Chapter Four: Linear Programming: Modeling Examples

PROBLEM SUMMARY

1. "Product mix" example
2. "Diet" example
3. "Investment" example
4. "Marketing" example
5. "Transportation" example
6. "Blend" example
7. Product mix (maximization)
8. Sensitivity analysis (4–7)
9. Diet (minimization)
10. Product mix (minimization)
11. Product mix (maximization)
12. Product mix (maximization)
13. Product mix (maximization)
14. Ingredients mix (minimization)
15. Transportation (maximization)
16. Product mix (maximization)
17. Ingredients mix (minimization)
18. Crop distribution (maximization)
19. Monetary allocation (maximization)
20. Diet (minimization), sensitivity analysis
21. Transportation (maximization)
22. Transportation (minimization)
23. Warehouse scheduling (minimization)
24. School busing (minimization)
25. Sensitivity analysis (4–24)
26. Ingredients mixture (minimization)
27. Interview scheduling (maximization)
28. Investments mixture (maximization)
29. Insurance poly mix (maximization)
30. Product mix (maximization)
31. Advertising mix (minimization), sensitivity analysis
32. Blend (maximization)
33. Multiperiod borrowing (minimization)
34. Multiperiod production scheduling (minimization)
35. Blend (maximization), sensitivity analysis
36. Assignment (minimization), sensitivity analysis
37. Transportation (minimization)
38. Scheduling (minimization)
39. Production line scheduling (maximization)
40. Network flow (minimization)
41. Investment example" in chapter (maximization)
42. Blend (maximization)
43. Trim loss (minimization)
44. Multiperiod investment (maximization)
45. Multiperiod sales and inventory (maximization)
46. Multiperiod production and inventory (minimization)
47. Employee assignment (maximization)
48. Data envelopment analysis
49. Data envelopment analysis
50. Network flow (maximization)
51. Multiperiod workforce planning (minimization)
52. Integer solution (4–43)
53. Machine scheduling (maximization), sensitivity analysis
54. Cargo storage (maximization)
55. Broadcast scheduling (maximization)
56. Product mix (maximization)
57. Product mix/advertising (maximization)
58. Scheduling (minimization)
59. Truck purchasing/leasing (minimization)

or,

$$20{,}000 - 12{,}000x_2 = 0$$
$$20{,}000x_1 - 9{,}000x_3 = 0$$

The new solution is $x_1 = 3.068$, $x_2 = 5.114$, $x_3 = 6.818$ and $Z = 184{,}090$. This results in approximately 61,362 exposures per type of advertising (with some slight differences due to computer rounding).

5. The slack variables for the three ≤ warehouse constraints would be added to the constraints as follows,

$$x_{1A} + x_{1B} + x_{1C} + s_1 = 300$$
$$x_{2A} + x_{2B} + x_{2C} + s_2 = 200$$
$$x_{3A} + x_{3B} + x_{3C} + s_3 = 200$$

These three slacks would then be added to the objective function with the storage cost coefficients of \$9 for s_1, \$6 for s_2 and \$7 for s_3.

This change would not result in a new solution.

The model must be reformulated with three new variables reflecting the shipments from the new warehouse at Memphis (4) to the three stores, x_{4A}, x_{4B}, and x_{4C}. These variables must be included in the objective function with the cost coefficients of \$18, 9 and 12 respectively. A new supply constraint must be added,

$$x_{4A} + x_{4B} + x_{4C} \le 200$$

The solution to this reformulated model is,

$$x_{1C} = 200$$
$$x_{2B} = 50$$
$$x_{3A} = 150$$
$$x_{4B} = 200$$
$$Z = 6{,}550$$

Yes, the warehouse should be leased.

The shadow price for the Atlanta warehouse shows the greatest decrease in cost, \$6 for every additional set supplied from this source. However, the upper limit of the sensitivity range is 200, the same as the current supply value. Thus, if the supply is increased at Atlanta by even one television set the shadow price will change.

6. This change would not affect the solution at all since there is no surplus with any of the three constraints.

Component 1 has the greatest dual price of \$20. For each barrel of component 1 the company can acquire, profit will increase by \$20, up to the limit of the sensitivity range which is an increase of 1,700 bbls. or 6,200 total bbls. of component 1. For example an increase of one bbl. of component 1 from 4,500 to 4,501 results in a increase in total cost to \$76,820.

7. (a) maximize $Z = \$190x_1 + 170x_2 + 155x_3$
subject to

$$3.5x_1 + 5.2x_2 + 2.8x_3 \le 500$$
$$1.2x_1 + 0.8x_2 + 1.5x_3 \le 240$$
$$40x_1 + 55x_2 + 20x_3 \le 6{,}500$$
$$x_1, x_2, x_3 \ge 0$$

(b) $x_1 = 41.27$, $x_2 = 0$, $x_3 = 126.98$, $Z = \$27{,}523.81$
$s_1 = s_2 = 0$, $s_3 = 2{,}309.52$

8. (a) It would not affect the model. The slack apples are multiplied by the revenue per apple of \$.08 to determine the extra total revenue, i.e., (2,309.52)(\$.08) = \$184.76.

(b) This change requires a new variable, x_4, and that the constraint for apples be changed from ≤ to =. No, the Friendly's should not produce cider. The new solution would be $x_1 = 135$, $x_2 = 0$, $x_3 = 0$, $x_4 = 18.33$ and $Z = \$26{,}475$. This reduction in profit occurs because the requirement that all 6,500 apples be used forces resources to be used for cider that would be more profitable to be used to produce the other products. If the final model constraint for apples is, ≤ rather than =, the previous solution in 1(b) results.

