Chapter Three: Linear Programming: Computer Solution and Sensitivity Analysis

PROBLEM SUMMARY

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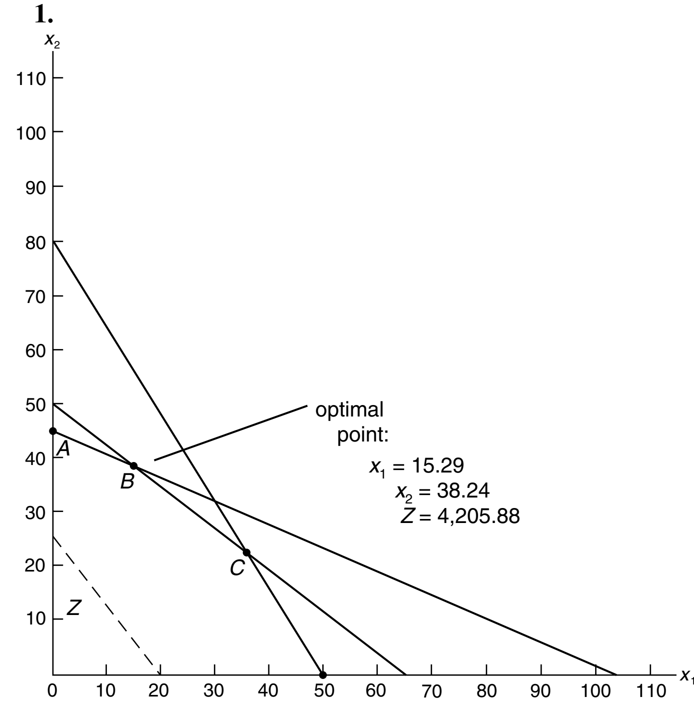
PROBLEM SOLUTIONS

1.



8. **(a)** x_1 = no. of units of A

x_2 = no. of units of B

maximize $Z = 9x_1 + 7x_2$

subject to

$$12x_1 + 4x_2 \leq 60$$
$$4x_1 + 8x_2 \leq 40$$
$$x_1, x_2 \geq 0$$

(b) maximize $Z = 9x_1 + 7x_2 + 0s_1 + 0s_2$

subject to

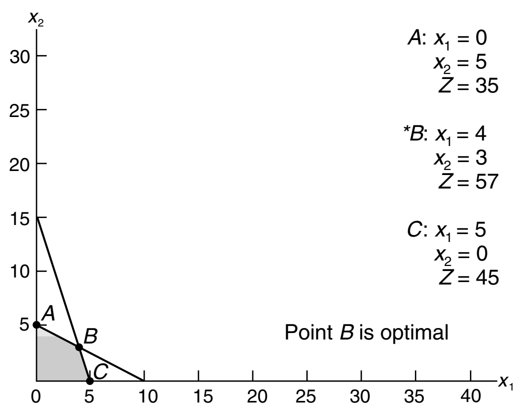
$$12x_1 + 4x_2 + s_1 = 60$$
$$4x_1 + 8x_2 + s_2 = 40$$
$$x_1, x_2, s_1, s_2 \geq 0$$

(c) In order for the optimal solution point to change from B to A the slope of the objective function must be at least as flat as the slope of the constraint line, $4x_1 + 8x_2 = 40$, which is –1/2. Thus, the profit for product B would have to be,

$$-9/c_2 = -1/2$$
$$c_2 = 18$$

If the profit for product B is increased to \$15 the optimal solution point will not change, although Z would change from \$57 to \$81. If the profit for product B is increased to \$20 the solution point will change from B to A, $x_1 = 0$, $x_2 = 5$, $Z = \$100$.

9.



