Chapter Four: Linear Programming: Modeling Examples

PROBLEM SUMMARY

1. “Product mix” example
2. “Diet” example
3. “Investment” example
4. “Marketing” example
5. “Transportation” example
6. “Blend” example
7. Product mix (maximization)
8. Sensitivity analysis (4–7)
9. Diet (minimization)
10. Product mix (minimization)
11. Product mix (maximization)
12. Product mix (maximization)
13. Product mix (maximization)
14. Ingredients mix (minimization)
15. Transportation (maximization)
16. Product mix (maximization)
17. Ingredients mix (minimization)
18. Crop distribution (maximization)
19. Monetary allocation (maximization)
20. Diet (minimization), sensitivity analysis
21. Transportation (maximization)
22. Transportation (minimization)
23. Warehouse scheduling (minimization)
24. School busing (minimization)
25. Sensitivity analysis (4–24)
26. Ingredients mixture (minimization)
27. Interview scheduling (maximization)
28. Investments mixture (maximization)
29. Insurance poly mix (maximization)
30. Product mix (maximization)
31. Advertising mix (minimization), sensitivity analysis
32. Blend (maximization)
33. Multiperiod borrowing (minimization)
34. Multiperiod production scheduling (minimization)
35. Blend (maximization), sensitivity analysis
36. Assignment (minimization), sensitivity analysis
37. Transportation (minimization)
38. Scheduling (minimization)
39. Production line scheduling (maximization)
40. Network flow (minimization)
41. Investment example” in chapter (maximization)
42. Blend (maximization)
43. Trim loss (minimization)
44. Multiperiod investment (maximization)
45. Multiperiod sales and inventory (maximization)
46. Multiperiod production and inventory (minimization)
47. Employee assignment (maximization)
48. Data envelopment analysis
49. Data envelopment analysis
50. Network flow (maximization)
51. Multiperiod workforce planning (minimization)
52. Integer solution (4–43)
53. Machine scheduling (maximization), sensitivity analysis
54. Cargo storage (maximization)
55. Broadcast scheduling (maximization)
56. Product mix (maximization)
57. Product mix/advertising (maximization)
58. Scheduling (minimization)
59. Truck purchasing/leasing (minimization)
or,

\[
\begin{align*}
20,000 - 12,000x_2 &= 0 \\
20,000x_1 - 9,000x_3 &= 0 \\
\end{align*}
\]

The new solution is \(x_1 = 3.068, x_2 = 5.114, x_3 = 6.818\) and \(Z = 184,090\). This results in approximately 61,362 exposures per type of advertising (with some slight differences due to computer rounding).

5. The slack variables for the three \(\leq\) warehouse constraints would be added to the constraints as follows,

\[
\begin{align*}
x_1A + x_1B + x_1C + s_1 &= 300 \\
x_2A + x_2B + x_2C + s_2 &= 200 \\
x_3A + x_3B + x_3C + s_3 &= 200
\end{align*}
\]

These three slacks would then be added to the objective function with the storage cost coefficients of $9 for \(s_1\), $6 for \(s_2\) and $7 for \(s_3\).

This change would not result in a new solution.

The model must be reformulated with three new variables reflecting the shipments from the new warehouse at Memphis (4) to the three stores, \(x_{4A}\), \(x_{4B}\), and \(x_{4C}\). These variables must be included in the objective function with the cost coefficients of $18, 9 and 12 respectively.

A new supply constraint must be added,

\[
x_{4A} + x_{4B} + x_{4C} \leq 200
\]

The solution to this reformulated model is,

\[
\begin{align*}
x_{1C} &= 200 \\
x_{2B} &= 50 \\
x_{3A} &= 150 \\
x_{4B} &= 200 \\
Z &= 6,650
\end{align*}
\]

Yes, the warehouse should be leased.

The shadow price for the Atlanta warehouse shows the greatest decrease in cost, $6 for every additional set supplied from this source. However, the upper limit of the sensitivity range is 200, the same as the current supply value. Thus, if the supply is increased at Atlanta by even one television set the shadow price will change.

6. This change would not affect the solution at all since there is no surplus with any of the three constraints.

Component 1 has the greatest dual price of $20. For each barrel of component 1 the company can acquire, profit will increase by $20, up to the limit of the sensitivity range which is an increase of 1,700 bbls. or 6,200 total bbls. of component 1. For example an increase of one bbl. of component 1 from 4,500 to 4,501 results in a increase in total cost to $76,820.

7. (a) maximize \(Z = 190x_1 + 170x_2 + 155x_3\) subject to

\[
\begin{align*}
3.5x_1 + 5.2x_2 + 2.8x_3 &\leq 500 \\
1.2x_1 + 0.8x_2 + 1.5x_3 &\leq 240 \\
40x_1 + 55x_2 + 20x_3 &\leq 6,500 \\
x_1, x_2, x_3 &\geq 0
\end{align*}
\]

(b) \(x_1 = 41.27, x_2 = 0, x_3 = 126.98, Z = 27,523.81\)

\(s_1 = s_2 = 0, s_3 = 2,309.52\)

8. (a)It would not affect the model. The slack apples are multiplied by the revenue per apple of $0.08 to determine the extra total revenue, i.e.,

\(2,309.52(0.08) = 184.76\).