9. (a) x_1 = no. of eggs
x_2 = no. of bacon strips
x_3 = no. of cups of cereal

minimize $Z = 4x_1 + 3x_2 + 2x_3$
subject to

$$2x_1 + 4x_2 + x_3 \ge 16$$
$$3x_1 + 2x_2 + x_3 \ge 12$$
$$x_1, x_2, x_3 \ge 0$$

(b) $x_1 = 2$
$x_2 = 3$
$Z = \$0.17$

10.(a) x_i = number of boats of type i, i = 1 (bass boat), 2 (ski boat), 3 (speed boat)

In order to break even total revenue must equal total cost:

$$23{,}000x_1 + 18{,}000x_2 + 26{,}000x_3 = 12{,}500x_1 + 8{,}500x_2 + 13{,}700x_3 + 2{,}800{,}000$$

or,

$$10{,}500x_1 + 9{,}500x_2 + 12{,}300x_3 = 2{,}800{,}000$$

minimize $Z = 12{,}500x_1 + 8{,}500x_2 + 13{,}700x_3$
subject to

$$\begin{aligned} 10{,}500x_1 + 9{,}500x_2 + 12{,}300x_3 &= 2{,}800{,}000 \\ x_1 &\geq 70 \\ x_2 &\geq 50 \\ x_3 &\geq 50 \\ x_1 &\leq 120 \\ x_2 &\leq 120 \\ x_3 &\leq 120 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

(b) $x_1 = 70.00$
$x_2 = 120.00$
$x_3 = 75.203$
$Z = \$2{,}925{,}284.553$

11.(a) x_1 = no. of clocks
x_2 = no. of radios
x_3 = no. of toasters

maximize $Z = 8x_1 + 10x_2 + 7x_3$
subject to

$$\begin{aligned} 7x_1 + 10x_2 + 5x_3 &\leq 2{,}000 \\ 2x_1 + 3x_2 + 2x_3 &\leq 660 \\ x_1 &\leq 200 \\ x_2 &\leq 300 \\ x_3 &\leq 150 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

(b) $x_1 = 178.571$
$x_3 = 150.00$
$Z = \$2{,}478.571$

12.(a) x_1 = no. of gallons of Yodel
x_2 = no. of gallons of Shotz
x_3 = no. of gallons of Rainwater

maximize $Z = 1.50x_1 + 1.60x_2 + 1.25x_3$
subject to

$$\begin{aligned} x_1 + x_2 + x_3 &= 1{,}000 \\ 1.50x_1 + .90x_2 + .50x_3 &\leq 2{,}000 \\ x_1 &\leq 400 \\ x_2 &\leq 500 \\ x_3 &\leq 300 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

(b) $x_1 = 400$
$x_2 = 500$
$x_3 = 100$
$Z = \$1{,}525.00$

13.(a) x_1 = no. of lb of Super Two at Fresno
x_2 = no. of lb of Super Two at Dearborn
x_3 = no. of lb of Green Grow at Fresno
x_4 = no. of lb of Green Grow at Dearborn

maximize $Z = 7x_1 + 5x_2 + 5x_3 + 4x_4$
subject to

$$\begin{aligned} 2x_1 + 4x_2 + 2x_3 + 3x_4 &\leq 45{,}000 \\ x_1 + x_2 &\leq 6{,}000 \\ x_3 + x_4 &\leq 7{,}000 \\ x_1 + x_3 &\leq 5{,}000 \\ x_2 + x_4 &\leq 6{,}000 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

(b) $x_1 = 5{,}000$
$x_2 = 1{,}000$
$x_4 = 5{,}000$
$Z = \$60{,}000$

14.(a) x_i = ore i (i = 1,2,3,4,5,6)

minimize $Z = 27x_1 + 25x_2 + 32x_3 + 22x_4 +$
$20x_5 + 24x_6$
subject to

$$\begin{aligned} .19x_1 + .43x_2 + .17x_3 + .20x_4 + .12x_6 &\geq .21 \\ .15x_1 + .10x_2 + .12x_4 + .24x_5 + .18x_6 &\leq .12 \\ .12x_1 + .25x_2 + .10x_5 + .16x_6 &\leq .07 \\ .14x_1 + .07x_2 + .53x_3 + .18x_4 + 0.31x_5 + .25x_6 &\geq .30 \\ .14x_1 + .07x_2 + .53x_3 + .18x_4 + .31x_5 + .25x_6 &\leq .65 \\ .60x_1 + .85x_2 + .70x_3 + .50x_4 + .65x_5 + .71x_6 &= 1.00 \end{aligned}$$

(b) $x_2 = .1153$ ton – ore 2
$x_3 = .8487$ ton – ore 3
$x_4 = .0806$ ton – ore 4
$x_5 = .4116$ ton – ore 5
$Z = \$40.05$

15.(a) x_{ij} = number of trucks assigned to route from warehouse i to terminal j, where i = 1 (Charlotte), 2 (Memphis), 3 (Louisville) and j = a (St. Louis), b (Atlanta), c (New York)

maximize $Z = 1.800x_{1a} + 2{,}100x_{1b} + 1{,}600x_{1c}$
$+ 1{,}000x_{2a} + 700x_{2b} + 900x_{2c} + 1{,}400x_{3a}$
$+ 800x_{3b} + 2{,}200x_{3c}$
subject to

$$\begin{aligned} x_{1a} + x_{1b} + x_{1c} &= 30 \\ x_{2a} + x_{2b} + x_{2c} &= 30 \\ x_{3a} + x_{3b} + x_{3c} &= 30 \\ x_{1a} + x_{2a} + x_{3a} &\leq 40 \\ x_{1b} + x_{2b} + x_{3b} &\leq 60 \\ x_{1c} + x_{2c} + x_{3c} &\leq 50 \\ x_{ij} &\geq 0 \end{aligned}$$

