Chapter Six: Transportation, Transshipment, and Assignment Problems

PROBLEM SUMMARY

1. Balanced transportation

2. Balanced transportation

3. Balanced transportation

4. Unbalanced transportation

5. Unbalanced transportation

6. Unbalanced transportation

7. Unbalanced transportation, multiple optimal

8. Sensitivity analysis (6–7)

9. Unbalanced transportation, multiple optimal

10. Unbalanced transportation

11. Unbalanced transportation

12. Balanced transportation

13. Balanced transportation

14. Sensitivity analysis (6–13)

15. Unbalanced transportation, multiple optimal

16. Sensitivity analysis (6–15)

17. Shortage costs (6–15)

18. Unbalanced transportation

19. Unbalanced transportation, multiple optimal

20. Balanced transportation

21. Unbalanced transportation, multiple optimal

22. Sensitivity analysis (6–21)

23. Unbalanced transportation

24. Sensitivity analysis (6–23)

25. Sensitivity analysis (6–23)

26. Unbalanced transportation

27. Unbalanced transportation

28. Unbalanced transportation

29. Unbalanced transportation, production scheduling

30. Unbalanced transportation

31. Sensitivity analysis (6–30)

32. Shortage costs

33. Multiperiod scheduling

34. Balanced transportation

35. Transshipment

36. Transshipment

37. Transshipment

38. Transshipment

39. Transshipment

40. Unbalanced assignment, LP formulation

41. Assignment

42. Assignment

43. Assignment

44. Unbalanced assignment, multiple optimal

45. Assignment, multiple optimal

46. Assignment

47. Unbalanced assignment, multiple optimal

48. Assignment or transportation

49. Prohibited routes (6–48)

50. Unbalanced assignment

51. Unbalanced assignment, multiple optimal

52. Unbalanced assignment

53. Unbalanced assignment (maximization)

54. Unbalanced assignment

55. Assignment

56. Unbalanced assignment

PROBLEM SOLUTIONS

1. St. Louis - Chicago = 250
Richmond - Chicago = 50
Richmond - Atalnta = 350

2. $x_{13} = 2$
$x_{14} = 10$
$x_{22} = 9$
$x_{23} = 8$
$x_{31} = 10$
$x_{32} = 1$

3. $x_{A3} = 100$
$x_{B1} = 135$
$x_{B2} = 45$
$x_{C2} = 130$
$x_{C3} = 70$
$Z = \$2,345$

4. Minimize $Z = 6x_{A1} + 9x_{A2} + 100x_{A3} + 12x_{B1} + 3x_{B2} + 5x_{B3} + 4x_{C1} + 8x_{C2} + 11x_{C3}$

subject to

$$\begin{aligned} x_{A1} + x_{A2} + x_{A3} &\le 130 \\ x_{B1} + x_{B2} + x_{B3} &\le 70 \\ x_{C1} + x_{C2} + x_{C3} &\le 100 \\ x_{A1} + x_{B1} + x_{C1} &= 80 \\ x_{A2} + x_{B2} + x_{C2} &= 110 \\ x_{A3} + x_{B3} + x_{C3} &= 60 \\ x_{ij} &\ge 0 \end{aligned}$$

$x_{A2} = 80$
$x_{B2} = 10$
$x_{B3} = 60$
$x_{C1} = 80$
$x_{C2} = 20$
$Z = \$1,530$

5. $x_{11} = 70$
$x_{13} = 20$
$x_{22} = 10$
$x_{23} = 20$
$x_{32} = 100$
$Z = \$1,240$

6. $x_{A2} = 20$
$x_{A3} = 60$
$x_{B2} = 70$
$x_{C1} = 80$
$x_{C2} = 20$
$Z = \$1,290$