(b)This change requires a new variable, \(x_4\), and that the constraint for apples be changed from \(\leq\) to \(=\). No, the Friendly’s should not produce cider. The new solution would be \(x_1 = 135, x_2 = 0, x_3 = 0, x_4 = 18.33\) and \(Z = 26,475\). This reduction in profit occurs because the requirement that all 6,500 apples be used forces resources to be used for cider that would be more profitable to be used to produce the other products. If the final model constraint for apples is \(\leq\) rather than \(=\), the previous solution in 1(b) results.

9. (a) \(x_1 = \) no. of eggs
\(x_2 = \) no. of bacon strips
\(x_3 = \) no. of cups of cereal

minimize \(Z = 4x_1 + 3x_2 + 2x_3\) subject to

\[
\begin{align*}
2x_1 + 4x_2 + x_3 &\geq 16 \\
3x_1 + 2x_2 + x_3 &\geq 12 \\
x_1, x_2, x_3 &\geq 0
\end{align*}
\]

(b) \(x_1 = 2\)
\(x_2 = 3\)
\(Z = 0.17\)

10.(a) \(x_i = \) number of boats of type \(i, i = 1\) (bass boat), 2 (ski boat), 3 (speed boat)

In order to break even total revenue must equal total cost:

\[
\begin{align*}
23,000x_1 + 18,000x_2 + 26,000x_3 &= 12,500x_1 + 8,500x_2 + 13,700x_3 + 2,800,000
\end{align*}
\]
or,
\[ 10,500x_1 + 9,500x_2 + 12,300x_3 = 2,800,000 \]
minimize \( Z = 12,500x_1 + 8,500x_2 + 13,700x_3 \)
subject to
\[ 10,500x_1 + 9,500x_2 + 12,300x_3 = 2,800,000 \]
\[ x_1 \geq 70 \]
\[ x_2 \geq 50 \]
\[ x_3 \geq 50 \]
\[ x_1 \leq 120 \]
\[ x_2 \leq 120 \]
\[ x_3 \leq 120 \]
\( x_1, x_2, x_3 \geq 0 \)

(b) \( x_1 = 70.00 \)
\( x_2 = 120.00 \)
\( x_3 = 75.203 \)
\( Z = \$2,925,284.553 \)

11.(a) \( x_1 = \) no. of clocks
\( x_2 = \) no. of radios
\( x_3 = \) no. of toasters
maximize \( Z = 8x_1 + 10x_2 + 7x_3 \)
subject to
\[ 7x_1 + 10x_2 + 5x_3 \leq 2,000 \]
\[ 2x_1 + 3x_2 + 2x_3 \leq 660 \]
\[ x_1 \leq 200 \]
\[ x_2 \leq 300 \]
\[ x_3 \leq 150 \]
\( x_1, x_2, x_3 \geq 0 \)

(b) \( x_1 = 178.571 \)
\( x_3 = 150.00 \)
\( Z = \$2,478.571 \)

12.(a) \( x_1 = \) no. of gallons of Yodel
\( x_2 = \) no. of gallons of Shotz
\( x_3 = \) no. of gallons of Rainwater
maximize \( Z = 1.50x_1 + 1.60x_2 + 1.25x_3 \)
subject to
\[ x_1 + x_2 + x_3 = 1,000 \]
\[ 1.50x_1 + .90x_2 + .50x_3 \leq 2,000 \]
\[ x_1 \leq 400 \]
\[ x_2 \leq 500 \]
\[ x_3 \leq 300 \]
\( x_1, x_2, x_3 \geq 0 \)

(b) \( x_1 = 400 \)
\( x_2 = 500 \)
\( x_3 = 100 \)
\( Z = \$1,525.00 \)

13.(a) \( x_1 = \) no. of lb of Super Two at Fresno
\( x_2 = \) no. of lb of Super Two at Dearborn
\( x_3 = \) no. of lb of Green Grow at Fresno
\( x_4 = \) no. of lb of Green Grow at Dearborn
maximize \( Z = 7x_1 + 5x_2 + 5x_3 + 4x_4 \)
subject to
\[ 2x_1 + 4x_2 + 2x_3 + 3x_4 \leq 45,000 \]
\[ x_1 + x_2 \leq 6,000 \]
\[ x_3 + x_4 \leq 7,000 \]
\[ x_1 + x_3 \leq 5,000 \]
\[ x_2 + x_4 \leq 6,000 \]
\( x_1, x_2, x_3, x_4 \geq 0 \)

(b) \( x_1 = 5,000 \)
\( x_2 = 1,000 \)
\( x_4 = 5,000 \)
\( Z = \$60,000 \)

14.(a) \( x_i = \) ore \( i \) \( (i = 1,2,3,4,5,6) \)
minimize \( Z = 27x_1 + 25x_2 + 32x_3 + 22x_4 + 20x_5 + 24x_6 \)
subject to
\[ .19x_1 + .43x_2 + .17x_3 + .20x_4 + .12x_5 \geq .21 \]
\[ .15x_1 + .10x_2 + .12x_4 + .24x_5 + .18x_6 \leq .12 \]
\[ .12x_1 + .25x_2 + .10x_5 + .16x_6 \leq .07 \]
\[ .14x_1 + .07x_2 + .53x_3 + .18x_4 + .031x_5 + .25x_6 \geq .30 \]
\[ .14x_1 + .07x_2 + .53x_3 + .18x_4 + .31x_5 + .25x_6 \leq .65 \]
\[ .60x_1 + .85x_2 + .70x_3 + .50x_4 + .65x_5 + .71x_6 = 1.00 \]