(b) $x_{1b} = 30$
$x_{2a} = 30$
$x_{3c} = 30$
$Z = \$159,000$

16. **(a)** x_1 = no. of sofas
x_2 = no. of tables
x_3 = no. of chairs

maximize $Z = 400x_1 + 275x_2 + 190x_3$
subject to

$$\begin{aligned} 7x_1 + 5x_2 + 4x_3 &\le 2{,}250 \\ 12x_1 + 7x_3 &\le 1{,}000 \\ 6x_1 + 9x_2 + 5x_3 &\le 240 \\ x_1 + x_2 + x_3 &\le 650 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

(b) $x_1 = 40$
$Z = \$16,000$

17. **(a)** x_{ij} = lbs. of seed i used in mix j, where i = t (tall fescue), m (mustang fescue), b (bluegrass) and j = 1,2,3.

minimize $Z = 1.70\ (x_{t1} + x_{t2} + x_{t3}) + 2.80\ (x_{m1} + x_{m2} + x_{m3}) + 3.25\ (x_{b1} + x_{b2} + x_{b3})$
subject to

$$\begin{aligned} .50x_{t1} - .50x_{m1} - .50x_{b1} &\le 0 \\ -.20x_{t1} + .80x_{m1} - .20x_{b1} &\ge 0 \\ -.30x_{t2} - .30x_{m2} + .70x_{b2} &\ge 0 \\ -.30x_{t2} + .70x_{m2} - .30x_{b2} &\ge 0 \\ .80x_{t2} - .20x_{m2} - .20x_{b2} &\le 0 \\ .50x_{t3} - .50x_{m3} - .50x_{b3} &\ge 0 \\ .30x_{t3} - .70x_{m3} - .70x_{b3} &\le 0 \\ -.10x_{t3} - .10x_{m3} + .90x_{b3} &\ge 0 \\ x_{t1} + x_{m1} + x_{b1} &\ge 1{,}200 \\ x_{t2} + x_{m2} + x_{b2} &\ge 900 \\ x_{t3} + x_{m3} + x_{b3} &\ge 2{,}400 \\ x_{ij} &\ge 0 \end{aligned}$$

(b) $x_{t1} = 600$
$x_{t2} = 180$
$x_{t3} = 1,680$
$x_{m1} = 600$
$x_{m2} = 450$
$x_{m3} = 480$
$x_{b1} = 0$
$x_{b2} = 270$
$x_{b3} = 240$
$Z = \$10,123.50$

18. **(a)** x_{ij} = acres of crop i planted on plot j, where i = c (corn), p (peas), s (soybeans) and j = 1,2,3

maximize $Z = 600(x_{c1} + x_{c2} + x_{c3}) + 450(x_{p1} + x_{p2} + x_{p3}) + 300(x_{s1} + x_{s2} + x_{s3})$

subject to

$$\begin{aligned} x_{c1} + x_{p1} + x_{s1} &\ge 300 \\ x_{c1} + x_{p1} + x_{s1} &\le 500 \\ x_{c2} + x_{p2} + x_{s2} &\ge 480 \\ x_{c2} + x_{p2} + x_{s2} &\le 800 \\ x_{c3} + x_{p3} + x_{s3} &\ge 420 \\ x_{c3} + x_{p3} + x_{s3} &\le 700 \\ x_{c1} + x_{c2} + x_{c3} &\le 900 \\ x_{p1} + x_{p2} + x_{p3} &\le 700 \\ x_{s1} + x_{s2} + x_{s3} &\le 1{,}000 \\ 800(x_{c1} + x_{p1} + x_{s1}) - 500(x_{c2} + x_{p2} + x_{s2}) &= 0 \\ 700(x_{c2} + x_{p2} + x_{s2}) - 800(x_{c3} + x_{p3} + x_{s3}) &= 0 \\ 700(x_{c1} + x_{p1} + x_{s1}) - 500(x_{c3} + x_{p3} + x_{s3}) &= 0 \\ x_{ij} &\ge 0 \end{aligned}$$

(b) $x_{c1} = 500$
$x_{c2} = 100$
$x_{c3} = 300$
$x_{p2} = 700$
$x_{s3} = 400$
$Z = \$975,000$

19. **(a)** x_1 = \$ allocated to job training
x_2 = \$ allocated to parks
x_3 = \$ allocated to sanitation
x_4 = \$ allocated to library

maximize $Z = .02x_1 + .09x_2 + .06x_3 + .04x_4$
subject to

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4{,}000{,}000 \\ x_1 &\le 1{,}600{,}000 \\ x_2 &\le 1{,}600{,}000 \\ x_3 &\le 1{,}600{,}000 \\ x_4 &\le 1{,}600{,}000 \\ x_2 - x_3 - x_4 &\le 0 \\ x_1 - x_3 &\ge 0 \\ x_1, x_2, x_3, x_4 &\ge 0 \end{aligned}$$

(b) $x_1 = 800,000$
$x_2 = 1,600,000$
$x_3 = 800,000$
$x_4 = 800,000$
$Z = 240,000$

20. **(a)** minimize $Z = .80x_1 + 3.70x_2 + 2.30x_3 + .90x_4 + .75x_5 + .40x_6 + .83x_7$

subject to

$$520x_1 + 500x_2 + 860x_3 + 600x_4 + 50x_5 + 460x_6 + 240x_7 \geq 1{,}500$$
$$520x_1 + 500x_2 + 860x_3 + 600x_4 + 50x_5 + 460x_6 + 240x_7 \leq 2{,}000$$
$$4.4x_1 + 3.3x_2 + .3x_3 + 3.4x_4 + .5x_5 + 2.2x_6 + .2x_7 \geq 5$$
$$30x_1 + 5x_2 + 75x_3 + 3x_4 + 10x_7 \geq 20$$
$$30x_1 + 5x_2 + 75x_3 + 3x_4 + 10x_7 \leq 60$$
$$17x_1 + 85x_2 + 82x_3 + 10x_4 + 6x_5 + 10x_6 + 16x_7 \geq 30$$
$$30x_4 + 70x_6 + 22x_7 \geq 40$$
$$180x_1 + 90x_2 + 350x_3 + 20x_7 \leq 30$$
$$x_i \geq 0$$

