Chapter Eight: Project Management

PROBLEM SUMMARY

1. Network construction and analysis

2. Network construction and analysis

3. Network construction and analysis

4. Network analysis

5. Network, earliest and latest event times, slack

6. Network, earliest and latest event times, slack

7. Network, earliest and latest event times, slack

8. Network, earliest and latest event times, slack

9. Network, earliest and latest event times, slack

10. Network construction and analysis

11. Network analysis

12. Network analysis

13. Network analysis

14. Network analysis

15. Probability analysis

16. Network analysis, probability analysis

17. Network analysis, probability analysis

18. Network analysis, probability analysis

19. Probability analysis

20. Network analysis, probability analysis

21. Network construction, probability analysis

22. Network construction, probability analysis

23. Network construction, probability analysis

24. Project crashing, linear programming model formulation

25. Project crashing, linear programming model formulation

26. General linear programming model formulation

27. General linear programming model formulation

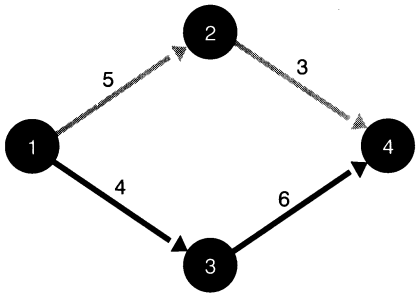
28. Project crashing, linear programming model formulation

29. Project crashing, computer

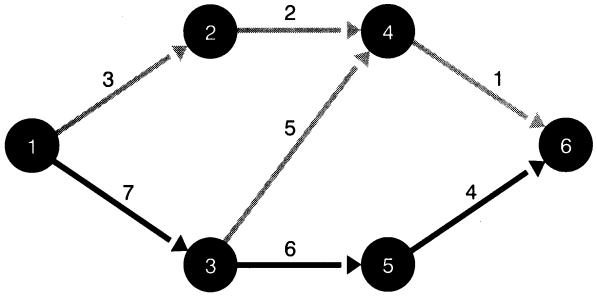
30. Project crashing, computer

PROBLEM SOLUTIONS

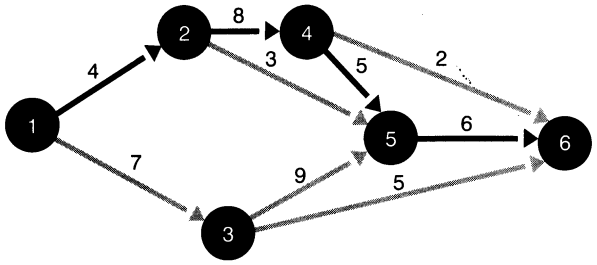
1. Paths: $1 \rightarrow 2 \rightarrow 4 = 5 + 3 = 8$; $1 \rightarrow 3 \rightarrow 4 = 4 + 6 = 10^*$; path $1 \rightarrow 3 \rightarrow 4$ is critical.



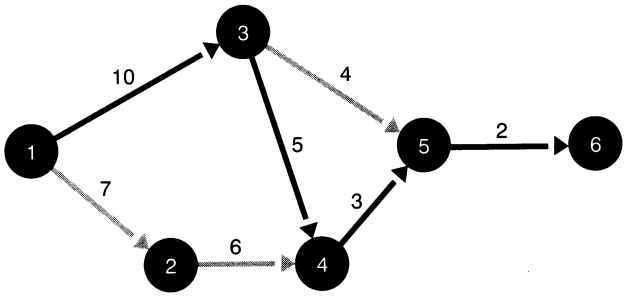
2. Paths: $1 \rightarrow 2 \rightarrow 4 \rightarrow 6 = 3 + 2 + 1 = 6$; $1 \rightarrow 3 \rightarrow 4 \rightarrow 6 = 7 + 5 + 1 = 13$; $1 \rightarrow 3 \rightarrow 5 \rightarrow 6 = 7 + 6 + 4 = 17^*$, critical path



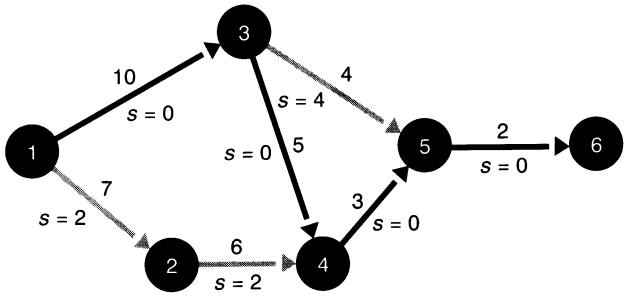
3. Paths: $1 \rightarrow 2 \rightarrow 4 \rightarrow 6$, $4 + 8 + 2 = 14$; $1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6$, $4 + 8 + 5 + 6 = 23^*$; $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$, $4 + 3 + 6 = 13$; $1 \rightarrow 3 \rightarrow 5 \rightarrow 6$, $7 + 9 + 6 = 22$; $1 \rightarrow 3 \rightarrow 6$, $7 + 5 = 12$



4. Paths: $1 \rightarrow 3 \rightarrow 5 \rightarrow 6 = 10 + 4 + 2 = 16$; $1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 = 10 + 5 + 3 + 2 = 20^*$, critical path; $1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6 = 7 + 6 + 3 + 2 = 18$



5. The critical path activities have no slack.

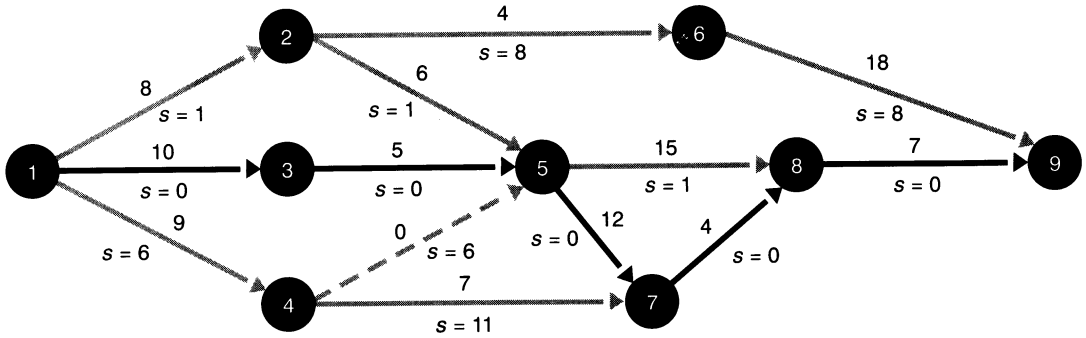


Activity	Time	ES	EF	LS	LF	Slack
1 – 2	7	0	7	2	9	2
1 – 3	10	0	10	0	10	0
2 – 4	6	7	13	9	15	2
3 – 4	5	10	15	10	15	0
3 – 5	4	10	14	14	18	4
4 – 5	3	15	18	15	18	0
5 – 6	2	18	20	18	20	0

Critical path = 1 – 3 – 4 – 5 –6

The critical path activities have no slack.

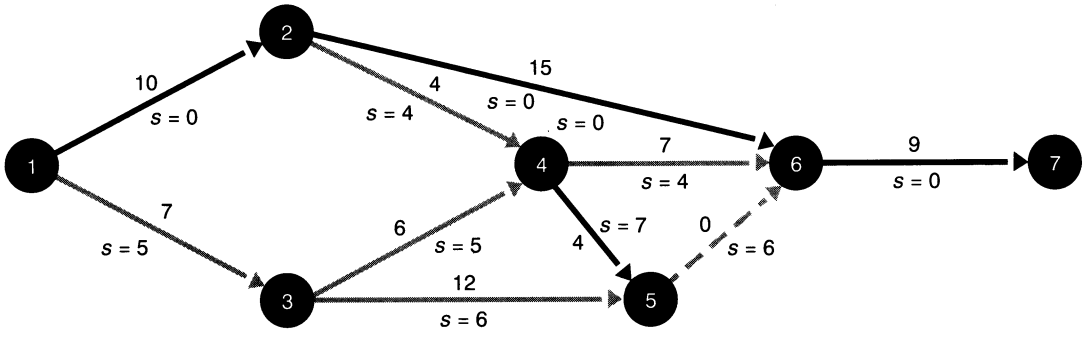
6.