(b) \( x_2 = .1153 \) ton – ore 2
\( x_3 = .8487 \) ton – ore 3
\( x_4 = .0806 \) ton – ore 4
\( x_5 = .4116 \) ton – ore 5
\( Z = \$40.05 \)

15.(a) \( x_{ij} = \) number of trucks assigned to route from warehouse \( i \) to terminal \( j \), where \( i = 1 \) (Charlotte), 2 (Memphis), 3 (Louisville) and \( j = a \) (St. Louis), b (Atlanta), c (New York)
maximize \( Z = 1,800x_{1a} + 2,100x_{1b} + 1,600x_{1c} + 1,000x_{2a} + 700x_{2b} + 900x_{2c} + 1,400x_{3a} + 800x_{3b} + 2,200x_{3c} \)
subject to
\[ x_{1a} + x_{1b} + x_{1c} = 30 \]
\[ x_{2a} + x_{2b} + x_{2c} = 30 \]
\[ x_{3a} + x_{3b} + x_{3c} = 30 \]
\[ x_{1a} + x_{2a} + x_{3a} \leq 40 \]
\[ x_{1b} + x_{2b} + x_{3b} \leq 60 \]
\[ x_{1c} + x_{2c} + x_{3c} \leq 50 \]
\( x_{ij} \geq 0 \)
(b) \( x_{1b} = 30 \)
\( x_{2a} = 30 \)
\( x_{3c} = 30 \)
\( Z = $159,000 \)

16. (a) \( x_1 = \) no. of sofas
\( x_2 = \) no. of tables
\( x_3 = \) no. of chairs

maximize \( Z = 400x_1 + 275x_2 + 190x_3 \)

subject to
\[ 7x_1 + 5x_2 + 4x_3 \leq 2,250 \]
\[ 12x_1 + 7x_3 \leq 1,000 \]
\[ 6x_1 + 9x_2 + 5x_3 \leq 240 \]
\[ x_1 + x_2 + x_3 \leq 650 \]
\[ x_1, x_2, x_3 \geq 0 \]

(b) \( x_1 = 40 \)
\( Z = $16,000 \)

17. (a) \( x_{ij} = \) lbs. of seed \( i \) used in mix \( j \), where \( i = t \) (tall fescue), \( m \) (mustang fescue), \( b \) (bluegrass) and \( j = 1,2,3 \).

minimize \( Z = 1.70(x_{t1} + x_{t2} + x_{t3}) + 2.80 \)
\[ (x_{m1} + x_{m2} + x_{m3}) + 3.25(x_{b1} + x_{b2} + x_{b3}) \]

subject to
\[ .50x_{t1} - .50x_{m1} - .50x_{b1} \leq 0 \]
\[ -.20x_{t1} + .80x_{m1} - .20x_{b1} \geq 0 \]
\[ -.30x_{t2} - .30x_{m2} + .70x_{b2} \geq 0 \]
\[ -.30x_{t2} + .70x_{m2} - .30x_{b2} \geq 0 \]
\[ .80x_{t3} - .20x_{m3} - .20x_{b3} \leq 0 \]
\[ .50x_{t3} - .50x_{m3} - .50x_{b3} \geq 0 \]
\[ .30x_{t3} - .70x_{m3} - .70x_{b3} \leq 0 \]
\[ -.10x_{t3} - .10x_{m3} + .90x_{b3} \geq 0 \]
\[ x_{t1} + x_{m1} + x_{b1} \geq 1,200 \]
\[ x_{t2} + x_{m2} + x_{b2} \geq 900 \]
\[ x_{t3} + x_{m3} + x_{b3} \geq 2,400 \]
\[ x_{ij} \geq 0 \]

(b) \( x_{t1} = 600 \)
\( x_{t2} = 180 \)
\( x_{t3} = 1,680 \)
\( x_{m1} = 600 \)
\( x_{m2} = 450 \)
\( x_{m3} = 480 \)
\( x_{b1} = 0 \)
\( x_{b2} = 270 \)
\( x_{b3} = 240 \)
\( Z = $10,123.50 \)

18. (a) \( x_{ij} = \) acres of crop \( i \) planted on plot \( j \), where \( i = c \) (corn), \( p \) (peas), \( s \) (soybeans) and \( j = 1,2,3 \).

maximize \( Z = 600(x_{c1} + x_{c2} + x_{c3}) + 450(x_{p1} + x_{p2} + x_{p3}) + 300(x_{s1} + x_{s2} + x_{s3}) \)

subject to
\[ x_{c1} + x_{p1} + x_{s1} \geq 300 \]
\[ x_{c1} + x_{p1} + x_{s1} \leq 500 \]
\[ x_{c2} + x_{p2} + x_{s2} \geq 480 \]
\[ x_{c2} + x_{p2} + x_{s2} \leq 800 \]
\[ x_{c3} + x_{p3} + x_{s3} \geq 420 \]
\[ x_{c3} + x_{p3} + x_{s3} \leq 700 \]
\[ x_{c1} + x_{c2} + x_{c3} \leq 900 \]
\[ x_{p1} + x_{p2} + x_{p3} \leq 700 \]
\[ x_{s1} + x_{s2} + x_{s3} \leq 1,000 \]
\[ 800(x_{c1} + x_{p1} + x_{s1}) - 500(x_{c2} + x_{p2} + x_{s2}) = 0 \]
\[ 700(x_{c2} + x_{p2} + x_{s2}) - 800(x_{c3} + x_{p3} + x_{s3}) = 0 \]
\[ 700(x_{c1} + x_{p1} + x_{s1}) - 500(x_{c3} + x_{p3} + x_{s3}) = 0 \]
\[ x_{ij} \geq 0 \]