7. Minimize $Z = 14x_{A1} + 9x_{A2} + 16x_{A3} + 18x_{A4} + 11x_{B1} + 8x_{B2} + Mx_{B3} + 16x_{B4} + 16x_{C1} + 12x_{C2} + 10x_{C3} + 22x_{C4}$

subject to

$$\begin{aligned} x_{A1} + x_{A2} + x_{A3} + x_{A4} &\le 150 \\ x_{B1} + x_{B2} + x_{B3} + x_{B4} &\le 210 \\ x_{C1} + x_{C2} + x_{C3} + x_{C4} &\le 320 \\ x_{A1} + x_{B1} + x_{C1} &= 130 \\ x_{A2} + x_{B2} + x_{C2} &= 70 \\ x_{A3} + x_{B3} + x_{C3} &= 180 \\ x_{A4} + x_{B4} + x_{C4} &= 240 \\ x_{ij} &\ge 0 \end{aligned}$$

$x_{A2} = 70$
$x_{A4} = 80$
$x_{B1} = 50$
$x_{B4} = 160$
$x_{C1} = 80$
$x_{C3} = 180$
$Z = \$8,260$

8. There is no effect. The Gary mill has 60 tons left over as surplus with the current solution to Problem 11. Reducing the capacity at Gary to 30 still leaves a surplus of 30 tons.

9. Minimize $Z = 100x_{A1} + 10x_{A2} + 5x_{A3} + 12x_{B1} + 9x_{B2} + 4x_{B3} + 7x_{C1} + 3x_{C2} + 11x_{C3} + 9x_{D1} + 5x_{D2} + 7x_{D3}$

subject to

$$\begin{aligned} x_{A1} + x_{A2} + x_{A3} &= 90 \\ x_{B1} + x_{B2} + x_{B3} &= 50 \\ x_{C1} + x_{C2} + x_{C3} &= 80 \\ x_{D1} + x_{D2} + x_{D3} &= 60 \\ x_{A1} + x_{B1} + x_{C1} + x_{D1} &\le 120 \\ x_{A2} + x_{B2} + x_{C2} + x_{D2} &\le 100 \\ x_{A3} + x_{B3} + x_{C3} + x_{D3} &\le 110 \\ x_{ij} &\ge 0 \end{aligned}$$

$x_{A1} = 90$
$x_{B1} = 30$
$x_{B3} = 20$
$x_{C2} = 80$
$x_{D1} = 40$
$x_{D2} = 20$
$Z = \$1,590$

10. Minimize $Z = 9x_{TN} + 14x_{TP} + 12x_{TC} + 17x_{TB} + 11x_{MN} + 10x_{MP} + 100x_{MC} + 10x_{MB} + 12x_{FN} + 8x_{FP} + 15x_{FC} + 7x_{FB}$

subject to

$$\begin{aligned} x_{TN} + x_{TP} + x_{TC} + x_{TB} &\le 200 \\ x_{MN} + x_{MP} + x_{MC} + x_{MB} &\le 200 \\ x_{FN} + x_{FP} + x_{FC} + x_{FB} &\le 200 \\ x_{TN} + x_{MN} + x_{FN} &= 130 \\ x_{TP} + x_{MP} + x_{FP} &= 170 \\ x_{TC} + x_{MC} + x_{FC} &= 100 \\ x_{TB} + x_{MB} + x_{FB} &= 150 \\ x_{ij} &\ge 0 \end{aligned}$$

Tampa - NY = 100
Tampa - Chicago = 100
Miami - NY = 30
Miami - Philadelphia = 120
Fresno - Philadelphia = 50
Fresno - Boston = 50
$Z = \$5,080$

11. Minimize $Z = 7x_{1A} + 8x_{1B} + 5x_{1C} + 6x_{2A} + 100x_{2B} + 6x_{3C} + 10x_{3A} + 4x_{3B} + 5x_{3C} + 3x_{4A} + 9x_{4B} + 100x_{4C}$

subject to

$$x_{1A} + x_{1B} + x_{1C} \leq 5$$
$$x_{2A} + x_{2B} + x_{2C} \leq 25$$
$$x_{3A} + x_{3B} + x_{3C} \leq 20$$
$$x_{4A} + x_{4B} + x_{4C} \leq 25$$
$$x_{1A} + x_{2A} + x_{3A} + x_{4A} = 10$$
$$x_{1B} + x_{2B} + x_{3B} + x_{4B} = 20$$
$$x_{1C} + x_{2C} + x_{3C} + x_{4C} = 15$$
$$x_{ij} \geq 0$$