Activity	Time	ES	EF	LS	LF	Slack
1 – 2	8	0	8	1	9	1
1 – 3	10	0	10	0	10	0
1 – 4	9	0	9	6	15	6
2 – 5	6	8	14	9	15	1
2 – 6	4	8	12	16	20	8
3 – 5	5	10	15	10	15	0
4 – 5	0	9	9	15	15	6
4 – 7	7	9	16	20	27	11
5 – 7	12	15	27	15	27	0
5 – 8	15	15	30	16	31	1
6 – 9	18	12	30	20	38	8
7 – 8	4	27	31	27	31	0
8 – 9	7	31	38	31	38	0

Critical Path = 1 – 3 – 5 – 7 – 8 – 9
Project duration = 38

7.



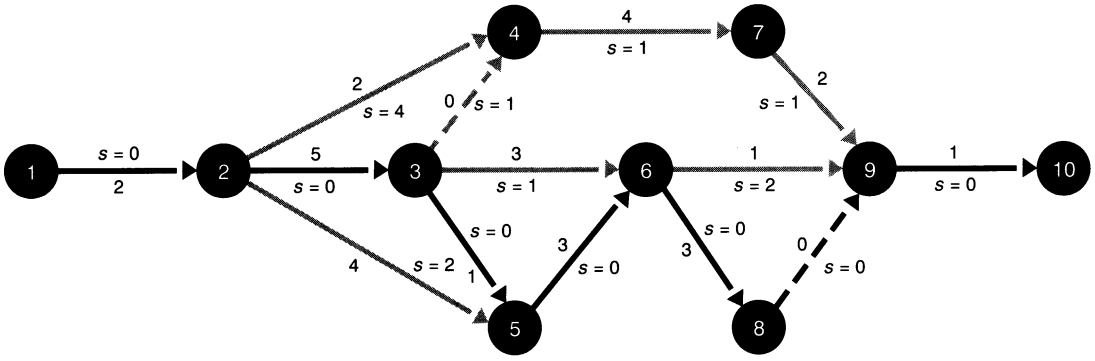
Activity	Time	ES	EF	LS	LF	Slack
1 – 2	10	0	10	0	10	0
1 – 3	7	0	7	5	12	5
2 – 4	4	10	14	14	18	4
2 – 6	15	10	25	10	25	0
3 – 4	6	7	13	12	18	5
3 – 5	12	7	19	13	25	6
4 – 5	4	14	18	21	25	7
4 – 6	7	14	21	18	25	4
5 – 6	0	19	19	25	25	6
6 – 7	9	25	34	25	34	0

Critical Path = 1 – 2 – 6 – 7
Project completion time = 34

8.

Activity	Time	ES	EF	LS	LF	Slack
1 – 2	2	0	2	0	2	0
2 – 3	5	2	7	2	7	0
2 – 4	2	2	4	6	8	4
2 – 5	4	2	6	4	8	2
3 – 4	0	7	7	8	8	1
3 – 5	1	7	8	7	8	0
3 – 6	3	7	10	8	11	1
4 – 7	4	7	11	8	12	1
5 – 6	3	8	11	8	11	0
6 – 8	3	11	14	11	14	0
6 – 9	1	11	12	13	14	2
7 – 9	2	11	13	12	14	1
8 – 9	0	14	14	14	14	0
9 – 10	1	14	15	14	15	0

Time until General is ready to battle = 15 days
Critical path = 1 – 2 – 3 – 5 – 6 – 8 – 9 – 10



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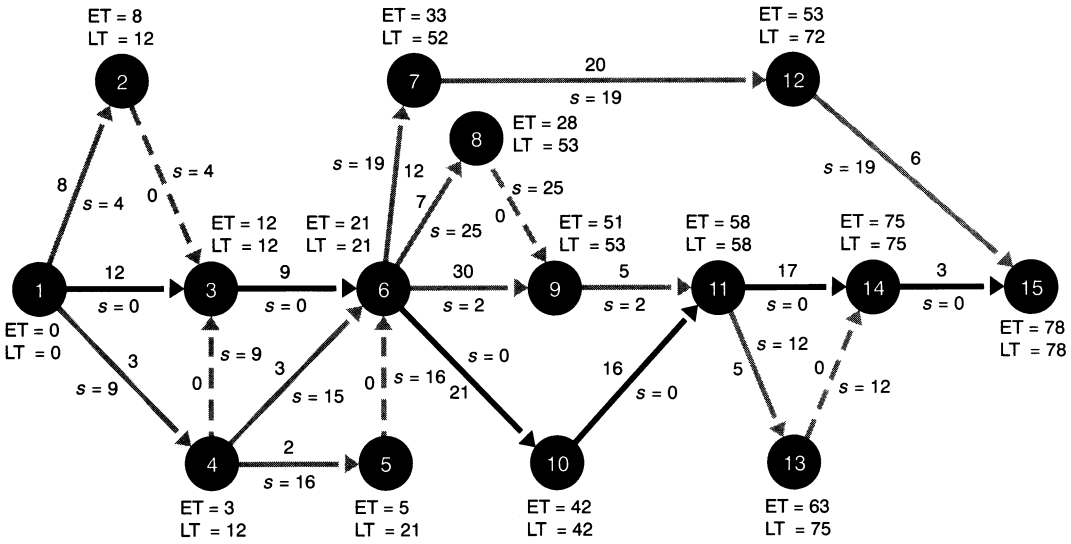
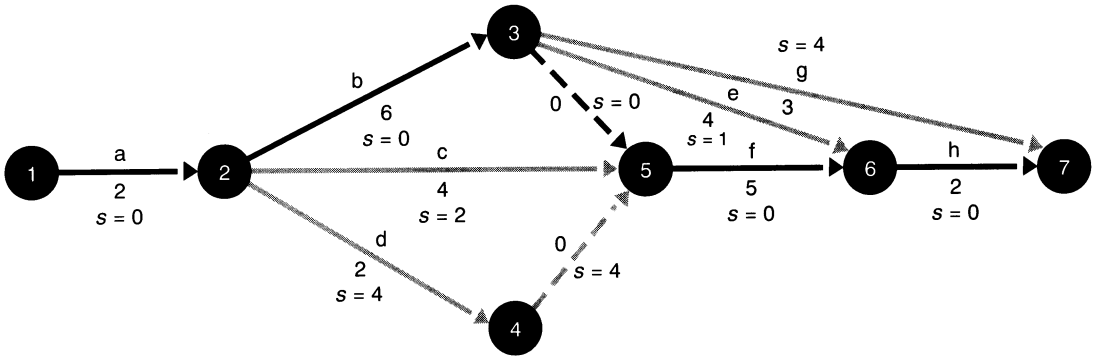


Activity	Time	ES	EF	LS	LF	Slack
1 – 2	8	0	8	4	12	4
1 – 3	12	0	12	0	12	0
1 – 4	3	0	3	9	12	9
2 – 3	0	8	8	12	12	4
3 – 6	9	12	21	12	21	0
4 – 3	0	3	3	12	12	9
4 – 5	2	3	5	19	21	16
4 – 6	3	3	6	18	21	15
5 – 6	0	5	5	21	21	16
6 – 7	12	21	33	40	52	19
6 – 8	7	21	28	46	53	25
6 – 9	30	21	51	23	53	2
6 – 10	21	21	42	21	42	0
7 – 12	20	33	53	52	72	19
8 – 9	0	28	28	53	53	25
9 – 11	5	51	56	53	58	2
10 – 11	16	42	58	42	58	0
11 – 13	5	58	63	70	75	12
11 – 14	17	58	75	58	75	0
12 – 15	6	53	59	72	78	19
13 – 14	0	63	63	75	75	12
14 – 15	3	75	78	75	78	0

Critical path = 1 – 3 – 6 – 10 – 11 – 14 – 15
Project completion time = 78 weeks

10.