(b) \( x_{c1} = 500 \)
\( x_{c2} = 100 \)
\( x_{c3} = 300 \)
\( x_{p2} = 700 \)
\( x_{s3} = 400 \)
\( Z = $975,000 \)

19. (a) \( x_1 = \) $ allocated to job training
\( x_2 = \) $ allocated to parks
\( x_3 = \) $ allocated to sanitation
\( x_4 = \) $ allocated to library

maximize \( Z = .02x_1 + .09x_2 + .06x_3 + .04x_4 \)

subject to
\[ x_1 + x_2 + x_3 + x_4 \leq 4,000,000 \]
\[ x_1 \leq 1,600,000 \]
\[ x_2 \leq 1,600,000 \]
\[ x_3 \leq 1,600,000 \]
\[ x_4 \leq 1,600,000 \]
\[ x_2 - x_3 - x_4 \leq 0 \]
\[ x_1 - x_3 \geq 0 \]
\[ x_1, x_2, x_3, x_4 \geq 0 \]

(b) \( x_1 = 800,000 \)
\( x_2 = 1,600,000 \)
\( x_3 = 800,000 \)
\( x_4 = 800,000 \)
\( Z = 240,000 \)
20. (a) minimize \( Z = 0.80x_1 + 3.70x_2 + 2.30x_3 + 0.90x_4 + 0.75x_5 + 0.40x_6 + 0.83x_7 \)

subject to

\[
\begin{align*}
520x_1 + 500x_2 + 860x_3 + 600x_4 + 50x_5 + 460x_6 + 240x_7 & \geq 1,500 \\
520x_1 + 500x_2 + 860x_3 + 600x_4 + 50x_5 + 460x_6 + 240x_7 & \leq 2,000 \\
4.4x_1 + 3.3x_2 + .3x_3 + 3.4x_4 + .5x_5 + 2.2x_6 + .2x_7 & \geq 5 \\
30x_1 + 5x_2 + 75x_3 + 3x_4 + 10x_7 & \geq 20 \\
30x_1 + 5x_2 + 75x_3 + 3x_4 + 10x_7 & \leq 60 \\
17x_1 + 85x_2 + 82x_3 + 10x_4 + 6x_5 + 10x_6 + 16x_7 & \geq 30 \\
30x_4 + 70x_6 + 22x_7 & \geq 40 \\
180x_1 + 90x_2 + 350x_3 + 20x_7 & \leq 30 \\
x_i & \geq 0
\end{align*}
\]

(b) \( x_4 = 1.667 \)
\( x_6 = 0.304 \)
\( x_7 = 1.500 \)
\( Z = 2.867 \)

(c) The model becomes infeasible and cannot be solved. Limiting each food item to one-half pound is too restrictive. In fact, experimentation with the model will show that one food item in particular, dried beans, is restrictive. All other food items can be limited except dried beans.

21. (a) \( x_{ij} \) = number of units of product \( i \) (\( i = 1,2,3 \)) produced on machine \( j \) (\( j = 1,2,3,4 \))

maximize \( Z = 7.8x_{11} + 7.8x_{12} + 8.2x_{13} + 7.9x_{14} + 6.7x_{21} + 8.9x_{22} + 9.2x_{23} + 6.3x_{24} + 8.4x_{31} + 8.1x_{32} + 9.0x_{33} + 5.8x_{34} \)

subject to

\[
\begin{align*}
35x_{11} + 40x_{21} + 38x_{31} & \leq 9,000 \\
41x_{12} + 36x_{22} + 37x_{32} & \leq 14,400 \\
34x_{13} + 32x_{23} + 33x_{33} & \leq 12,000 \\
39x_{14} + 43x_{24} + 40x_{34} & \leq 15,000 \\
x_{11} + x_{12} + x_{13} + x_{14} & = 400 \\
x_{21} + x_{22} + x_{23} + x_{24} & = 570 \\
x_{31} + x_{32} + x_{33} + x_{34} & = 320 \\
x_{ij} & \geq 0
\end{align*}
\]

(b) \( x_{11} = 15.385 \)
\( x_{14} = 384.615 \)
\( x_{22} = 400.00 \)
\( x_{23} = 170.00 \)
\( x_{31} = 121.212 \)
\( x_{33} = 198.788 \)
\( Z = 11,089.73 \)

22. (a) Minimize \( Z = 69x_{11} + 71x_{12} + 72x_{13} + 74x_{14} + 76x_{21} + 74x_{22} + 75x_{23} + 79x_{24} + 86x_{31} + 89x_{32} + 80x_{33} + 82x_{34} \)

subject to

\[
\begin{align*}
x_{11} + x_{12} + x_{13} + x_{14} & \leq 220 \\
x_{21} + x_{22} + x_{23} + x_{24} & \leq 170 \\
x_{31} + x_{32} + x_{33} + x_{34} & \leq 280 \\
x_{11} + x_{21} + x_{31} & = 110 \\
x_{12} + x_{22} + x_{32} & = 160 \\
x_{13} + x_{23} + x_{33} & = 90 \\
x_{14} + x_{24} + x_{34} & = 180
\end{align*}
\]