$x_{1C} = 5$
$x_{2C} = 10$
$x_{3B} = 20$
$x_{4A} = 10$
$Z = \$195$

12. $x_{1A} = 70$
$x_{2B} = 20$
$x_{2C} = 10$
$x_{3A} = 20$
$x_{3B} = 100$
$x_{3D} = 100$
$Z = \$13,200$

13. $x_{A2} = 1,800$
$x_{A4} = 950$
$x_{A6} = 750$
$x_{B1} = 1,600$
$x_{B3} = 1,500$
$x_{B5} = 1,250$
$x_{B6} = 650$
$Z = \$3,292.50$

14. No effect

15. $x_{1B} = 250$
$x_{1D} = 170$
$x_{2A} = 520$
$x_{2C} = 90$
$x_{3C} = 130$
$x_{3D} = 210$
$Z = \$21,930$

16.(1)

(1)	(2)
$x_{1B} = 250$	$x_{1B} = 250$
$x_{1D} = 350$	$x_{1D} = 50$
$x_{2A} = 520$	$x_{2A} = 520$
$x_{2C} = 90$	$x_{2C} = 90$
$x_{3C} = 310$	$x_{3C} = 310$
$x_{3D} = 30$	$x_{3D} = 30$
	$x_{4D} = 300$
$Z = \$29,130$	$Z = \$24,930$

Select alternative 2; add a warehouse at Charlotte

17. $x_{1B} = 250$
$x_{1D} = 170$
$x_{2A} = 520$
$x_{2C} = 90$
$x_{3C} = 130$
$x_{3D} = 210$
$x_{4C} = 180$
$Z = \$26,430$

Total transportation cost = \$21,930
Total shortage cost = \$4,500

18. GA - 1 to NC - W = 2
GA - 1 to VA - SW = 10
GA - 2 to NC - SW = 6
GA - 2 to VA - C = 4
SC - 1 to NC - SW = 1
SC - 1 to NC - P = 6
SC - 1 to VA - T = 7
FL - 1 to NC - E = 2
FL - 1 to NC - W = 6
FL - 2 to VA - C = 5
$Z = \$841,000$

19. $x_{1B} = 60$
$x_{2A} = 45$
$x_{2B} = 25$
$x_{2C} = 35$
$x_{3B} = 5$
$Z = \$1,605$

20. $x_{11} = 30$
$x_{12} = 5$
$x_{14} = 2$
$x_{22} = 20$
$x_{33} = 14$
$x_{44} = 26$
$x_{54} = 10$
$x_{55} = 30$
$x_{63} = 6$
$x_{64} = 2$
$x_{66} = 20$
$Z = 364$ miles

21. North A = 250
South B = 200
South C = 40
East A = 150
East C = 160
West D = 210
Central B = 100
Central D = 190
$Z = 20,700$ min.

22. North A = 250
South B = 200
South C = 140
East A = 100
East C = 210
West D = 210
Central B = 150
Central D = 140
Z = 21,200 min.

The overall travel time increased by 500 minutes, which divided by all 1,400 students is only an increase of .357 minutes per student. This does not seem to be a significantly large increase.

23. A - 3 = 8
A - 4 = 18
B - 3 = 12
B - 5 = 27
D - 3 = 5
D - 6 = 35
E - 1 = 25
E - 2 = 15
E - 3 = 4
Z = $1,528 (multiple optimal)

24. If Easy Time purchased all the baby food demanded at each store from the distributor total profit would be $1,246, which is less than buying it from the other locations as determined in problem *123*. This profit is computed by multiplying the profit at each store by the demand. In order to determine if some of the demand should be met by the distributor a new source (F) must be added to problem *23*. This source represents the distributor and has an available supply of 150 cases, the total demand from all the stores. The new optimal solution is shown as follows.