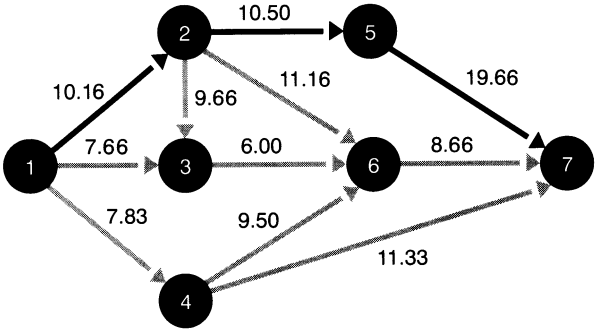
Activity		Time	ES	EF	LS	LF	Slack
(a)	1 – 2	2	0	2	0	2	0
(b)	2 – 3	6	2	8	2	8	0
(c)	2 – 5	4	2	6	4	8	2
(d)	2 – 4	2	2	4	6	8	4
(e)	3 – 6	4	8	12	9	13	1
(f)	5 – 6	5	8	13	8	13	0
dummy	3 – 5	0	8	8	8	8	0
(g)	3 – 7	3	8	11	12	15	4
(h)	6 – 7	2	13	15	13	15	0
dummy	4 – 5	0	4	4	8	8	4

Critical path = a – b – dummy – f – h or 1 – 2 – 3 – 5 – 6 – 7
Project completion time = 15

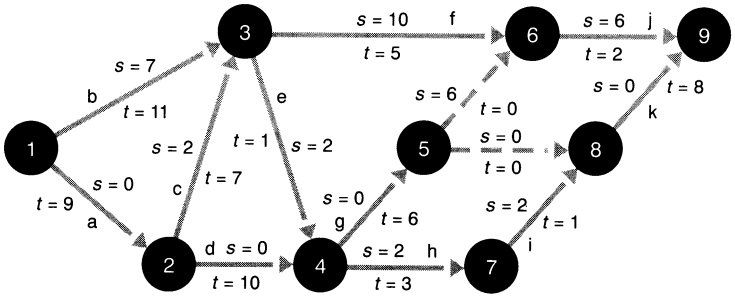
11.

Activity	*a*	*m*	*b*	*t*	ES	EF	LS	LF	Slack	σ
1 – 2	6	10	15	10.16	0	10.16	0	10.16	0	1.50
1 – 3	2	7	16	7.66	0	7.66	18.00	25.66	18.00	2.33
1 – 4	4	8	11	7.83	0	7.83	14.33	22.16	14.33	1.16
2 – 3	3	10	15	9.66	10.16	19.83	16.00	25.66	5.83	2.00
2 – 5	7	9	20	10.50	10.16	20.66	10.16	20.66	0	2.16
2 – 6	4	12	15	11.16	10.16	21.33	20.50	31.66	10.33	1.83
3 – 6	3	6	9	6.00	19.83	25.83	25.66	31.66	5.83	1.00
4 – 6	5	9	16	9.50	7.83	17.33	22.16	31.66	14.33	1.83
5 – 7	3	20	35	19.66	20.66	40.33	20.66	40.33	0	5.33
4 – 7	4	12	16	11.33	7.83	19.16	29.00	40.33	21.16	2.00
6 – 7	2	9	14	8.66	25.83	34.50	31.66	40.33	5.83	2.00

Expected project completion time = 40.33
σ = 5.95
Critical path = 1 – 2 – 5 – 7

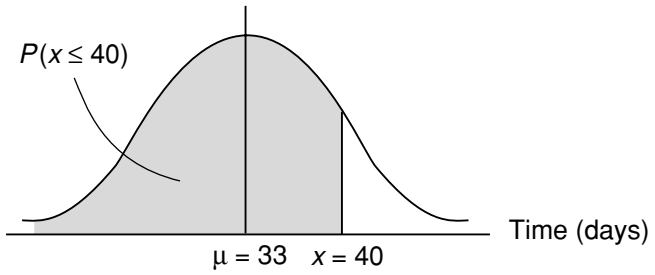


12.



Activity		*a*	*m*	*b*	*t*	ES	EF	LS	LF	Slack	σ
(a)	1 – 2	5	8	17	9	0	9	0	9	0	2
(b)	1 – 3	3	12	15	11	0	11	7	18	7	2
(c)	2 – 3	4	7	10	7	9	16	11	18	2	1
(d)	2 – 4	5	8	23	10	9	19	9	19	0	3
(e)	3 – 4	1	1	1	1	16	17	18	19	2	0
(f)	3 – 6	1	4	13	5	16	21	26	31	10	2
(g)	4 – 5	3	6	9	6	19	25	19	25	0	1
(h)	4 – 7	1	2.5	7	3	19	22	21	24	2	1
dummy	5 – 6	0	0	0	0	25	25	31	31	6	0
dummy	5 – 8	0	0	0	0	25	25	25	25	0	0
(i)	7 – 8	1	1	1	1	22	23	24	25	2	0
(j)	6 – 9	2	2	2	2	25	27	31	33	6	0
(k)	8 – 9	5	8	11	8	25	33	25	33	0	1

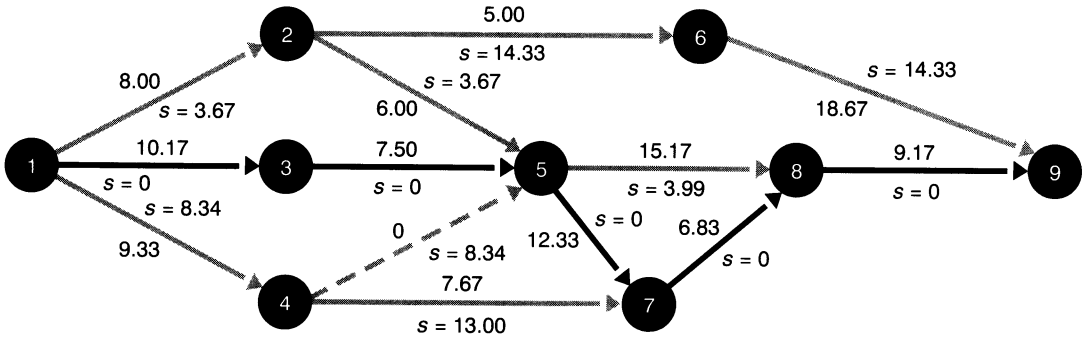
Critical path = a – d – g – dummy – k or 1– 2 – 4 – 5 – 8 – 9
Expected project completion time = 33 weeks
σ = 3.87



$$Z = \frac{x - \mu}{\sigma} = \frac{40 - 33}{3.87} = 1.81$$

$P(x \le 40) = .9649$

13. (a, b, c, d)