Ash: \( 0.03x_{11} - 0.01x_{21} - 0.02x_{31} \leq 0 \)
\( 0.04x_{12} + 0.02x_{22} - 0.01x_{32} \leq 0 \)
\( 0.04x_{13} + 0.02x_{23} - 0.01x_{33} \leq 0 \)
\( 0.03x_{14} - 0.01x_{24} - 0.02x_{34} \leq 0 \)

Sulfur: \( 0.01x_{11} - 0.01x_{21} - 0.02x_{31} \leq 0 \)
\( 0.01x_{12} + 0.01x_{22} - 0.02x_{32} \leq 0 \)
\( -0.01x_{13} + 0.03x_{23} - 0.04x_{33} \leq 0 \)
\( -0.01x_{14} - 0.02x_{24} - 0.03x_{34} \leq 0 \)
\( x_{ij} \geq 0 \)

(b) \( x_{11} = 34 \)
\( x_{13} = 11 \)
\( x_{14} = 45 \)
\( x_{23} = 35 \)
\( x_{24} = 135 \)
\( x_{31} = 76 \)
\( x_{32} = 160 \)
\( x_{33} = 44 \)
\( Z = 44,054 \)

23. (a) \( x_{ij} \) = space (ft²) rented in month \( i \) for \( j \) months, where \( i = 1,2,...,6 \) and \( j = 1,2,...,6 \)

Minimize \( Z = 1.70x_{11} + 1.40x_{12} + 1.20x_{13} + 1.10x_{14} + 1.05x_{15} + 1.00x_{16} + 1.70x_{21} + 1.40x_{22} + 1.20x_{23} + 1.10x_{24} + 1.05x_{25} + 1.70x_{31} + 1.40x_{32} + 1.20x_{33} + 1.10x_{34} + 1.70x_{41} + 1.40x_{42} + 1.20x_{43} + 1.70x_{51} + 1.40x_{52} + 1.70x_{61} \)

subject to:

\[
\begin{align*}
x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} & = 47,000 \\
x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{22} + x_{23} + x_{24} + x_{25} & = 35,000 \\
x_{13} + x_{14} + x_{15} + x_{16} + x_{22} + x_{23} + x_{24} & = 52,000 \\
x_{14} + x_{15} + x_{16} + x_{22} + x_{23} + x_{24} & = 27,000 \\
x_{15} + x_{16} + x_{24} + x_{25} + x_{33} + x_{34} + x_{42} & = 19,000 \\
x_{16} + x_{25} + x_{34} + x_{43} + x_{52} + x_{61} & = 15,000
\end{align*}
\]
(b) $x_{11} = 12,000$
$x_{13} = 25,000$
$x_{14} = 8,000$
$x_{15} = 2,000$
$x_{33} = 2,000$
$x_{34} = 15,000$
$Z = $80,200

(c) $x_{16} = 52,000$
$Z = $52,000
It is much cheaper to rent all the space for the entire six month period in April and have excess or “surplus” space.

24. (a) $x_{ij}$ = no. of students bused from district $i$ to school $j$, where $i = n, s, e, w, c$ and $j = c, w, s$

minimize $Z = 8x_{nc} + 11x_{nw} + 14x_{ns} + 12x_{sc} + 9x_{sw} + 0x_{ss} + 9x_{cs} + 16x_{cw} + 10x_{cs} + 8x_{wc} + 0x_{ww} + 9x_{ws} + 0x_{cc} + 8x_{cw} + 12x_{cs}$

subject to
$x_{nc} + x_{sw} + x_{ss} = 700$
$x_{sc} + x_{sw} + x_{ss} = 300$
$x_{ec} + x_{ew} + x_{es} = 900$
$x_{wc} + x_{ww} + x_{ws} = 600$
$x_{cc} + x_{cw} + x_{cs} = 500$
$x_{nc} + x_{sc} + x_{ec} + x_{wc} + x_{cc} \leq 1,200$
$x_{nw} + x_{sw} + x_{es} + x_{ww} + x_{cs} \leq 1,200$
$x_{ns} + x_{ss} + x_{es} + x_{ws} + x_{cs} \leq 1,200$
$x_{ij} \geq 0$

(b) $x_{nc} = 700$
$x_{ss} = 300$
$x_{es} = 900$
$x_{ww} = 600$
$x_{cc} = 500$
$Z = 14,600$

25. (a) Add the following 3 constraints to the original formulation:

$x_{ss} \leq 150$
$x_{ww} \leq 300$
$x_{cc} \leq 250$
$x_{nc} = 700$
$x_{nw} = 0$
$x_{sw} = 150$
$x_{sc} = 150$
$x_{es} = 900$
$x_{wc} = 250$
$x_{ww} = 300$
$x_{ws} = 50$
$x_{cc} = 250$
$x_{cw} = 250$
$Z = 20,400$

(b) Change the 3 demand constraints in the (a) formulation from $\leq 1,200$ to $= 1,000$.