(b) $x_4 = 1.667$
$x_6 = 0.304$
$x_7 = 1.500$
$Z = \$2.867$

(c) The model becomes infeasible and cannot be solved. Limiting each food item to one-half pound is too restrictive. In fact, experimentation with the model will show that one food item in particular, dried beans, is restrictive. All other food items can be limited except dried beans.

21. **(a)** x_{ij} = number of units of products i ($i = 1,2,3$) produced on machine j ($j = 1,2,3,4$)

maximize $Z = \$7.8x_{11} + 7.8x_{12} + 8.2x_{13} + 7.9x_{14} + 6.7x_{21} + 8.9x_{22} + 9.2x_{23} + 6.3x_{24} + 8.4x_{31} + 8.1x_{32} + 9.0x_{33} + 5.8x_{34}$

subject to

$$35x_{11} + 40x_{21} + 38x_{31} \leq 9{,}000$$
$$41x_{12} + 36x_{22} + 37x_{32} \leq 14{,}400$$
$$34x_{13} + 32x_{23} + 33x_{33} \leq 12{,}000$$
$$39x_{14} + 43x_{24} + 40x_{34} \leq 15{,}000$$
$$x_{11} + x_{12} + x_{13} + x_{14} = 400$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 570$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 320$$
$$x_{ij} \geq 0$$

(b) $x_{11} = 15.385$
$x_{14} = 384.615$
$x_{22} = 400.00$
$x_{23} = 170.00$
$x_{31} = 121.212$
$x_{33} = 198.788$
$Z = \$11{,}089.73$

22. **(a)** Minimize $Z = 69x_{11} + 71x_{12} + 72x_{13} + 74x_{14} + 76x_{21} + 74x_{22} + 75x_{23} + 79x_{24} + 86x_{31} + 89x_{32} + 80x_{33} + 82x_{34}$

subject to

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 220$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 170$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 280$$
$$x_{11} + x_{21} + x_{31} = 110$$
$$x_{12} + x_{22} + x_{32} = 160$$
$$x_{13} + x_{23} + x_{33} = 90$$
$$x_{14} + x_{24} + x_{34} = 180$$

Ash: $.03x_{11} - .01x_{21} - .02x_{31} \leq 0$
$$.04x_{12} + .0x_{22} - .01x_{32} \leq 0$$
$$.04x_{13} + .0x_{23} - .01x_{33} \leq 0$$
$$.03x_{14} - .01x_{24} - .02x_{34} \leq 0$$

Sulfur: $.01x_{11} - .01x_{21} - .02x_{31} \leq 0$
$$.01x_{12} + .01x_{22} - .02x_{32} \leq 0$$
$$-.01x_{13} + .03x_{23} - .04x_{33} \leq 0$$
$$.0x_{14} - .02x_{24} - .03x_{34} \leq 0$$
$$x_{ij} \geq 0$$

(b) $x_{11} = 34$
$x_{13} = 11$
$x_{14} = 45$
$x_{23} = 35$
$x_{24} = 135$
$x_{31} = 76$
$x_{32} = 160$
$x_{33} = 44$
$Z = \$44{,}054$

23. **(a)** x_{ij} = space (ft^2) rented in month i for j months, where $i = 1,2...,6$, and $j = 1,2,...,6$

Minimize $Z = 1.70x_{11} + 1.40x_{12} + 1.20x_{13} + 1.10x_{14} + 1.05x_{15} + 1.00x_{16} + 1.70x_{21} + 1.40x_{22} + 1.20x_{23} + 1.10x_{24} + 1.05x_{25} + 1.70x_{31} + 1.40x_{32} + 1.20x_{33} + 1.10x_{34} + 1.70x_{41} + 1.40x_{42} + 1.20x_{43} + 1.70x_{51} + 1.40x_{52} + 1.70x_{61}$

subject to:

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 47{,}000$$
$$x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{22} + x_{23} + x_{24} + x_{25} = 35{,}000$$
$$x_{13} + x_{14} + x_{15} + x_{16} + x_{22} + x_{23} + x_{24} + x_{25} + x_{31} + x_{32} + x_{33} + x_{34} = 52{,}000$$
$$x_{14} + x_{15} + x_{16} + x_{23} + x_{24} + x_{25} + x_{32} + x_{33} + x_{34} + x_{41} + x_{42} + x_{43} = 27{,}000$$
$$x_{15} + x_{16} + x_{24} + x_{25} + x_{33} + x_{34} + x_{42} + x_{43} + x_{51} + x_{52} = 19{,}000$$
$$x_{16} + x_{25} + x_{34} + x_{43} + x_{52} + x_{61} = 15{,}000$$

(b) $x_{11} = 12{,}000$
$x_{13} = 25{,}000$
$x_{14} = 8{,}000$
$x_{15} = 2{,}000$
$x_{33} = 2{,}000$
$x_{34} = 15{,}000$
$Z = \$80{,}200$

(c) $x_{16} = 52{,}000$
$Z = \$52{,}000$
It is much cheaper to rent all the space for the entire six month period in April and have excess or "surplus" space.