A - 3 = 8
A - 4 = 18
B - 3 = 12
B - 5 = 27
D - 3 = 5
D - 6 = 35
E - 1 = 25
E - 2 = 15
E - 3 = 5
Z = $1,528

25. Solve the model as a linear programming model to obtain the shadow prices. Among the 5 purchase locations, the store at Albany has the highest shadow price of $3. The sensitivity range for supply at Albany is $25 \leq q_1 \leq 43$. Thus, as much as 17 additional cases can be purchased from Albany which would increase profit by $51 for a total of $1,579.

26. Charlotte - Atlanta = 30
Memphis - St. Louis = 30
Louisville - NY = 30
Z = 159,000

27. 1 - C = 2
1 - E = 5
2 - C = 10
3 - E = 5
4 - D = 8
5 - A = 9
6 - B = 6
Z = 1,275

28. Jan - Jan = 180
Jan - May = 30
Feb - Feb = 260
Feb - March = 40
March - March = 300
April - April = 210
April - May = 90
May - May = 280
May - June = 20
June - June = 300
Z = $180,645

29. R_J - Jan = 300
O_J - Jan = 110
R_F - Feb = 300
O_F - Feb = 20
O_F - March = 120
R_M - March = 180
R_M - April = 120
O_M - March = 200
R_A - April = 300
O_A - April = 200
R_M - May = 300
O_M - May = 130
R_J - June = 300
O_J - June = 80
Z = $301,004

30. Sacramento - St. Paul = 13
Sacramento - Topeka = 5
Bakersfield - Denver = 8
Bakersfield - St. Paul = 2
San Antonio - Topeka = 10
Montgomery - Denver = 12
Jacksonville - Akron = 15
Jacksonville - Topeka = 5
Ocala - Louisville = 15
Z = $278,000

It is cheaper for National Foods to continue to operate its own trucking firm.

31. Increasing the supply at Sacramento, Jacksonville and Ocala to 25 tons would have little effect, reducing the overall monthly shipping cost to $276,000, which is still higher than the $245,000 the company is currently spending with its own trucks.

Alternatively, increasing the supply at San Antonio and Montgomery to 25 tons per month reduces the monthly shipping cost to $242,500 which is less than the company's cost with their own trucks.

32. L.A. - Singapore = 150
L.A. - Taipei = 300
Savannah - Hong Kong = 400
Savannah - Taipei = 200
Galveston - Singapore = 350
Order shortage in Hong Kong = 200
Z = $723,500
Penalty cost = $160,000

33.

Period of Use		Period of Production 1	2	3	4	Capacity
1	Beginning Inventory	300				300
	Regular	8,700	300			9,000
	Overtime		1,000			1,000
	Subcontract					3,000
2	Regular		10,000			10,000
	Overtime		700	800		1,500
	Subcontract			200		3,000
3	Regular			12,000		12,000
	Overtime			2,000		2,000
	Subcontract			1,000	2,000	3,000
4	Regular				12,000	12,000
	Overtime				2,000	2,000
	Subcontract				3,000	3,000
	Demand	9,000	12,000	16,000	19,000	