$x_{nc} = 400$
$x_{nw} = 300$
$x_{sw} = 150$
$x_{sc} = 150$
$x_{es} = 850$
$x_{wc} = 300$
$x_{ww} = 300$
$x_{cc} = 250$
$x_{cw} = 250$
$Z = 21,200$

26. (a) $x_1 = no. \text{ of lb of oats}$
$x_2 = no. \text{ of lb of corn}$
$x_3 = no. \text{ of lb of soybean}$
$x_4 = no. \text{ of lb of vitamin supplement}$

minimize $Z = .50x_1 + 1.20x_2 + .60x_3 + 2.00x_4$

subject to
$x_1 \leq 300$
$x_2 \leq 400$
$x_3 \leq 200$
$x_4 \leq 100$
$
x_3/(x_1 + x_2 + x_3 + x_4) \geq .30$
$
x_4/(x_1 + x_2 + x_3 + x_4) \geq .20$
$
x_2/x_1 \leq 2/1$
$
x_1 \leq x_3$
$x_1 + x_2 + x_3 + x_4 \geq 500$
$x_1, x_2, x_3, x_4 \geq 0$

(b) $x_1 = 200$
$x_2 = 0$
$x_3 = 200$
$x_4 = 100.00$
$Z = $420

27. (a) $x_1 = \text{no. of day contacts by phone}$
$x_2 = \text{no. of day contacts in person}$
$x_3 = \text{no. of night contacts by phone}$
$x_4 = \text{no. of night contacts in person}$

maximize $Z = 2x_1 + 4x_2 + 3x_3 + 7x_4$

subject to
$x_2 + x_4 \leq 300$
$6x_1 + 15x_2 \leq 1,200$
$5x_3 + 12x_4 \leq 2,400$
$x_1, x_2, x_3, x_4 \geq 0$

(b) $x_1 = 200$
$x_3 = 480$
$Z = 1,840$
28. (a) \( x_{ij} \) = dollar amount invested in alternative \( i \) in year \( j \), where \( i = p \) (product research and development), \( a \) (manufacturing operations improvements), \( s \) (advertising and sales promotion) and \( j = 1, 2, 3, 4 \) (denoting year): 

maximize \( Z = 1.2x_{a4} + 1.3x_{m3} + 1.5x_{p3} \)

subject to

\[
\begin{align*}
  x_{a1} &\geq 30,000 \\
  x_{m1} &\geq 40,000 \\
  x_{p1} &\geq 50,000 \\
  x_{a1} + x_{m1} + x_{p1} + s_1 &= 500,000 \\
  x_{a2} + x_{m2} + x_{p2} &= s_2 + 1.2x_{a1} \\
  x_{a3} + x_{m3} &= s_3 + 1.2x_{a2} + 1.3x_{m1} \\
  x_{a4} + x_{m4} &= s_4 + 1.2x_{a3} + 1.3x_{m2} + 1.5x_{p1} \\
  x_{ij}, s_j &\geq 0
\end{align*}
\]

Note: Since it is assumed that any amount of funds can be invested in each alternative—i.e., there is no minimum investment required—and funds can always be invested in as short a period as one year yielding a positive return, it is apparent that the \( s_j \) variables for uninvested funds will be driven to zero in every period. Thus, these variables could be omitted from the model formulation for this problem.

(b) \( x_{a1} = 410,000 \quad x_{m1} = 40,000 \)

\( x_{a2} = 492,000 \quad x_{p1} = 50,000 \)

\( x_{a3} = 642,400 \quad Z = 1,015,056 \)

\( x_{a4} = 845,880 \)

29. (a) \( x_1 \) = no. of homeowner’s policies

\( x_2 \) = no. of auto policies

\( x_3 \) = no. of life policies

maximize \( Z = 35x_1 + 20x_2 + 58x_3 \)

subject to

\[
\begin{align*}
  14x_1 + 12x_2 + 35x_3 &\leq 35,000 \\
  6x_1 + 3x_2 + 12x_3 &\leq 20,000 \\
  x_1, x_2, x_3 &\geq 0
\end{align*}
\]

(b) \( x_1 = 2,500 \quad Z = 887,500 \)

30. (a) \( x_1 \) = no. of issues of \emph{Daily Life}

\( x_2 \) = no. of issues of \emph{Agriculture Today}

\( x_3 \) = no. of issues of \emph{Surf’s Up}

maximize \( Z = 2.25x_1 + 4.00x_2 + 1.50x_3 \)

subject to

\[
\begin{align*}
  x_1 + x_2 + x_3 &\geq 5,000 \\
  .01x_1 + .03x_2 + .02x_3 &\leq 120 \\
  .2x_1 + .5x_2 + .3x_3 &\leq 3,000 \\
  x_1 &\leq 3,000 \\
  x_2 &\leq 2,000 \\
  x_3 &\leq 6,000 \\
  x_1, x_2, x_3 &\geq 0
\end{align*}
\]

(b) \( x_1 = 3,000 \quad x_2 = 2,000 \quad x_3 = 1,500 \quad Z = 17,000 \)

31. (a) \( x_1 \) = no. of television commercials

\( x_2 \) = no. of newspaper ads

\( x_3 \) = no. of radio commercials

minimize \( Z = 15,000x_1 + 4,000x_2 + 6,000x_3 \)

subject to

\[
\begin{align*}
  25,000x_1 + 10,000x_2 + 15,000x_3 &\geq 100,000 \\
  (15,000x_1 + 3,000x_2 + 12,000x_3)/2 &\geq 2/1 \\
  (10,000x_1 + 7,000x_2 + 3,000x_3)/2 &\geq 2/1 \\
  (15,000x_1 + 4,000x_2 + 9,000x_3)/2 &\geq 2/1 \\
  25,000x_1 + 10,000x_2 + 15,000x_3 &\geq .30 \\
  x_2 &\leq 7 \\
  x_1, x_2, x_3 &\geq 0
\end{align*}
\]

(b) \( x_2 = 2.5 \quad x_3 = 5.0 \quad Z = 40,000 \)

(c) This reformulation of the model would result in a fourth variable \( x_4 \), with model parameters inserted accordingly. However, it would have no effect on the solution.