Activity	*a*	*m*	*b*	*t*	ES	EF	LS	LF	Slack	σ
1 – 2	4	8	12	8.00	0	8.00	3.66	11.66	3.66	1.33
1 – 3	6	10	15	10.16	0	10.16	0	10.16	0	1.50
1 – 4	2	10	14	9.33	0	9.33	8.33	17.66	8.33	2.00
2 – 5	3	6	9	6.00	8.00	14.00	11.66	17.66	3.66	1.00
2 – 6	1	4	13	5.00	8.00	13.00	22.33	27.33	14.33	2.00
3 – 5	3	6	18	7.50	10.16	17.66	10.16	17.66	0	2.50
4 – 5	0	0	0	0	9.33	9.33	17.66	17.66	8.33	0
4 – 7	2	8	12	7.66	9.33	17.00	22.33	30.00	13.00	1.66
5 – 8	9	15	22	15.16	17.66	32.83	21.66	36.83	4.00	2.16
5 – 7	5	12	21	12.33	17.66	30.00	17.66	30.00	0	2.66
7 – 8	5	6	12	6.83	30.00	36.83	30.00	36.83	0	1.16
6 – 9	7	20	25	18.66	13.00	46.00	27.33	46.00	14.33	3.00
8 – 9	3	0	20	9.16	36.83	46.00	36.83	46.00	0	2.83

e) Critical path = 1 – 3 – 5 – 7 – 8 – 9

f) Expected project completion time = 46 months
σ = 5 months

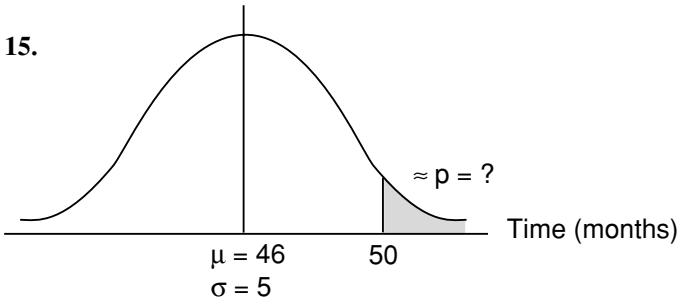
14. a, b, c, d)

Activity	*a*	*m*	*b*	*t*	ES	EF	LS	LF	Slack	σ
1 – 2	1	2	6	2.50	0	2.50	0	2.50	0	.833
2 – 4	1	3	5	3.00	2.50	5.50	7.50	10.50	5.00	.66
2 – 3	3	5	10	5.50	2.50	8.00	2.50	8.00	0	1.16
2 – 5	3	6	14	6.83	2.50	9.33	2.66	9.50	.16	1.83
3 – 4	0	0	0	0	8.00	8.00	10.50	10.50	2.50	0
3 – 5	1	1.5	2	1.50	8.00	9.50	8.00	9.50	0	.16
3 – 6	2	3	7	3.50	8.00	11.50	9.00	12.50	1.00	.83
4 – 7	2	4	9	4.50	8.00	12.50	10.50	15.00	2.50	1.16
5 – 6	1	3	5	3.00	9.50	12.50	9.50	12.50	0	.66
7 – 9	1	2	3	2.00	12.50	14.50	15.00	17.00	2.50	.33
6 – 9	1	1	5	1.66	12.50	14.16	15.33	17.00	2.83	.66
6 – 8	2	4	9	4.50	12.50	17.00	12.50	17.00	0	1.16
8 – 9	0	0	0	0	17.00	17.00	17.00	17.00	0	0
9 – 10	1	1	1	1.00	17.00	18.00	17.00	18.00	0	0

e) Critical path = 1 – 2 – 3 – 5 – 6 – 8 – 9 – 10

f) Expected project completion time = 18 days
σ = 1.97

15.



$$Z = \frac{x - \mu}{\sigma}$$

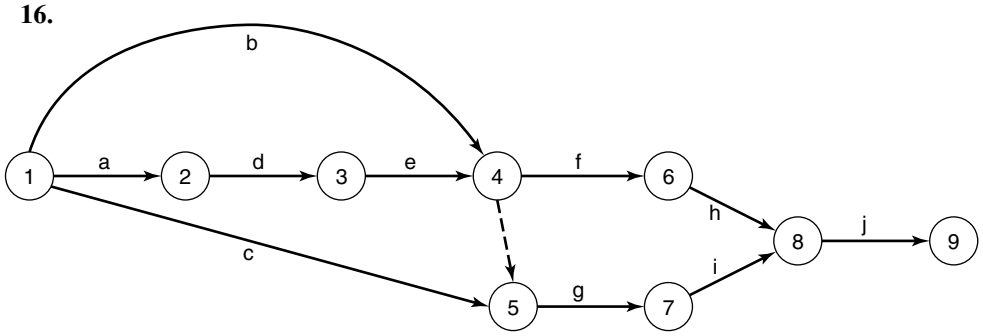
$$= \frac{50 - 46}{5}$$

$$= 4/5 = .80$$

From normal table, $p = .2881$

.5000 – .2881 = .2119 probability that the project will exceed 50 months

16.



	Activity Time	ES	EF	LS	LF	Slack	Standard Deviation
Project	23						1.7
a	3	0	3	0	3	0	0.667
b	3.167	0	3.167	7.667	10.833	7.667	0.5
c	4.167	0	4.167	6.667	10.833	6.667	0.833
d	2.833	3	5.833	3	5.833	0	0.5
e	5	5.833	10.833	5.8333	10.833	0	1
f	1.833	10.833	12.667	15.167	17	4.333	0.167
g	5.833	10.833	16.667	10.833	16.667	0	0.833
h	3.833	12.667	16.5	17	20.833	4.333	0.5
i	4.167	16.667	20.833	16.667	20.833	0	0.5
j	2.167	20.833	23	20.833	23	0	0.5

Probability the project will be completed in 21 days?

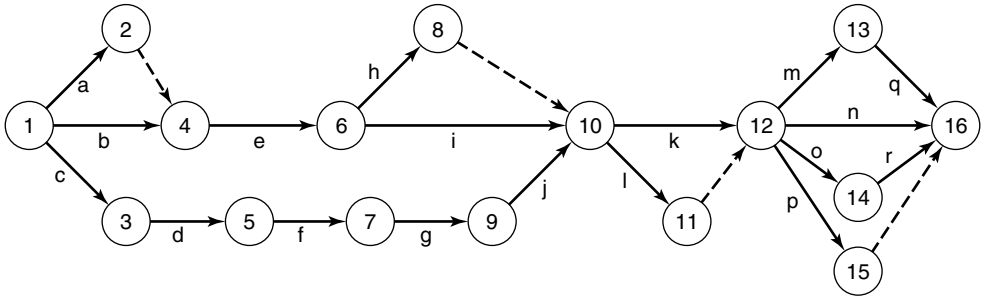
$$Z = \frac{x - \mu}{\sigma}$$

$$Z = \frac{21 - 23}{1.7} = .986$$

$P(x \leq 21) = .379$

17.

	Activity Time	ES	EF	LS	ES	Slack	Standard Deviation
Project	160.833						8.54
a	24.833	0	24.833	45.5	70.333	45.5	2.167
b	22.833	0	22.833	47.5	70.333	47.5	2.5
c	40.167	0	40.167	0	40.167	0	5.167
d	30.833	40.167	71	40.167	71	0	4.167
e	21	24.833	45.833	70.333	91.3333	45.5	3
f	17.167	71	88.167	71	88.167	0	2.5
g	11.833	88.167	100	88.167	100	0	2.167
h	19.167	45.833	65	91.333	110.5	45.5	2.5
i	15.167	45.833	61	95.333	110.5	49.5	2.167
j	10.5	100	110.5	100	110.5	0	1.167
k	28	110.5	138.5	110.5	138.5	0	3.333
l	10.167	110.5	120.667	128.333	138.5	17.8333	1.5
m	7	138.5	145.5	148	155	9.5	1
n	14.333	138.5	152.833	146.5	160.833	8	1.667
o	14.5	138.5	153	138.5	153	0	2.167
p	4.167	138.5	142.667	156.667	160.833	18.1667	0.5
q	5.833	145.5	151.333	155	160.833	9.5	0.5
r	7.833	153	160.833	153	160.833	0	0.833