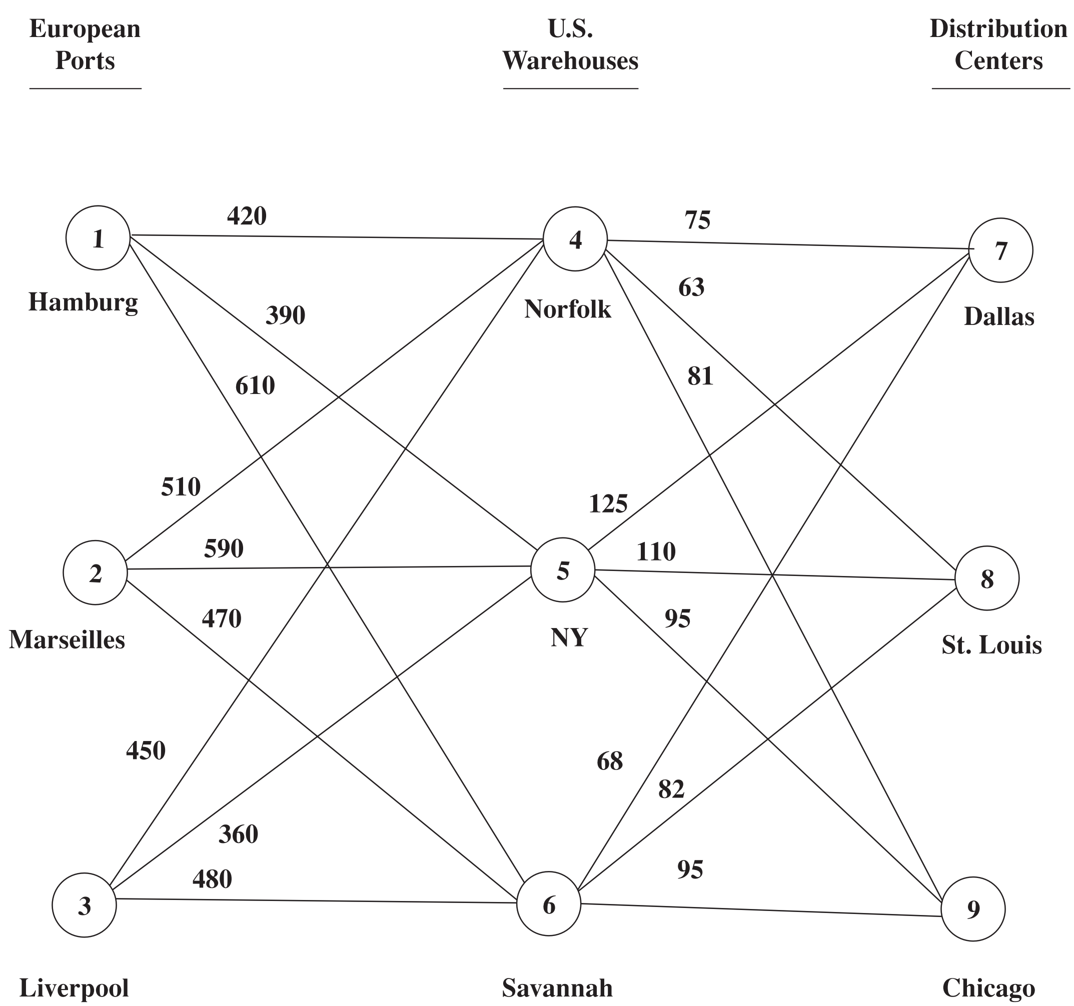
Z = $1,198,500 (multiple optimal solutions)

34. Al - Eagles (2) and Bengals (5)
Barbara - Saints (5) and Jets (1)
Carol - Cowboys (1) and Packers (2)
Dave - Redskins (1) and Cardinals (7)
$Z = 24$
Multiple optimal solutions exist

Carol seems to have received the best allocation but overall the allocation seems relatively fair.

35. x_{14} (Hamburg - Norfolk) = 42
x_{59} (NY - Chicago) = 50
x_{26} (Marseilles - Savannah) = 63
x_{35} (Liverpool - NY) = 37
x_{48} (Norfolk - St. Louis) = 42
x_{15} (Hamburg - NY) = 13
x_{67} (Savannah - Dallas) = 60
x_{68} (Savannah - St. Louis) = 3