32. (a) \( x_{ij} \) = lbs. of coffee \( i \) used in blend \( j \) per week, where \( i = b \) (Brazilian), \( o \) (Mocha), \( c \) (Colombian), \( m \) (mild) and \( j = s \) (special), \( d \) (dark), \( r \) (regular)

maximize \( Z = 4.5x_{bs} + 3.75x_{os} + 3.60x_{cs} + 4.8x_{ms} + 3.25x_{md} + 2.5x_{od} + 2.35x_{cd} + 3.55x_{md} + 1.75x_{rd} + 1.00x_{cr} + 0.85x_{er} + 2.05x_{mr} \)

subject to
33. (a) $x_1$ = $ amount borrowed for six months in July

$y_i$ = $ amount borrowed in month $i$ ($i = 1, 2, ..., 6$) for one month
$c_i$ = $ amount carried over from month $i$ to $i + 1$

minimize $Z = .11x_1 + .05 \sum_{i=1}^{6} y_i$

subject to

- July: $x_1 + y_1 + 20,000 - c_1 = 60,000$
- August: $c_1 + y_2 + 30,000 - c_2 = 60,000 + y_1$
- September: $c_2 + y_3 + 40,000 - c_3 = 80,000 + y_2$
- October: $c_3 + y_4 + 50,000 - c_4 = 30,000 + y_3$
- November: $c_4 + y_5 + 80,000 - c_5 = 30,000 + y_4$
- December: $c_5 + y_6 + 100,000 - c_6 = 100,000 + y_5$
- End: $x_3 + y_6 \leq c_6$
- $x_1, y_1, c_i \geq 0$

(b) Solution
$x_1 = 70,000$
$y_3 = 40,000$
$y_4 = 20,000$
$y_1 = y_2 = y_5 = y_6 = 0$
$c_5 = 30,000$
$c_6 = 110,000$
$Z = $10,700

(c) Changing the six-month interest rate to 9% results in the following new solution:
$x_1 = 90,000$
$y_3 = 20,000$
$c_1 = 50,000$
$c_2 = 20,000$
$c_5 = 50,000$
$c_6 = 130,000$
$Z = $9,100

34. (a) $x_{ij}$ = production in month $i$ to meet demand in month $j$, where $i = 1, 2, ..., 7$ and $j = 4, 5, 6$ and 7

$y_j$ = overtime production in month $j$ where $j = 4, 5, 6, 7$

Minimize $Z = 150x_{14} + 100x_{24} + 50x_{34} + 0x_{44} + 200x_{15} + 150x_{25} + 100x_{35} + 50x_{45} + 0x_{55} + 50x_{56} + 250x_{16} + 200x_{26} + 150x_{36} + 0x_{66} + 50x_{67} + 300x_{17} + 250x_{27} + 200x_{37} + 0x_{77} + 400y_4 + 400y_5 + 400y_6 + 400y_7$

Subject to

$1 = 50,000 + 200x_{14} + 0x_{15} + 0x_{16} + 0x_{17}$
$2 = 30,000 + 0x_{14} + 200x_{15} + 0x_{16} + 0x_{17}$
$3 = 20,000 + 0x_{14} + 0x_{15} + 200x_{16} + 0x_{17}$
$4 = 100,000 + 0x_{14} + 0x_{15} + 0x_{16} + 200x_{17}$
$5 = 30,000 + 0x_{14} + 0x_{15} + 0x_{16} + 0x_{17}$
$6 = 100,000 + 0x_{14} + 0x_{15} + 0x_{16} + 0x_{17}$
$7 = 100,000 + 0x_{14} + 0x_{15} + 0x_{16} + 0x_{17}$

(b) $x_{14} = 10$
$y_5 = 5$
$x_{34} = 10$
$y_6 = 10$
$x_{44} = 40$
$y_7 = 20$
$x_{35} = 20$
$Z = $31,500

<table>
<thead>
<tr>
<th>Month $i$</th>
<th>Capacity</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Capacity</th>
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<tr>
<td>7</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td></td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

| Overtime — | 5 | 10 | 20 |
| Demand     | 60| 85 | 100| 120|

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(c) \( x_{34} = 20 \)
\( x_{44} = 40 \)
\( x_{25} = 5 \)
\( x_{35} = 20 \)
\( x_{55} = 60 \)
\( x_{26} = 10 \)
\( x_{66} = 90 \)
\( x_{17} = 40 \)
\( x_{27} = 25 \)
\( x_{77} = 50 \)
\( y_7 = 5 \)
\( Z = \$26,000 \)

The new solution would be,
\[ \begin{align*}
&x_{cr} = 20 \\
&x_{pr} = 10 \\
&x_{ar} = 70 \\
&x_{bb} = 75 \\
&x_{pb} = 25 \\
&x_{cm} = 180 \\
&x_{pm} = 225 \\
&x_{pm} = 115 \\
&x_{am} = 330 \\
&Z = \$1,477.50
\end{align*} \]