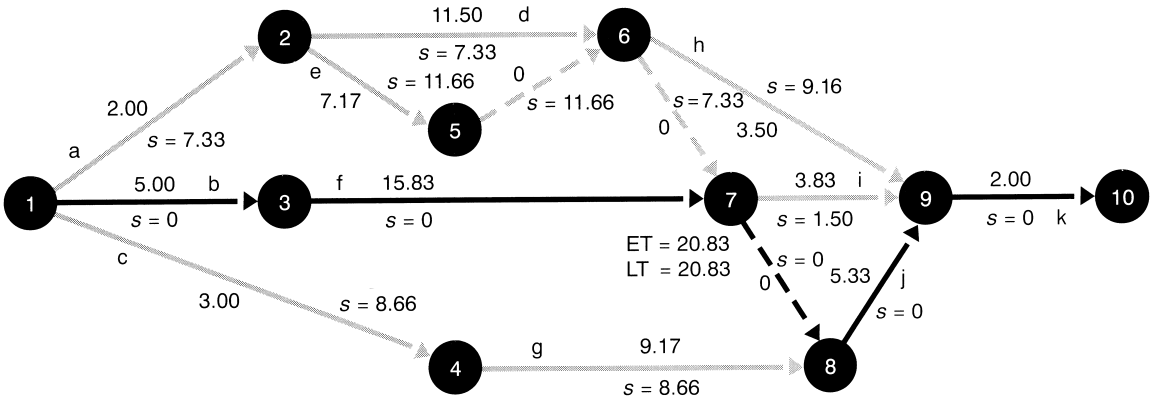


Critical path: c – d – f – g – j – k – o – r

$$Z = \frac{x - \mu}{\sigma} = \frac{180 - 160.83}{8.54} = 2.24$$

$P(x \leq 180 \text{ minutes}) = .5000 + .4875 = .9875$

18. (a, b)



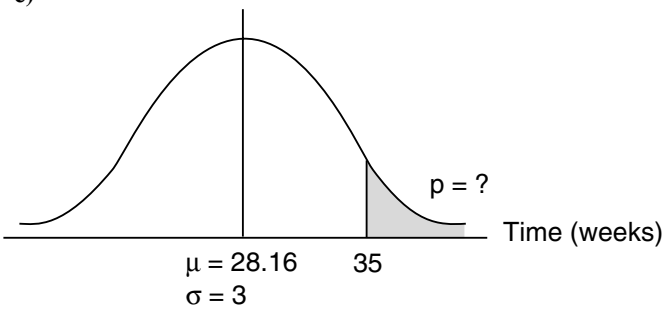
Activity		*a*	*m*	*b*	*t*	ES	EF	LS	LF	Slack	σ
(a)	1 – 2	1	2	3	2.00	0	2.00	7.33	9.33	7.33	.33
(b)	1 – 3	2	5	8	5.00	0	5.00	0	5.00	0	1.00
(c)	1 – 4	1	3	5	3.00	0	3.00	8.66	11.66	8.66	.66
(d)	2 – 6	4	10	25	11.50	2.00	13.50	9.33	20.83	7.33	3.50
(e)	2 – 5	3	7	12	7.16	2.00	9.16	13.66	20.83	11.66	1.50
(f)	3 – 7	10	15	25	15.83	5.00	20.83	5.00	20.83	0	2.50
(g)	4 – 8	5	9	14	9.16	3.00	12.16	11.66	20.83	8.66	1.50
dummy	5 – 6	0	0	0	0	9.16	9.16	20.83	20.83	11.66	0
dummy	6 – 7	0	0	0	0	13.50	13.50	20.83	20.83	7.33	0
(h)	6 – 9	2	3	7	3.50	13.50	17.00	22.66	26.16	9.16	.83
dummy	7 – 8	0	0	0	0	20.83	20.83	20.83	20.83	0	0
(i)	7 – 9	1	4	6	3.83	20.83	24.66	22.33	26.16	1.50	.83
(j)	8 – 9	2	5	10	5.33	20.83	26.16	20.83	26.16	0	1.33
(k)	9 – 10	2	2	2	2	26.16	28.16	26.16	28.16	0	0

c) Critical path = b – f – dummy – j – k or 1 – 3 – 7 – 8 – 9 – 10

d) Expected project completion time = 28.17

$\sigma = 3.00$

e)



$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{35 - 28.16}{3}$$

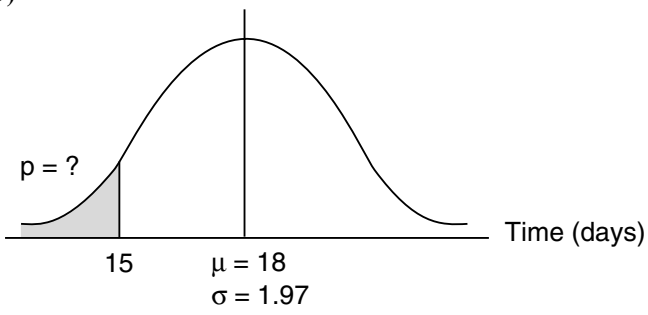
$$= \frac{6.84}{3.00}$$

$$= 2.28$$

From normal table, $p = .4887$

.5000 – .4887 = .0113 probability that the company will be fined

19)



$$Z = \frac{x - \mu}{\sigma}$$

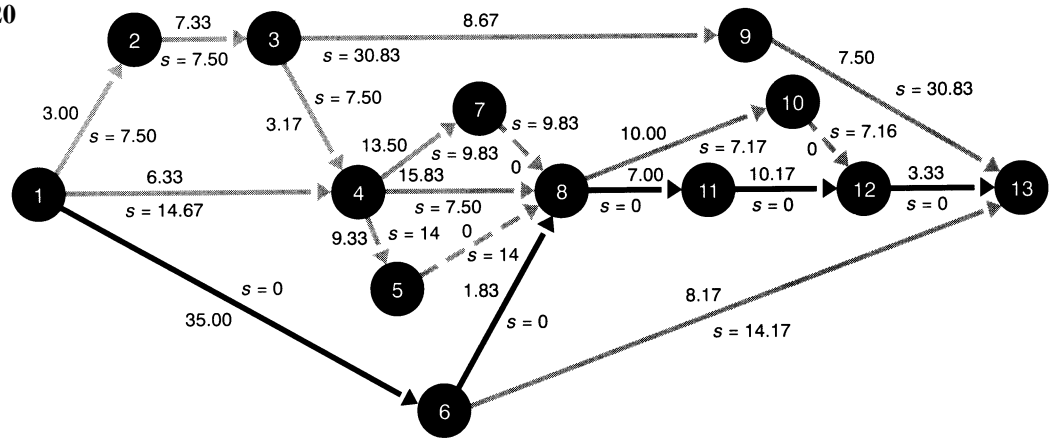
$$= \frac{15 - 18}{1.97}$$

$$= -1.52$$

From normal table, $p = .4357$

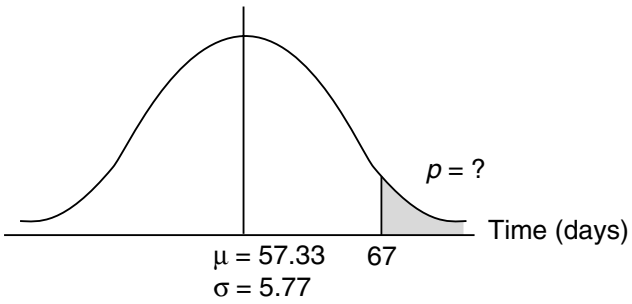
.5000 – .4357 = .0643 probability that the preparations would be in time