24. **(a)** x_{ij} = no. of students bused from district i to school j, where i = n, s, e, w, c and j = c,w,s

minimize $Z = 8x_{nc} + 11x_{nw} + 14x_{ns} + 12x_{sc} + 9x_{sw} + 0x_{ss} + 9x_{ec} + 16x_{ew} + 10x_{es} + 8x_{wc} + 0x_{ww} + 9x_{ws} + 0x_{cc} + 8x_{cw} + 12x_{cs}$

subject to

$$x_{nc} + x_{nw} + x_{ns} = 700$$
$$x_{sc} + x_{sw} + x_{ss} = 300$$
$$x_{ec} + x_{ew} + x_{es} = 900$$
$$x_{wc} + x_{ww} + x_{ws} = 600$$
$$x_{cc} + x_{cw} + x_{sc} = 500$$
$$x_{nc} + x_{sc} + x_{ec} + x_{wc} + x_{cc} \le 1{,}200$$
$$x_{nw} + x_{sw} + x_{ew} + x_{ww} + x_{cw} \le 1{,}200$$
$$x_{ns} + x_{ss} + x_{es} + x_{ws} + x_{cs} \le 1{,}200$$
$$x_{ij} \ge 0$$

(b) $x_{nc} = 700$
$x_{ss} = 300$
$x_{es} = 900$
$x_{ww} = 600$
$x_{cc} = 500$
$Z = 14{,} 600$

25. **(a)** Add the following 3 constraints to the original formulation:

$x_{ss} \le 150$
$x_{ww} \le 300$
$x_{cc} \le 250$

$x_{nc} = 700$
$x_{nw} = 0$
$x_{sw} = 150$
$x_{ss} = 150$
$x_{es} = 900$
$x_{wc} = 250$
$x_{ww} = 300$
$x_{ws} = 50$
$x_{cc} = 250$
$x_{cw} = 250$
$Z = 20{,}400$

(b) Change the 3 demand constraints in the (a) formulation from $\le 1{,}200$ to $= 1{,}000$.

$x_{nc} = 400$
$x_{nw} = 300$
$x_{sw} = 150$
$x_{ss} = 150$
$x_{ec} = 50$
$x_{es} = 850$
$x_{wc} = 300$
$x_{ww} = 300$
$x_{cc} = 250$
$x_{cw} = 250$
$Z = 21{,}200$

26. **(a)** x_1 = no. of lb of oats
x_2 = no. of lb of corn
x_3 = no. of lb of soybean
x_4 = no. of lb of vitamin supplement

minimize $Z = .50x_1 + 1.20x_2 + .60x_3 + 2.00x_4$
subject to

$$x_1 \le 300$$
$$x_2 \le 400$$
$$x_3 \le 200$$
$$x_4 \le 100$$
$$x_3/(x_1 + x_2 + x_3 + x_4) \ge .30$$
$$x_4/(x_1 + x_2 + x_3 + x_4) \ge .20$$
$$x_2/x_1 \le 2/1$$
$$x_1 \le x_3$$
$$x_1 + x_2 + x_3 + x_4 \ge 500$$
$$x_1, x_2, x_3, x_4 \ge 0$$

(b) $x_1 = 200$
$x_2 = 0$
$x_3 = 200$
$x_4 = 100.00$
$Z = \$420$

27. **(a)** x_1 = no. of day contacts by phone
x_2 = no. of day contacts in person
x_3 = no. of night contacts by phone
x_4 = no. of night contacts in person

maximize $Z = 2x_1 + 4x_2 + 3x_3 + 7x_4$
subject to

$$x_2 + x_4 \le 300$$
$$6x_1 + 15x_2 \le 1{,}200$$
$$5x_3 + 12x_4 \le 2{,}400$$
$$x_1, x_2, x_3, x_4 \ge 0$$

(b) $x_1 = 200$
$x_3 = 480$
$Z = 1{,}840$

28. **(a)** x_{ij} = dollar amount invested in alternative i in year j, where i = p (product research and development), m (manufacturing operations improvements), a (advertising and sales promotion) and j = 1,2,3,4 (denoting year):
s_j = slack, or uninvested funds in year j

maximize $Z = s_4 + 1.2x_{a4} + 1.3x_{m3} + 1.5x_{p3}$
subject to

$$x_{a1} \geq 30{,}000$$
$$x_{m1} \geq 40{,}000$$
$$x_{p1} \geq 50{,}000$$
$$x_{a1} + x_{m1} + x_{p1} + s_1 = 500{,}000$$
$$x_{a2} + x_{m2} + x_{p2} + s_2 = s_1 + 1.2x_{a1}$$
$$x_{a3} + x_{m3} + x_{s3} = s_2 + 1.2x_{a2} + 1.3x_{m1}$$
$$x_{a4} + x_{s4} = s_3 + 1.2x_{a3} + 1.3x_{m2} + 1.5x_{p1}$$
$$x_{ij}, s_j \geq 0$$

Note: Since it is assumed that any amount of funds can be invested in each alternative— i.e., there is no minimum investment required—and funds can always be invested in as short a period as one year yielding a positive return, it is apparent that the s_j variables for uninvested funds will be driven to zero in every period. Thus, these variables could be omitted from the model formulation for this problem.

