
6	77	700	29	24	72	10	+800
7	89	700	81	26	20	9	+2,900
8	85	700	59	25	72	10	+1,500
9	88	700	11	23	89	11	–600
10	87	700	59	25	66	10	+1,500
11	53	600	45	24	56	10	–600
12	22	400	49	25	08	8	–200
13	82	700	55	25	27	9	+2,200
14	49	500	24	24	83	11	–2,500
15	05	300	81	26	07	8	–3,600
16	78	700	92	27	36	9	+3,600
17	53	600	04	22	75	12	–3,000
18	79	700	61	25	44	9	+2,200
19	37	500	45	24	18	9	–3,500
20	65	600	37	24	30	9	0

Probability of at least breaking even = $\frac{10}{20}$ = .50

21.

Attractiveness	RN1 Range
1	1-27
2	28-62
3	63-76
4	77-85
5	86-99,00

Intelligence	RN2 Range
1	1-10
2	11-26
3	27-71
4	72-88
5	89-99,00

Personality	RN3 Range
1	1-15
2	16-45
3	46-78
4	79-85
5	86-99,00

Date	RN1	Attractiveness	RN2	Intelligence	RN3	Personality	Average Rating
1	95	5	30	3	59	3	3.67
2	93	5	28	3	72	3	3.67
3	09	1	54	3	66	3	2.33
4	95	5	36	3	98	5	4.33
5	56	2	26	2	60	3	2.33
6	79	4	14	2	50	3	3.00
7	61	2	81	4	84	4	3.33
8	14	1	24	2	75	3	2.00
9	85	4	49	3	08	1	2.67
10	09	1	53	3	45	2	2.00
11	60	2	98	5	90	5	4.00
12	86	5	74	4	55	3	4.00
13	69	3	08	1	10	1	1.67
14	96	5	06	1	62	3	3.00
15	78	4	22	2	99	5	3.67
16	61	2	18	2	45	2	2.00
17	04	1	23	2	63	3	2.00
18	16	1	20	2	99	5	2.67
19	82	4	88	4	85	4	4.00
20	03	1	65	3	33	2	2.00

Average overall rating of Salem dates = 2.92

22. There are several ways to access the accuracy of the results. First the student can determine the expected value for each characteristic and average them to see if this results in a value close to the simulated result.

E (Attractiveness) = 2.50

E (Intelligence) = 3.05

E (Personality) = 2.77

$$\text{Average rating} = \frac{2.50 + 3.05 + 2.77}{3} = 2.77$$

This is a relatively close to the simulated result of 2.92 which tends to verify that result.

Confidence limits can also be developed for the average rating. However, this is best/easiest done with Excel.

23. Capacity

$$F(x) = \frac{x^2}{360{,}000}$$

$$r = \frac{x^2}{360{,}000}$$

$$x = \sqrt{360{,}000r}$$

Demand	RN Range
0	1-3
100	4-15
200	16-35
300	36-70
400	71-90
500	91-99,00

Week	RN1	Capacity	RN2	Demand	Capacity>Demand?
1	.16	240.0	27	200	yes
2	.93	578.6	42	300	yes
3	.13	216.3	85	400	no
4	.27	311.8	46	300	yes
5	.53	436.8	19	200	yes
6	.18	254.6	31	200	yes
7	.24	293.9	1	0	yes
8	.06	146.9	19	200	no
9	.27	311.8	35	200	yes
10	.81	540.0	72	400	yes
11	.16	240.0	75	400	no
12	.37	365.0	66	300	yes
13	.74	516.1	90	400	yes
14	.39	374.7	95	500	no
15	.67	491.1	17	200	yes
16	.12	207.8	9	100	yes
17	.88	562.8	34	200	yes
18	.07	158.7	44	300	no
19	.51	428.5	70	300	yes
20	.89	566.0	95	500	yes

$$\text{Probability capacity} > \text{demand} = \frac{15}{20} = .75$$

24. White Sox:

Play Designation	Cumulative Probability	Random Numbers (*r*)
No advance	.03	01–03
Groundout	.42	04–42
Double play	.48	43–48
Long fly	.57	49–57
Very long fly	.65	58–65
Walk	.71	66–71
Infield single	.73	72–73
Outfield single	.83	74–83
Long single	.86	84–86
Double	.90	87–90
Long double	.95	91–95
Triple	.97	96–97
Home run	1.00	98–99, 00

Yankees:

Play Designation	Cumulative Probability	Random Numbers (*r*)
No advance	.04	01–04
Groundout	.42	05–42
Double play	.46	43–46
Long fly	.56	47–56
Very long fly	.62	57–62
Walk	.69	63–69
Infield single	.73	70–73
Outfield single	.83	74–83
Long single	.87	84–87
Double	.92	88–92
Long double	.95	93–95
Triple	.96	96
Home run	1.00	97–99, 00