20



Activity	*a*	*m*	*b*	*t*	ES	EF	LS	LF	Slack	σ
1 – 2	1	3	5	3.00	0	3.00	7.50	10.50	7.50	.66
1 – 4	4	6	10	6.33	0	6.33	14.66	21.00	14.66	1.00
1 – 6	20	35	50	35.00	0	35.00	0	35.00	0	5.00
2 – 3	4	7	12	7.33	3.00	10.33	10.50	17.83	7.50	1.33
3 – 4	2	3	5	3.16	10.33	13.50	17.83	21.00	7.50	0.50
4 – 7	8	12	25	13.50	13.50	27.00	23.33	36.83	9.83	2.83
4 – 8	10	16	21	15.83	13.50	29.33	21.00	36.83	7.50	1.83
4 – 5	5	9	15	9.33	13.50	22.83	27.50	36.83	14.00	1.66
3 – 9	6	8	14	8.66	10.33	19.00	41.16	49.83	30.83	1.33
5 – 8	0	0	0	0	22.83	22.83	36.83	36.83	14.00	0
6 – 8	1	2	2	1.83	35.00	36.83	35.00	36.83	0	.16
6 – 13	5	8	12	8.16	35.00	43.16	49.16	57.83	14.16	1.16
7 – 8	0	0	0	0	27.00	27.00	36.83	36.83	9.83	0
8 – 10	5	10	15	10.00	36.83	46.83	44.00	54.00	7.16	1.66
8 – 11	4	7	10	7.00	36.83	43.83	36.83	43.83	0	1.00
9 – 13	5	7	12	7.50	19.00	26.50	49.83	57.33	30.83	1.16
10 – 12	0	0	0	0	46.83	46.83	54.00	54.00	7.16	0
11 – 12	5	9	20	10.16	43.83	54.00	43.83	54.00	0	2.50
12 – 13	1	3	7	3.33	54.00	57.33	54.00	57.33	0	1.00

Expected project completion time = 57.37 days



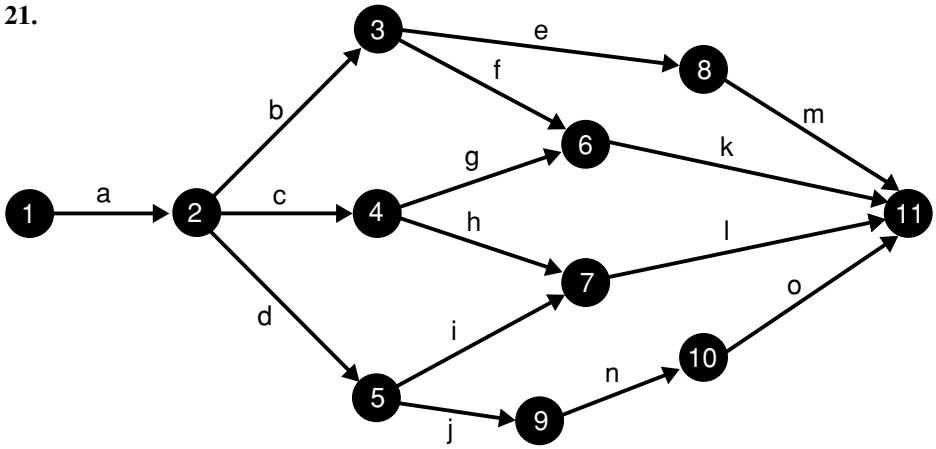
Expected completion time = 57.33

$\sigma = 5.77$

$$Z = \frac{x - \mu}{\sigma} = \frac{67 - 57.33}{5.77} = 1.68$$

$P(x \le 67) = .9535$

21.



Critical path = a – b – d – j – n – o

Expected project completion time = 45 weeks

$\sigma = 4.10$

Since probability .90, $Z = 1.29$

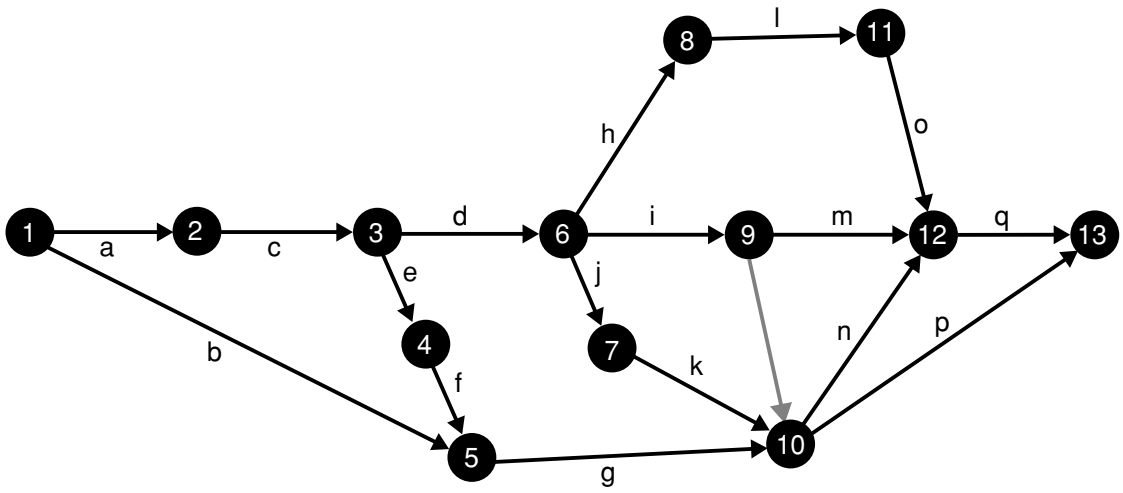
$$1.29 = \frac{x - 45}{4.10}$$

$$x - 45 = 5.29$$

$$x = 50.3$$

To be 90% certain of delivering the part on time, RusTech should probably specify at least 50.3 weeks in the contract bid.

22.



The activity schedule, critical path, expected project time and variance determined using QSB + as follows

Activity	Activity Duration	Activity Variance	ES	LS	EF	LF	Slack
a	15	336.111	0	0.000	31.666	31.667	Critical
b	8.833	1.361	31.667	81.1667	40.5	90	49.50
c	24.167	6.25	31.667	31.667	55.833	55.833	Critical
d	19.5	4.694	55.833	55.833	75.333	75.333	Critical
e	8.167	1.361	55.833	68.167	64	76.333	12.333
f	13.667	5.444	64	76.333	77.667	90	12.333
g	20.167	2.25	77.667	90	97.833	110.167	12.333
h	25	11.111	75.333	75.333	100.333	100.333	Critical
i	14.667	4	75.333	94.833	90	109.5	19.5
j	23	5.444	75.333	78.50	98.333	101.5	3.166
k	8.666	1.778	98.333	101.5	107	110.167	3.167
duminp	0	0	90	110.167	90	110.167	20.167
l	7.167	2.25	100.333	100.333	107.5	107.5	Critical
m	5	.444	90	109.5	95	114.5	19.5
n	4.333	1	107	110.167	111.333	114.5	3.167
o	7	1	107.5	107.5	114.5	114.5	Critical
p	5.5	.694	107	129.833	112.5	135.333	22.833
q	20.833	6.25	114.5	114.5	135.333	135.333	Critical

Expected Completion Time = 118.67 CPU seconds = 0
Total Variance = 391 Total Variance on Critical Path = 34.33

Critical path for PM Computers

Critical path: a ==> c ==> d ==> h ==> l ==> o ==> q.
Variance on this path = 34.33

Done.

QSB+ will also determine the probability that the marketing program will be completed within 6 months, or 180 days, as follows.

Probability Analysis for PM Computers

The following probability calculation assumes that activities are independent and so are all paths. It also assumes that your network has a large enough number of activities to assume the normality. Therefore, when the activities are not independent or the number of activities is not large, the analysis may be biased.