(b) x_{a1} = 410,000 $\quad x_{m1}$ = 40,000
x_{a2} = 492,000 $\quad x_{p1}$ = 50,000
x_{a3} = 642,400 $\quad Z$ = \$1,015,056
x_{a4} = 845,880

29. **(a)** x_1 = no. of homeowner's policies
x_2 = no. of auto policies
x_3 = no. of life policies

maximize $Z = 35x_1 + 20x_2 + 58x_3$
subject to

$$14x_1 + 12x_2 + 35x_3 \leq 35{,}000$$
$$6x_1 + 3x_2 + 12x_3 \leq 20{,}000$$
$$x_1, x_2, x_3 \geq 0$$

(b) x_1 = 2,500
Z = \$87,500

30. **(a)** x_1 = no. of issues of *Daily Life*
x_2 = no. of issues of *Agriculture Today*
x_3 = no. of issues of *Surf's Up*

maximize $Z = 2.25x_1 + 4.00x_2 + 1.50x_3$
subject to

$$x_1 + x_2 + x_3 \geq 5{,}000$$
$$.01x_1 + .03x_2 + .02x_3 \leq 120$$
$$.2x_1 + .5x_2 + .3x_3 \leq 3{,}000$$
$$x_1 \leq 3{,}000$$
$$x_2 \leq 2{,}000$$
$$x_3 \leq 6{,}000$$
$$x_1, x_2, x_3 \geq 0$$

(b) x_1 = 3,000
x_2 = 2,000
x_3 = 1,500
Z = \$17,000

31. **(a)** x_1 = no. of television commercials
x_2 = no. of newspaper ads
x_3 = no. of radio commercials

minimize $Z = 15{,}000x_1 + 4{,}000x_2 + 6{,}000x_3$
subject to

$$x_3/x_2 \geq 2/1$$
$$25{,}000x_1 + 10{,}000x_2 + 15{,}000x_3 \geq 100{,}000$$
$$(15{,}000x_1 + 3{,}000x_2 + 12{,}000x_3)/(10{,}000x_1 + 7{,}000x_2 + 3{,}000x_3) \geq 2/1$$
$$(15{,}000x_1 + 4{,}000x_2 + 9{,}000x_3)/(25{,}000x_1 + 10{,}000x_2 + 15{,}000x_3) \geq .30$$
$$x_2 \leq 7$$
$$x_1, x_2, x_3 \geq 0$$

(b) x_2 = 2.5
x_3 = 5.0
Z = 40,000

(c) This reformulation of the model would result in a fourth variable x_4, with model parameters inserted accordingly. However, it would have no effect on the solution.

32. **(a)** x_{ij} = lbs. of coffee i used in blend j per week, where i = b (Brazilian), o (Mocha), c (Colombian), m (mild) and j = s (special), d (dark), r (regular)

maximize $Z = 4.5x_{\mathrm{bs}} + 3.75x_{\mathrm{os}} + 3.60x_{\mathrm{cs}} + 4.8x_{\mathrm{ms}} + 3.25x_{\mathrm{bd}} + 2.5x_{\mathrm{od}} + 2.35x_{\mathrm{cd}} + 3.55x_{\mathrm{md}} + 1.75x_{\mathrm{br}} + 1.00x_{\mathrm{or}} + 0.85x_{\mathrm{cr}} + 2.05x_{\mathrm{mr}}$
subject to

$$.6x_{cs} - .4x_{bs} - .4x_{os} - .4x_{ms} \geq 0$$
$$-.3x_{bs} + .7x_{os} - .3x_{cs} - .3x_{ms} \geq 0$$
$$.4x_{bd} - .6x_{od} - .6x_{cd} - .6x_{md} \geq 0$$
$$-.1\,x_{bd} - .1x_{od} - .1x_{cd} + .9x_{md} \leq 0$$
$$-.6x_{br} - .6x_{or} - .6x_{cr} + .4x_{mr} \leq 0$$
$$.7x_{br} - .3x_{or} - .3x_{cr} - .3x_{mr} \geq 0$$
$$x_{bs} + x_{bd} + x_{br} \leq 110$$
$$x_{os} + x_{od} + x_{or} \leq 70$$
$$x_{cs} + x_{cd} + x_{cr} \leq 80$$
$$x_{ms} + x_{md} + x_{mr} \leq 150$$
$$x_{ij} \geq 0$$

(b) $x_{os} = 60$ Special: $x_{os} + x_{cs} + x_{ms} = 200$lbs.
$x_{cs} = 80$ Dark: $x_{bd} + x_{md} = 72$ lbs.
$x_{ms} = 60$ Regular: $x_{br} + x_{or} + x_{mr} = 138$ lbs.
$x_{bd} = 64.8$
$x_{md} = 7.2$
$x_{br} = 45.2$
$x_{or} = 10$
$x_{mr} = 82.8$
$Z = \$1,296$

33. **(a)** x_1 = \$ amount borrowed for six months in July
y_i = \$ amount borrowed in month i (i = 1, 2, ..., 6) for one month
c_i = \$ amount carried over from month i to $i + 1$

minimize $Z = .11x_1 + .05 \sum_{i=1}^{6} y_i$

subject to

July:	$x_1 + y_1 + 20{,}000 - c_1 = 60{,}000$
August:	$c_1 + y_2 + 30{,}000 - c_2 = 60{,}000 + y_1$
September:	$c_2 + y_3 + 40{,}000 - c_3 = 80{,}000 + y_2$
October:	$c_3 + y_4 + 50{,}000 - c_4 = 30{,}000 + y_3$
November	$c_4 + y_5 + 80{,}000 - c_5 = 30{,}000 + y_4$
December:	$c_5 + y_6 + 100{,}000 - c_6 = 100{,}000 + y_5$
End:	$x_3 + y_6 \leq c_6$
	$x_1, y_i, c_i \geq 0$

(b) Solution

$x_1 = 70{,}000$
$y_3 = 40{,}000$
$y_4 = 20{,}000$
$y_1 = y_2 = y_5 = y_6 = 0$
$c_5 = 30{,}000$
$c_6 = 110{,}000$
$Z = \$10{,}700$

(c) Changing the six-month interest rate to 9% results in the following new solution:

$x_1 = 90{,}000$
$y_3 = 20{,}000$
$c_1 = 50{,}000$
$c_2 = 20{,}000$
$c_5 = 50{,}000$
$c_6 = 130{,}000$
$Z = \$9{,}100$

34. **(a)** x_{ij} = production in month i to meet demand in month j, where i = 1,2,... 7 and j = 4,5,6 and 7
y_j = overtime production in month j where j = 4,5,6,7.

