Chapter Thirteen: Queuing Analysis

PROBLEM SUMMARY

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43. Multiple-server model analysis
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45. Single server model, finite queue
46. Multiple-server model, decision analysis
47. Multiple-server model, decision analysis
48. Multiple-server model, decision analysis

PROBLEM SOLUTIONS

1. a) Hair salon: multiple-server; first-come, first-served or appointment; calling population can be finite (appointments only) or infinite (off-the-street business)

b) Bank: multiple-server; first-come, first-served; infinite calling population

c) Laundromat: multiple-server; first-come, first-served; infinite calling population

d) Doctor's office: single- (or multiple-) server; appointment (usually); finite calling population

e) Advisor's office: single-server; first-come, first-served or appointment; finite calling population

f) Airport runway: single-server; first-come, first-served; finite calling population

g) Service station: multiple-server; first-come, first-served; infinite calling population

h) Copy center: single- or multiple-server; first-come, first-served; infinite calling population

i) Team trainer: single-server; first-come, first-served or appointment; finite calling population

j) Mainframe computer: multiple-server; first-come, first-served (or priority level); infinite calling population

2. The addition of a new counter created two queues. The multiple-server model is for a single queue with more than one server.

3. a) False. The operating characteristic values may be higher or lower depending on the magnitude of the standard deviation compared to the mean of the exponentially distributed service time.

b) True. Since there is no variability the operating characteristics would always be lower.

4. When arrivals are random, in the short run more customers may arrive than the serving system can accommodate.

5. When customers are served according to a prearranged schedule or alphabetically, or are picked at random.

6. When $\mu = \sigma$

7. $\lambda = 16$ per hour; $\mu = 24$ per hour; $P_0 = 1 - \dfrac{\lambda}{\mu} = 1 - \dfrac{16}{24} = .33$; $P_3 = \left(\dfrac{\lambda}{\mu}\right)^3 \cdot P_0 = (.67)^3 \cdot 33 = .099$; $L = \dfrac{\lambda}{\mu - \lambda} = \dfrac{16}{8} = 2.0$; $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{(16)^2}{24(8)} = 1.33$; $W = \dfrac{1}{\mu - \lambda} = \dfrac{1}{8} = .125$ hr (7.5 min); $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{16}{24(8)} = .083$ hr (5 min); $U = \dfrac{\lambda}{\mu} = \dfrac{16}{24} = .67$

8. $\lambda = 10$; $\mu = 12$; $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{10}{12(2)} = .41$ hr (24.6 min); $U = \dfrac{\lambda}{\mu} = \dfrac{10}{12} = .833$

9. $\lambda = 6$; $\mu = 10$; $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{(6)^2}{10(4)} = .9$ car; $W = \dfrac{1}{\mu - \lambda} = \dfrac{1}{4} = .25$ hr (15 min); $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{6}{10(4)} = .15$ hr (9 min);if the arrival rate is increased to 12 per hour, the arrival rate would exceed the service rate; thus, an infinite queue length would result.

10. The arrival rate must be on an hourly basis; $\lambda = \dfrac{60}{7.5} = 8$ per hour; $\mu = 10$ per hour; $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{(8)^2}{10(2)} = 3.2$ parts; $U = \dfrac{\lambda}{\mu} = \dfrac{8}{10} = .80$; $I - U = I - .80 = .20$

11. $U = \dfrac{\lambda}{\mu}$; $\mu = 10$ per hour; $U = .90$; therefore, $.90 = \dfrac{\lambda}{10}$, or $\lambda = 9$ per hour, or 1 part every 6.67 min.

12. $\lambda = 4$ per hour; $\mu = \dfrac{60}{12} = 5$ per hour

a. $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{(4)^2}{5(1)} = 3.2$

b. $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{4}{5(1)} = .80$ hr (48 min)

c. $W = \dfrac{1}{\mu - \lambda} = 1$ hr (60 min)

d. 45 min per hour is a $\dfrac{45}{60}$, or .75, utilization factor. U cannot exceed .75. $U = \dfrac{\lambda}{\mu} = \dfrac{4}{5} = .80$ presently. Therefore, one more air traffic controller must be hired

13. $\lambda = 12$ per hour; $\mu = \dfrac{60}{4} = 15$ per hour. One window: $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{12}{15(3)} = .26$ hr (16 min). Two windows: $\mu = 15$ per hour (does not change). However, the arrival rate for each window is now split. $\lambda = 6$ per hour; $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{6}{15(9)} = .044$ hr (2.67 min); $16 - 2.67 = 13.33$ min, reduction in waiting time; $13.33 \times \$2{,}000 = \$26{,}660$; cost of window $= \$20{,}000$; $\$26{,}660 > \$20{,}000$; therefore, the second drive-in window should be installed.

14. $\lambda = 28$ per hour; $\mu = \dfrac{60}{2} = 30$ per hour

a. $L = \dfrac{\lambda}{\mu - \lambda} = \dfrac{28}{2} = 14$; $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{(28)^2}{30(2)} = 13.1$; $W = \dfrac{1}{\mu - \lambda} = \dfrac{1}{2} = .5$ hr (30 min); $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{28}{30(2)} = .47$ hr (28.2 min); $U = \dfrac{\lambda}{\mu} = \dfrac{28}{30} = .93 = 93\%$

b. $W = \dfrac{1}{\mu - \lambda}$; let $W = 10$ min $= .167$ hr; $.167 = \dfrac{1}{\mu - 28}$, so $(\mu - 28)(.167) = 1$; $\mu - 28 = 6$, so $\mu = 34$ students per hour. $\dfrac{60}{34} = 1.76$ min required to approve a schedule in order to meet the dean's goal. Since each assistant will reduce the service time by .25 min, then 1 more assistant is all that is needed (i.e., 2.00 min – .25 = 1.75 min).