Expected completion time (50% chance)
= 135.333

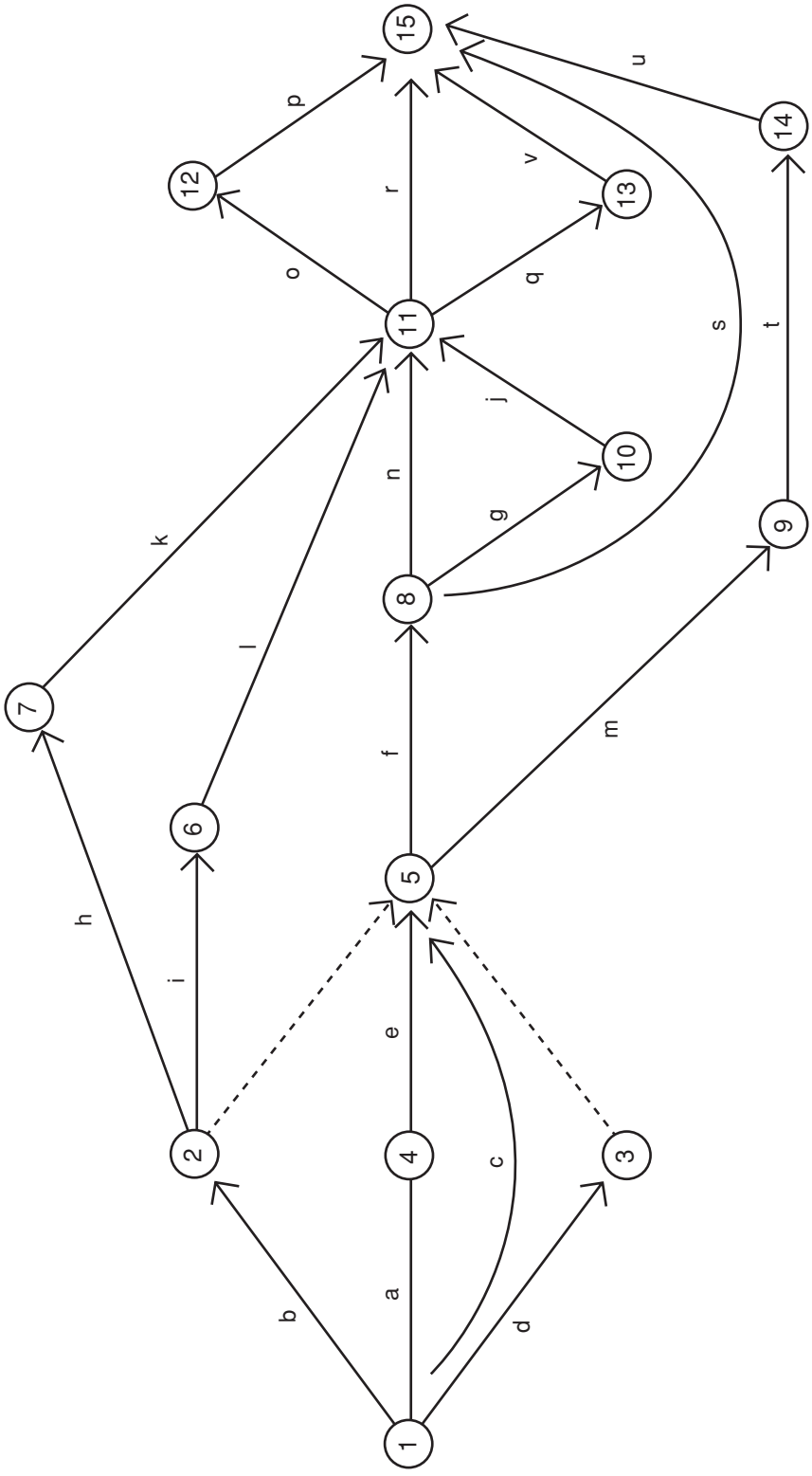
When do you expect to finish your project
(0 to end)? 180

Critical path: a ==> c ==> d ==> h ==> l ==> o ==> q.
Variance on this path = 367.6667

The probability to finish in 180 is 99.00719%.

Do you want to enter another scheduled completion time (Y/N)? n

23.



The critical path is: a - e - f - g - j - o - p

or

1 - 4 - 5 - 8 - 10 - 11 - 12 - 15

Project duration = 91.667

σ = 3.3082

From January 20 to April 29 is 101 days.

$$P(x \leq 101) = \frac{x - \mu}{Z}$$

$$= \frac{101 - 91.667}{3.3082}$$

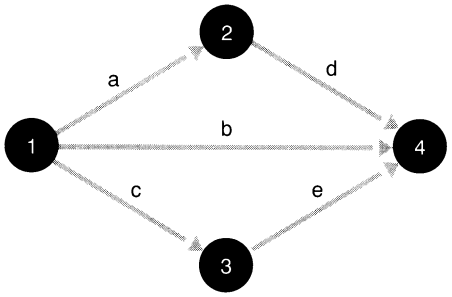
$$= 2.82$$

$$P(x \leq 101) = .9976$$

Activity "n," send out acceptance letters, has ES = 45.83 (March 6) and LF = 66.17 (March 20), so it appears the club would meet the deadline of March 30 to send out acceptance letters.

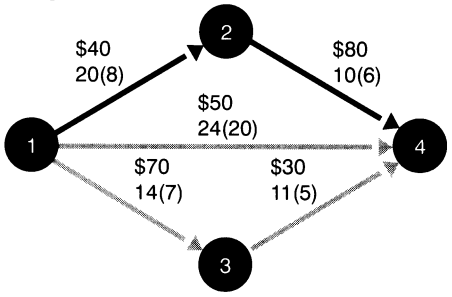
Activity "q," send out schedules, has ES = 66.16 (March 26) and LS = 84.67 (April 14) and LF = 91.67, so it seems likely the club would meet the deadline of April 15 for sending out game schedules.

24.

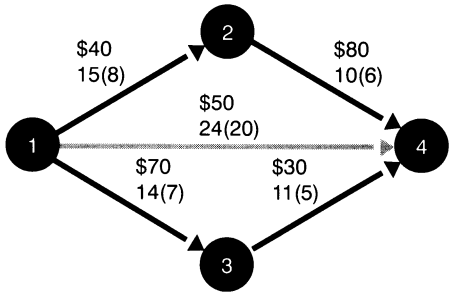


Activity	Total Allowable Crash Time (weeks)	Crash Cost per Week ($)
a	12	40
b	4	50
c	7	70
d	4	80
e	6	30

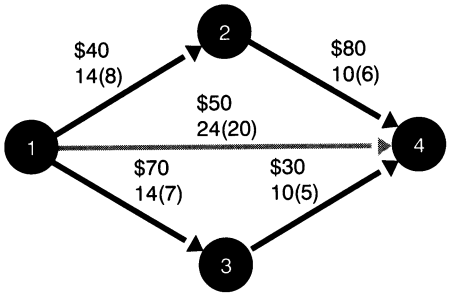
c) Present (normal) critical path = 1 → 2 → 4; normal critical path time = 30 weeks. Crash critical path (all crash times) = 1 → 4; maximum possible project crash time = 20 weeks. The following network shows normal and crash times, crash costs per week, and the critical path.



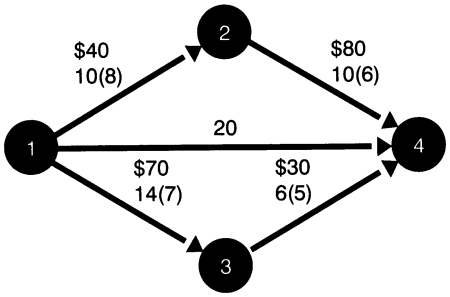
(1) To manually crash, the first step is to select the activity on the critical path with the minimum crash cost, which is activity $1 \rightarrow 2$. Reduce this activity to the point where another path becomes critical. Activity $1 \rightarrow 2$ can be reduced by *5 weeks* before path $1 \rightarrow 3 \rightarrow 4$ also becomes critical. Our revised network is shown below. Crashing cost = \$200; critical paths: $1 \rightarrow 2 \rightarrow 4$ and $1 \rightarrow 3 \rightarrow 4$; critical path time = 25.



(2) Next, both critical paths must be reduced simultaneously. The minimum crash cost activities on the two critical paths are activity $1 \rightarrow 2$ and activity $3 \rightarrow 4$. The most these activities can be crashed is *1 week* before the path $1 \rightarrow 4$ becomes critical. The revised network with these two activities reduced by one week is shown below. Crashing cost = 30 + 40 = \$70; critical paths: $1 \rightarrow 2 \rightarrow 4$, $1 \rightarrow 4$, $1 \rightarrow 3 \rightarrow 4$; critical path time = 24.