Minimize $Z = 150x_{14} + 100x_{24} + 50x_{34} + 0x_{44} + 200x_{15} + 150x_{25} + 100x_{35} + 50x_{45} + 0x_{55} + 50x_{56} + 250x_{16} + 200x_{26} + 150x_{36} + 0x_{66} + 50x_{67} + 300x_{17} + 250x_{27} + 200x_{37} + 0x_{77} + 400y_4 + 400y_5 + 400y_6 + 400y_7$

Subject to

$$x_{14} + x_{24} + x_{34} + x_{44} + y_4 = 60 + x_{45}$$
$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + y_5 = 85 + x_{56}$$
$$x_{16} + x_{26} + x_{36} + x_{56} + x_{66} + y_6 = 100 + x_{67}$$
$$x_{17} + x_{27} + x_{37} + x_{67} + x_{77} + y_7 = 120$$
$$x_{14} + x_{15} + x_{16} + x_{17} \leq 30$$
$$x_{24} + x_{25} + x_{26} + x_{27} \leq 30$$
$$x_{34} + x_{35} + x_{36} + x_{37} \leq 30$$
$$x_{44} \leq 40$$
$$x_{55} \leq 60$$
$$x_{66} \leq 90$$
$$x_{77} \leq 50$$
$$y_4 \leq 20$$
$$y_5 \leq 20$$
$$y_6 \leq 20$$
$$y_7 \leq 20$$

(b) $x_{14} = 10$
$x_{34} = 10$
$x_{44} = 40$
$x_{35} = 20$
$x_{55} = 60$
$x_{66} = 90$
$x_{17} = 20$
$x_{27} = 30$
$x_{77} = 50$
$y_5 = 5$
$y_6 = 10$
$y_7 = 20$
$Z = \$31{,}500$

		Production for Month *j*				
Month *i*	**Capacity**	**4**	**5**	**6**	**7**	**Capacity**
1	30	10			20	30
2	30				30	30
3	30	10	20			30
4	40	40				40
5	60		60			60
6	90			90		90
7	50				50	50
	Overtime	—	5	10	20	
	Demand	60	85	100	120	

(c) $x_{34} = 20$
$x_{44} = 40$
$x_{25} = 5$
$x_{35} = 20$
$x_{55} = 60$
$x_{26} = 10$
$x_{66} = 90$
$x_{17} = 40$
$x_{27} = 25$
$x_{77} = 50$
$y_7 = 5$
$Z = \$26,000$

35. **(a)** x_{ij} = amount of ingredient i in wiener type j, where i = c, b, p, a represent chicken, beef, pork, and additives, and j = r,b,m represent regular, beef, and all-meat, respectively

$$\begin{aligned}\text{maximize } Z = {} & .7x_{cr} + .6x_{br} + .4x_{pr} + .85x_{ar} \\ & + 1.05\, x_{cb} + .95x_{bb} + .75x_{pb} \\ & + 1.20x_{ab} + 1.55x_{cm} + 1.45x_{bm} \\ & + 1.25x_{pm} + 1.70x_{am}\end{aligned}$$

subject to

$$\begin{aligned}x_{cr} + x_{cb} + x_{cm} &\le 200 \\ x_{br} + x_{bb} + x_{bm} &\le 300 \\ x_{pr} + x_{pb} + x_{pm} &\le 150 \\ x_{ar} + x_{ab} + x_{am} &\le 400 \\ .90x_{br} + .90x_{pr} - .10x_{cr} - 10x_{ar} &\le 0 \\ .80x_{cr} - .20x_{br} - .20x_{pr} - .20x_{ar} &\ge 0 \\ .25x_{bb} - .75x_{cb} - .75x_{pb} - .75x_{ab} &\ge 0 \\ {}^*x_{am} &= 0 \\ .5x_{bm} + .5x_{pm} - .5x_{cm} - .5x_{am} &\le 0 \\ x_{ij} &\ge 0\end{aligned}$$

*Also feasible to delete x_{am} from the problem.

(b) $x_{cm} = 200$
$x_{bm} = 300$
$x_{pm} = 150$
$x_{mm} = 400$
$Z = \$1,612.50$

(c) This would require the addition of three constraints to the model,

$$\begin{aligned}x_{cr} + x_{br} + x_{pr} + x_{ar} &\ge 100 \\ x_{cb} + x_{bb} + x_{pb} + x_{ab} &\ge 100 \\ x_{cm} + x_{bm} + x_{pm} + x_{am} &\ge 100\end{aligned}$$

The new solution would be,

$x_{cr} = 20$
$x_{pr} = 10$
$x_{ar} = 70$
$x_{bb} = 75$
$x_{pb} = 25$
$x_{cm} = 180$
$x_{bm} = 225$
$x_{pm} = 115$
$x_{am} = 330$
$Z = \$1,477.50$

36. **(a)** This is an *assignment* problem.

x_1 = operator 1 to drill press
x_2 = operator 1 to lathe
x_3 = operator 1 to grinder
x_4 = operator 2 to drill press
x_5 = operator 2 to lathe
x_6 = operator 2 to grinder
x_7 = operator 3 to drill press
x_8 = operator 3 to lathe
x_9 = operator 3 to grinder

$$\begin{aligned}\text{minimize } Z = {} & 22x_1 + 18x_2 + 35x_3 + 41x_4 \\ & + 30x_5 + 28x_6 + 25x_7 + 36x_8 \\ & + 18x_9\end{aligned}$$

subject to

$$\begin{aligned}x_1 + x_2 + x_3 &= 1 \\ x_4 + x_5 + x_6 &= 1 \\ x_7 + x_8 + x_9 &= 1 \\ x_1 + x_4 + x_7 &= 1 \\ x_2 + x_5 + x_8 &= 1 \\ x_3 + x_6 + x_9 &= 1 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 &\ge 0\end{aligned}$$