35. (a) \( x_{ij} \) = amount of ingredient \( i \) in wiener type \( j \), where \( i = c, b, p, a \) represent chicken, beef, pork, and additives, and \( j = r, b, m \) represent regular, beef, and all-meat, respectively

maximize \( Z = .7x_{cr} + .6x_{br} + .4x_{pr} + .85x_{ar} + 1.05x_{cb} + .95x_{bb} + .75x_{pb} + 1.20x_{ab} + 1.55x_{cm} + 1.45x_{bm} + 1.25x_{pm} + 1.70x_{am} \)

subject to
\[ \begin{align*}
x_{cr} + x_{cb} + x_{cm} &\leq 200 \\
x_{br} + x_{bb} + x_{bm} &\leq 300 \\
x_{pr} + x_{pb} + x_{pm} &\leq 150 \\
x_{ar} + x_{ab} + x_{am} &\leq 400 \\
.90x_{cr} + .90x_{br} + .4x_{pr} + .85x_{ar} &\leq 100 \\
.80x_{cb} + .80x_{bb} + .20x_{pr} + .20x_{ar} &\leq 100 \\
.25x_{cb} + .75x_{cm} + .75x_{ab} + .75x_{am} &\leq 100 \\
x_{am} &\geq 0 \\
x_{ij} &\geq 0 \end{align*} \]

*Also feasible to delete \( x_{am} \) from the problem.

(b) \( x_{cm} = 200 \)
\( x_{bm} = 300 \)
\( x_{pm} = 150 \)
\( x_{am} = 400 \)
\( Z = \$1,612.50 \)

36. (a) This is an assignment problem.

\[ \begin{align*}
x_1 & = \text{operator 1 to drill press} \\
x_2 & = \text{operator 1 to lathe} \\
x_3 & = \text{operator 1 to grinder} \\
x_4 & = \text{operator 2 to drill press} \\
x_5 & = \text{operator 2 to lathe} \\
x_6 & = \text{operator 2 to grinder} \\
x_7 & = \text{operator 3 to drill press} \\
x_8 & = \text{operator 3 to lathe} \\
x_9 & = \text{operator 3 to grinder} \\
\end{align*} \]

minimize \( Z = 22x_1 + 18x_2 + 35x_3 + 41x_4 + 30x_5 + 28x_6 + 25x_7 + 36x_8 + 18x_9 \)

subject to
\[ \begin{align*}
x_1 + x_5 + x_9 &\leq 1 \\
40x_5 &\geq 1 \\
x_7 + x_8 + x_9 &\leq 1 \\
x_4 + x_5 + x_6 &\geq 1 \\
x_1 + x_4 + x_7 &\leq 1 \\
x_2 + x_5 + x_8 &\geq 1 \\
x_3 + x_6 + x_9 &\geq 1 \\
x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 &\geq 0 \end{align*} \]

(b) \( x_1 = 1 \)
\( x_5 = 1 \)
\( x_9 = 1 \)
\( Z = 70 \)

(c) This would require the model to be reformulated with three new variables, \( x_{10}, x_{11}, x_{12} \), representing Kelly’s assignment to the press, lathe, and grinder. The model would be reformulated as,

Minimize \( Z = 22x_1 + 18x_2 + 35x_3 + 41x_4 + 30x_5 + 28x_6 + 25x_7 + 36x_8 + 18x_9 + 20x_{10} + 20x_{11} + 20x_{12} \)
41. maximize \( Z = 0.085x_1 + 0.100x_2 + 0.065x_3 + 0.130x_4 \) 
subject to 
\[
\begin{align*}
x_4 &\leq 14,000 \\
x_2 &\leq x_1 + x_3 + x_4 \\
x_2 + x_3 &\geq 21,000 \\
-1.2x_1 + x_2 + x_3 -1.2x_4 &\geq 0 \\
x_1 + x_2 + x_3 + x_4 &\leq 70,000 \\
x_1, x_2, x_3, x_4 &\geq 0 
\end{align*}
\]
\[
x_1 = 17,818.182 \\
x_2 = 35,000 \\
x_3 = 3,181.818 \\
x_4 = 14,000 \\
Z = 7,041.36
\]

42. (a) \( x_{ij} \) = barrels of component \( i \) used in gasoline grade \( j \) per day, where \( i = 1, 2, 3, 4 \) and \( j = R \) (regular), \( P \) (premium), \( L \) (low lead) 

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Length (ft)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
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<td>2</td>
<td>2</td>
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<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Total used (ft)</td>
<td>21</td>
<td>24</td>
<td>23</td>
<td>25</td>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>

\( x_i = \) no. of standard-length boards to cut using pattern \( i \)

minimize \( Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \)
subject to 
\[
\begin{align*}
3x_1 + 2x_2 + 2x_3 + x_4 &\geq 700 \\
x_3 + 2x_4 + x_5 &\geq 1,200 \\
x_2 + x_5 + 2x_6 &\geq 300 \\
x_i &\geq 0 
\end{align*}
\]

(b) \( x_2 = 50 \) 
\( x_4 = 600 \) 
\( x_6 = 125 \) 
\( Z = 775 \)

43. (a) First, all possible patterns that contain the desired lengths must be determined.