$Z = \$77,362$ HND = 38
HNS = 17
MSD = 22

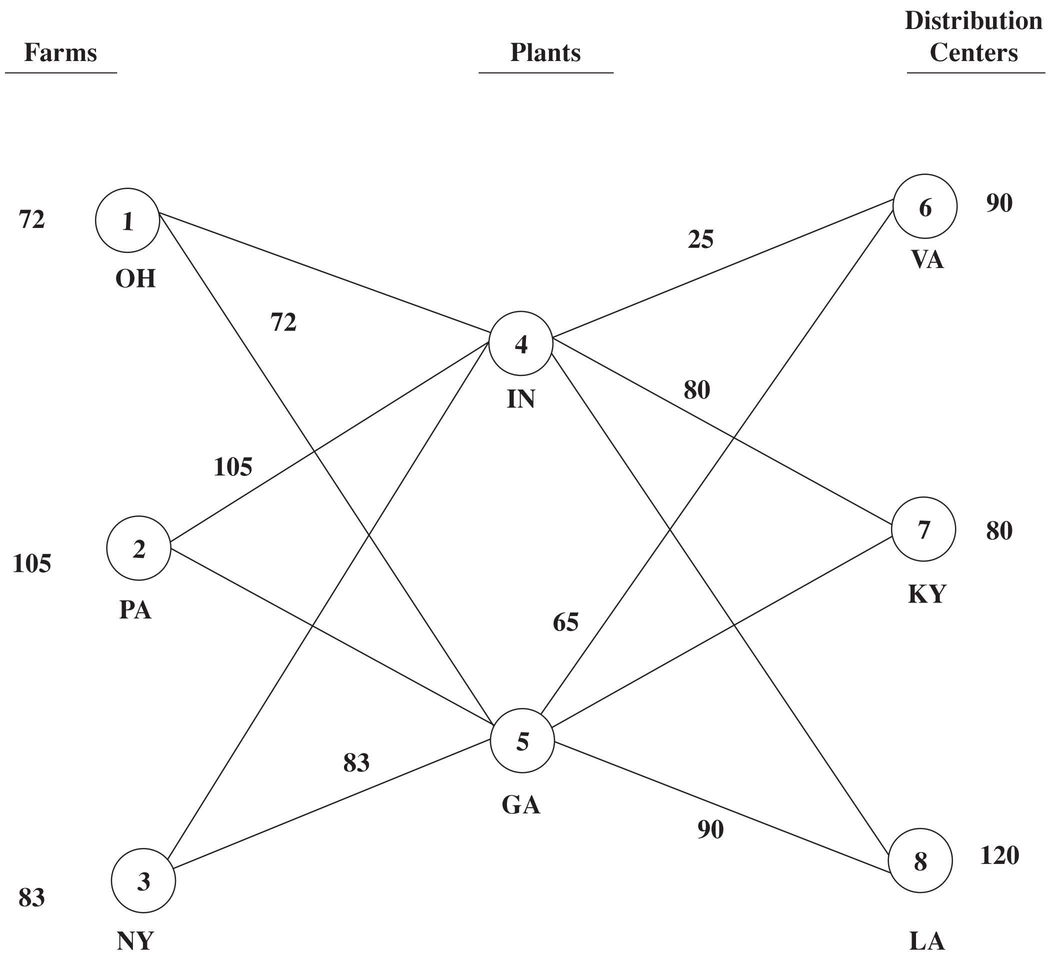


36. x_{16} (Mexico - Houston) = 18
x_{24} (Puerto Rico - Miami) = 11
x_{34} (Haiti - Miami) = 23
x_{47} (Miami - NY) = 20
x_{48} (Miami - St. Louis) = 12
x_{49} (Miami - LA) = 2
x_{69} (Houston - LA) = 18

Z = \$479 or \$479,000

37.(a) $x_{15} = 72$, $x_{24} = 105$, $x_{35} = 83$, $x_{46} = 25$, $x_{47} = 80$, $x_{56} = 65$, $x_{58} = 90$