(b) $x_1 = 1$
$x_5 = 1$
$x_9 = 1$
$Z = 70$

(c) This would require the model to be reformulated with three new variables, x_{10}, x_{11}, x_{12}, representing Kelly's assignment to the press, lathe, and grinder. The model would be reformulated as,

$$\begin{aligned}\text{Minimize } Z = {} & 22x_1 + 18x_2 + 35x_3 + 41x_4 + 30x_5 \\ & + 28x_6 + 25x_7 + 36x_8 + 18x_9 + \\ & 20x_{10} + 20x_{11} + 20x_{12}\end{aligned}$$

41. maximize $Z = .085x_1 + .100x_2 + .065x_3 + .130x_4$

subject to

$$x_4 \le 14{,}000$$
$$x_2 \le x_1 + x_3 + x_4$$
$$x_2 + x_3 \ge 21{,}000$$
$$-1.2x_1 + x_2 + x_3 - 1.2x_4 \ge 0$$
$$x_1 + x_2 + x_3 + x_4 \le 70{,}000$$
$$x_1, x_2, x_3, x_4 \ge 0$$

$x_1 = 17{,}818.182$
$x_2 = 35{,}000$
$x_3 = 3{,}181.818$
$x_4 = 14{,}000$
$Z = \$7{,}041.36$

42. (a) x_{ij} = barrels of component i used in gasoline grade j per day, where i = 1, 2, 3, 4 and j = R (regular), P (premium), L (low lead)

Regular: $x_{1R} + x_{2R} + x_{3R} + x_{4R}$
Premium: $x_{1P} + x_{2P} + x_{3P} + x_{4P}$
Low lead: $x_{1L} + x_{2L} + x_{3L} + x_{4L}$

maximize $Z = 9x_{1R} + 5x_{2R} + 0x_{3R} + 6x_{4R} + 9x_{1P} + 11x_{2P} + 6x_{3P} + 12x_{4P} + x_{1L} + 3x_{2L} - 2x_{3L} + 4x_{4L}$

subject to

$$x_{1R} + x_{2R} + x_{3R} + x_{4R} \ge 3{,}000$$
$$x_{1P} + x_{2P} + x_{3P} + x_{4P} \ge 3{,}000$$
$$x_{1L} + x_{2L} + x_{3L} + x_{4L} \ge 3{,}000$$
$$x_{1R} + x_{1P} + x_{1L} \le 5{,}000$$
$$x_{2R} + x_{2P} + x_{2L} \le 2{,}400$$
$$x_{3R} + x_{3P} + x_{3L} \le 4{,}000$$
$$x_{4R} + x_{4P} + x_{4L} \le 1{,}500$$
$$.6x_{1R} - .4x_{2R} - .4x_{3R} - .4x_{4R} \ge 0$$
$$-.2x_{1R} + .8x_{2R} - .2x_{3R} - .2x_{4R} \le 0$$
$$-.3x_{1R} - .3x_{2R} + .7x_{3R} - .3x_{4R} \ge 0$$
$$-.4x_{1P} - .4x_{2P} + .6x_{3P} - .4x_{4P} \ge 0$$
$$-.5x_{1L} + .5x_{2L} - .5x_{3L} - .5x_{4L} \le 0$$
$$.9x_{1L} - .1x_{2L} - .1x_{3L} - .1x_{4L} \ge 0$$
$$\text{all } x_{ij} \ge 0$$

(b) $x_{1R} = 2{,}000$
$x_{2R} = 100$
$x_{3R} = 900$
$x_{2P} = 2{,}300$
$x_{3P} = 3{,}100$
$x_{4P} = 1{,}500$
$x_{1L} = 3{,}000$
$Z = \$71{,}400$

43. (a) First, all possible patterns that contain the desired lengths must be determined.

	Pattern					
Length (ft)	**1**	**2**	**3**	**4**	**5**	**6**
7	3	2	2	1	0	0
9	0	0	1	2	1	0
10	0	1	0	0	1	2
Total used (ft)	21	24	23	25	19	20

x_i = no. of standard-length boards to cut using pattern i

minimize $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

subject to

$$3x_1 + 2x_2 + 2x_3 + x_4 = 700$$
$$x_3 + 2x_4 + x_5 = 1{,}200$$
$$x_2 + x_5 + 2x_6 = 300$$
$$x_i \ge 0$$

(b) $x_2 = 50$
$x_4 = 600$
$x_6 = 125$
$Z = 775$

(c)

	Pattern					
Length (ft)	**1**	**2**	**3**	**4**	**5**	**6**
7	3	2	2	1	0	0
9	0	0	1	2	1	0
10	0	1	0	0	1	2
Trim loss (ft)	4	1	2	0	6	5

x_i = no. of standard-length boards to cut using pattern i; coefficients of objective function = trim loss using pattern i

minimize $Z = 4x_1 + x_2 + 2x_3 + 0x_4 + 6x_5 + 5x_6$

subject to

$$3x_1 + 2x_2 + 2x_3 + x_4 \ge 700$$
$$x_3 + 2x_4 + x_5 \ge 1{,}200$$
$$x_2 + x_5 + 2x_6 \ge 300$$
$$x_i \ge 0$$

$x_2 = 300$
$x_4 = 600$
$Z = 300$