(3) Now all three paths are critical, and we want to crash to 20 weeks, the maximum amount. The minimum crash cost activities on each path are $1 \rightarrow 2$, $1 \rightarrow 4$, and $3 \rightarrow 4$. Crashing each of these activities by *4 weeks* results in the following revised network. Crashing cost = \$160 + 200 + 120 = \$480; critical paths: $1 \rightarrow 2 \rightarrow 4$, $1 \rightarrow 4$, $1 \rightarrow 3 \rightarrow 4$; critical path time = 20.



d) The total cost of crashing is \$750 (\$200 + 70 + 480 = 750). The normal project cost is \$3,950, and the cost of the crashed project is \$4,700.

e) Minimize $Z = x_4$

subject to

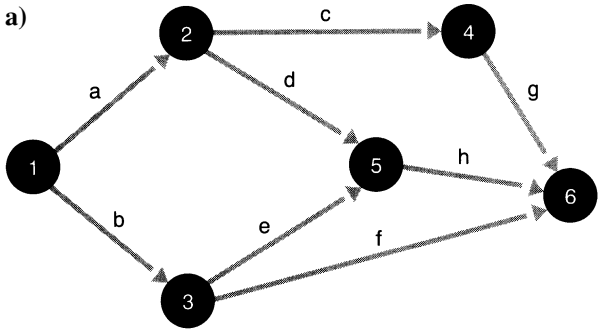
$$x_2 - x_1 \geq 20$$
$$x_3 - x_1 \geq 14$$
$$x_4 - x_1 \geq 24$$
$$x_4 - x_2 \geq 10$$
$$x_4 - x_3 \geq 11$$
$$x_i, x_j \geq 0$$

f) Minimize $Z = 40y_{12} + 50y_{14} + 70y_{13} + 80y_{24} + 30y_{34}$

subject to

$$y_{12} \leq 12$$
$$y_{14} \leq 4$$
$$y_{13} \leq 7$$
$$y_{24} \leq 4$$
$$y_{34} \leq 6$$
$$x_1 + 20 - y_{12} \leq x_2$$
$$x_1 + 14 - y_{13} \leq x_3$$
$$x_1 + 24 - y_{14} \leq x_4$$
$$x_2 + 10 - y_{24} \leq x_4$$
$$x_3 + 11 - y_{34} \leq x_4$$
$$x_4 \leq 20$$
$$x_i, x_j, y_{ij} \geq 0$$

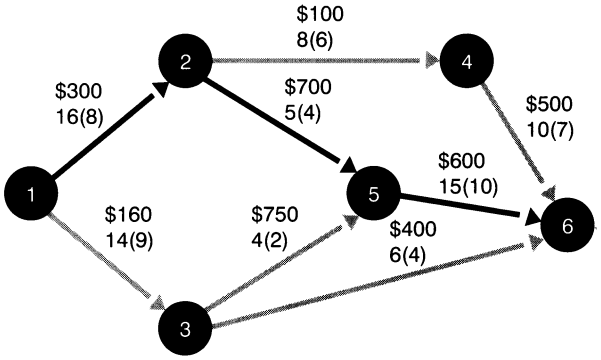
25. a)



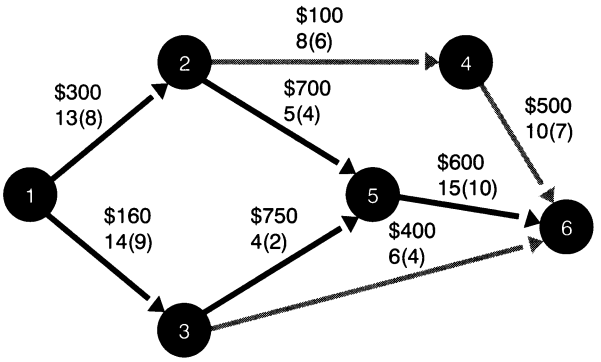
b)

Activity	Total Allowable Crash Times (weeks)	Crash Cost per Week (\$)
a	8	300
b	5	160
c	2	100
d	1	700
e	2	750
f	2	400
g	3	500
h	5	600

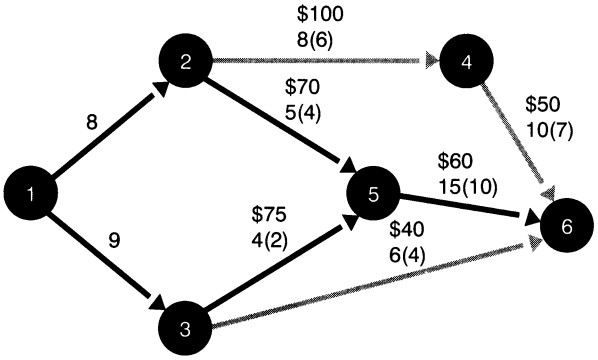
Normal critical path = $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$; normal critical path time = 36 weeks. The following network shows normal and crash times, crash costs per week, and the critical path.



(1) To manually crash, the first step is to select the activity on the critical path with the minimum crash cost, which is activity $1 \rightarrow 2$. This activity can be reduced to the point where another path becomes critical. Activity $1 \rightarrow 2$ can be reduced *3 weeks* before path $1 \rightarrow 3 \rightarrow 5 \rightarrow 6$ becomes critical. (Notice that reducing activity $1 \rightarrow 2$ reduces not only the critical path but also $1 \rightarrow 2 \rightarrow 4 \rightarrow 6$, thus keeping it from becoming critical.) The revised network is shown below. Crashing cost = \$900; critical paths: $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$, $1 \rightarrow 3 \rightarrow 5 \rightarrow 6$; critical path time = 33.



(2) Next, both critical paths must be reduced simultaneously. The minimum crash cost activities on these two paths are $1 \rightarrow 2$ and $1 \rightarrow 3$. The most these activities can be crashed is *5 weeks,* which results in the desired crashed network time of 28 weeks. The revised network with these two activities reduced by five weeks is shown below. Crashing cost = \$1,500 + \$800 = \$2,300; critical paths: $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$, $1 \rightarrow 3 \rightarrow 5 \rightarrow 6$; critical path time = 28. The total cost of crashing is \$3,200 (\$900 + \$2,300 = \$3,200).



c) Minimize $Z = x_6$

subject to

$$x_2 - x_1 \geq 16$$
$$x_3 - x_1 \geq 14$$
$$x_4 - x_2 \geq 8$$
$$x_5 - x_2 \geq 5$$
$$x_5 - x_3 \geq 4$$
$$x_6 - x_3 \geq 6$$
$$x_6 - x_4 \geq 10$$
$$x_6 - x_5 \geq 15$$
$$x_i, x_j \geq 0$$

d) The minimum project duration is 22 weeks.

$$\text{Minimize } Z = 300y_{12} + 160y_{13} + 100y_{24} + 700y_{25} + 750y_{35} + 400y_{36} + 500y_{46} + 600y_{56}$$

subject to

$$y_{12} \leq 8$$
$$y_{13} \leq 5$$
$$y_{24} \leq 2$$
$$y_{25} \leq 1$$
$$y_{35} \leq 2$$
$$y_{36} \leq 2$$
$$y_{46} \leq 3$$
$$y_{56} \leq 5$$
$$x_1 + 16 - y_{12} \leq x_2$$
$$x_1 + 14 - y_{13} \leq x_3$$
$$x_2 + 8 - y_{24} \leq x_4$$
$$x_2 + 5 - y_{25} \leq x_5$$
$$x_3 + 4 - y_{35} \leq x_5$$
$$x_3 + 6 - y_{36} \leq x_6$$
$$x_4 + 10 - y_{46} \leq x_6$$
$$x_5 + 15 - y_{56} \leq x_6$$
$$x_6 \leq 22$$
$$x_i, x_j, y_{ij} \geq 0$$