Z = \$4,871,000



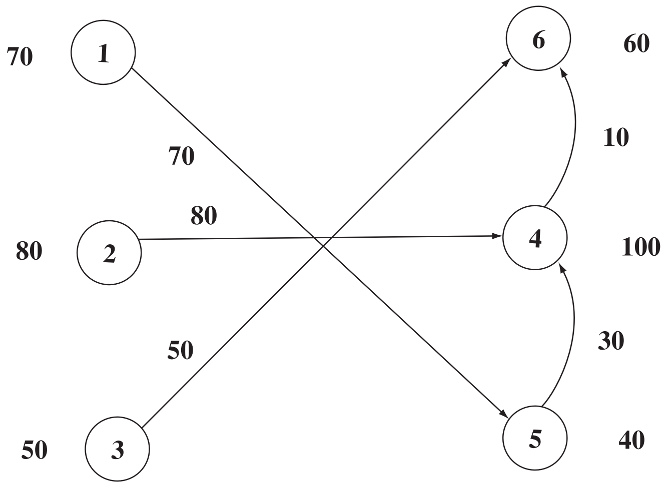
(b) Adding a capacity constraint at plants in Indiana and Georgia

$x_{14} = 15$, $x_{15} = 57$, $x_{24} = 105$, $x_{35} = 83$, $x_{46} = 40$, $x_{47} = 80$, $x_{56} = 50$, $x_{58} = 90$

Z = \$4,886,000

38. $x_{1C} = 15$ $\quad x_{BA} = 40$
$x_{2B} = 57$ $\quad x_{CB} = 80$
$x_{3A} = 105$

$Z = 1{,}490$ or \$14,900



39. x_{37} (Italy - Texas) = 2.1

x_{15} (Germany - Mexico) = 5.2
x_{26} (Belgium - Panama) = 6.3

x_{59} (Mexico - Ohio) = 5.2
x_{68} (Panama - Virginia) = 3.7
x_{69} (Panama - Ohio) = 2.6

Z = \$27.12 million

40. 1 - 1
2 - 4
3 - 2
5 - 3
$Z = 78$

41. 1 - C
2 - A
3 - B
4 - D
Z = 37 min.

42.(a) 1 - B
2 - D
3 - C
4 - A
Z = \$32

(b) Minimize $Z = 12x_{1A} + 11x_{1B} + 8x_{1C} + 14x_{1D} + 10x_{2A} + 9x_{2B} + 10x_{2C} + 8x_{2D} + 14x_{3A} + 100x_{3B} + 7x_{3C} + 11x_{3D} + 6x_{4A} + 8x_{4B} + 10x_{4C} + 9x_{4D}$

subject to

$$x_{1A} + x_{1B} + x_{1C} + x_{1D} = 1$$
$$x_{2A} + x_{2B} + x_{2C} + x_{2D} = 1$$
$$x_{3A} + x_{3B} + x_{3C} + x_{3D} = 1$$
$$x_{4A} + x_{4B} + x_{4C} + x_{4D} = 1$$
$$x_{1A} + x_{2A} + x_{3A} + x_{4A} = 1$$
$$x_{1B} + x_{2B} + x_{3B} + x_{4B} = 1$$
$$x_{1C} + x_{2C} + x_{3C} + x_{4C} = 1$$
$$x_{1D} + x_{2D} + x_{3D} + x_{4D} = 1$$
$$x_{ij} \geq 0$$

43. 1 - B
2 - D
3 - A
4 - C
5 - E
Z = 51 days

44.

1 - B	or	1 - E
2 - E		2 - A
3 - A		3 - B
4 - C		4 - C
5 - D		5 - D
6 - F		6 - F

Z = \$36

45.

1 - C	or	1 - D
2 - A		2 - A
3 - B		3 - B
4 - D		4 - C

Z = \$26

46. 1 - C
2 - F
3 - E
4 - A
5 - D
6 - B
Z = 85 defects

47.

A - 3	or	A - 6
B - 2		B - 2
C - 6		C - 5
D - 1		D - 3
E - 5		E - 1
F - 4		F - 4

Z = 14 miles

48. 1, 4 and 7 - Columbia
2, 6 and 8 - Atlanta
3, 5 and 9 - Nashville
Z = 985 (multiple optimal solutions)