Inning 1:

Team	*r*	Play	Outs	rbi
White Sox	39	Groundout	1	
	73	Infield single		
	72	Infield single		
	75	Outfield single		1
	37	Groundout	1	
	02		1	
				1
Yankees	87	Single		
	98	Home run		2
	10	Groundout	1	
	47	Fly	1	
	93	Double		
	21	Groundout	1	
				2

Inning 2:

Team	*r*	Play	Outs	rbi
White Sox	95	Double		
	97	Triple		1
	69	Walk		
	41	Groundout	1	1
	91	Long double		1
	80	Outfield single		1
	67	Walk		
	59	Very long fly	1	
	63	Very long fly	1	
				4
Yankees	78	Outfield single		
	87	Long single		
	47	Long fly	1	1
	56	Long fly	1	
	22	Groundout	1	
				1

Inning 3:

Team	*r*	Play	Outs	rbi
White Sox	19	Groundout	1	
	16	Groundout	1	
	78	Outfield single		
	03	Out	1	
				0
Yankees	04	Out	1	
	61	Very long fly	1	
	23	Groundout	1	
				0

Inning 4:

Team	*r*	Play	Outs	rbi
White Sox	15	Groundout	1	
	58	Very long fly	1	
	93	Long double		
	78	Outfield single		1
	61	Very long fly	1	
				1
Yankees	42	Groundout	1	
	77	Outfield single		
	65	Walk		
	71	Infield single		
	18	Groundout	1	1
	12	Groundout	1	
				1

Inning 5:

Team	*r*	Play	Outs	rbi
White Sox	17	Groundout	1	
	48	Out	1	
	89	Double		
	18	Groundout	1	
				0
Yankees	83	Outfield single		
	08	Groundout	1	
	90	Double		1
	05	Groundout	1	
	89	Double		1
	18	Groundout	1	
				2

Inning 6:

Team	*r*	Play	Outs	rbi
White Sox	08	Groundout	1	
	26	Groundout	1	
	47	Out	1	
				0
Yankees	94	Long double		
	06	Groundout	1	
	72	Infield single		1
	62	Very long fly	1	
	47	Long fly	1	
				1

Inning 7:

Team	*r*	Play	Outs	rbi
White Sox	68	Walk		
	60	Very long fly	1	
	88	Double		
	17	Groundout	1	1
	36	Groundout	1	
				1
Yankees	77	Outfield single		
	43	Double play	2	
	28	Groundout	1	
				0

Inning 8:

Team	*r*	Play	Outs	rbi
White Sox	31	Groundout	1	
	06	Groundout	1	
	68	Walk		
	39	Groundout	1	
				0
Yankees	71	Infield single		
	22	Groundout	1	
	76	Outfield single		1
	81	Outfield single		
	88	Double		1
	94	Long double		2
	76	Outfield single		1
	23	Groundout	1	
	47	Out	1	
				5

Inning 9:

Team	*r*	Play	Outs	rbi
White Sox	25	Groundout	1	
	79	Outfield single		
	08	Groundout	1	
	15	Groundout	1	
				0

Line score:

	1	2	3	4	5	6	7	8	9	Total
White Sox	1	4	0	1	0	0	1	0	0	7
Yankees	2	1	0	1	2	1	0	5	X	12

25. average maintenance cost for the life of the car = $3,594.73

P(cost ≤ $3,000) = .435

26. average profit = $813.11

probability of breaking even = .569

27. average rating = 2.91

probability of rating better than 3.0 (i.e., $P(x \geq 3.0)$) = .531

28. average surplus capacity = 47.86

probability of sufficient capacity = .599

29. (a) In Crystal Ball include the parameters (μ, σ) for each fund return distribution in a spreadsheet cell. Select 6 other spreadsheet cells for the investment combination returns. For example, if the return parameters for investment 1 is in cell D6, the return parameters for investment 2 are in cell D7 and the total return for the investment combination (1,2) is in cell C12, the formula in C12 is "=50,000* (1+D6)^3 + 50,000(1+D67)^3"

The simulation results are as follows,

Combination	Return	*P* (Return ≥ 120,000)
1,2	μ = 157,572, σ = 21,459	(.974)
1,3	μ = 166,739, σ = 28,599	(.959)
1,4	μ = 161,888, σ = 23,016	(.981)
2,3	μ = 148,692, σ = 22,693	(.911)
2,4	μ = 144,841, σ = 13,683	(.980)
3,4	μ = 129,429, σ = 20,886	(.653)

(b) The highest probability of exceeding $120,000 is virtually tied at .98 between (1,4) and (2,4).

30. *Q:* μ = 1,860.83, σ = 435.83
TC: μ = 1,465.2, σ = 326.85

31. (a) Select an Excel spreadsheet cell to include the normal distribution parameters (μ = 800, σ = 270) for conference rooms, for example, cell D4.