(a) A: $12(0) + 4(5) + s_1 = 60$

$$s_1 = 40$$
$$4(0) + 8(5) + s_2 = 40$$
$$s_2 = 0$$

B: $12(4) + 4(3) = 60$

$$s_1 = 0$$
$$4(4) + 8(3) + s_2 = 40$$
$$s_2 = 0$$

C: $12(5) + 4(0) + s_1 = 60$

$$s_1 = 0$$
$$4(5) + 8(0) + s_2 = 40$$
$$s_2 = 20$$

(b) The constraint line $12x_1 + 4x_2 = 60$ would move inward resulting in a new location for point B at $x_1 = 2$, $x_2 = 4$, which would still be optimal.

10. **(a)** For c_1 the upper limit is computed as,

$$-c_1/7 = -3$$
$$c_1 = 21$$

and the lower limit is,

$$-c_1/7 = -1/2$$
$$c_1 = 3.50$$

For c_2 the upper limit is,

$$-9/c_2 = -1/2$$
$$c_2 = 18$$

and the lower limit is,

$$-9/c_2 = -3$$
$$c_2 = 3$$

Summarizing,

$$3.50 \leq c_1 \leq 21$$
$$3 \leq c_2 \leq 18$$

(b)

***** Input Data *****

Max. $Z = 9x_1 + 7x_2$

Subject to

c_1 $12x_1 + 4x_2 \leq 60$
c_2 $4x_1 + 8x_2 \leq 40$

***** Program Output *****

Final Optimal Solution At Simplex Tableau : 2

$Z = 57.000$

Variable	Value	Reduced Cost
x_1	4.000	0.000
x_2	3.000	0.000
Constraint	**Slack/Surplus**	**Shadow Price**
c_1	0.000	0.550
c_2	0.000	0.600

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x_1	3.500	9.000	21.000	12.000	5.500
x_2	3.000	7.000	18.000	11.000	4.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
c_1	20.000	60.000	120.000	60.000	40.000
c_2	20.000	40.000	120.000	80.000	20.000

***** End of Output *****

(c) The shadow price for line 1 time is $0.55 per hour, while the shadow price for line 2 time is $0.60 per hour. The company would prefer to obtain more line 2 time since it would result in the greatest increase in profit.

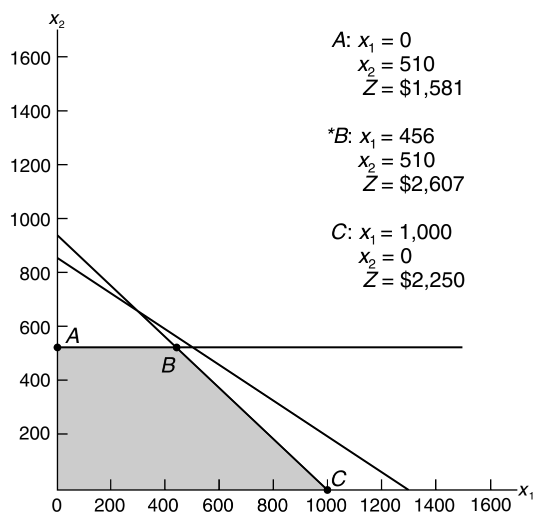
11. **(a)** x_1 = no. of yards of denim
x_2 = no. of yards of corduroy
maximize $Z = \$2.25x_1 + 3.10x_2$
subject to

$$5.0x_1 + 7.5x_2 \le 6{,}500$$
$$3.0x_1 + 3.2x_2 \le 3{,}000$$
$$x_2 \le 510$$
$$x_1, x_2 \ge 0$$

(b) maximize $Z = \$2.25x_1 + 3.10x_2 + 0s_1 + 0s_2 + 0s_3$
subject to

$$5.0x_1 + 7.5x_2 + s_1 = 6{,}500$$
$$3.0x_1 + 3.2x_2 + s_2 = 3{,}000$$
$$x_2 + s_3 = 510$$
$$x_1, x_2, s_1, s_2, s_3 \ge 0$$

12.



(a) $5.0(456) + 7.5(510) + s_1 = 6{,}500$
$s_1 = 6{,}500 - 6{,}105$
$s_1 = 395$ lbs.
$3.0(456) + 3.2(510) + s_2 = 3{,}000$
$s_2 = 0$ hrs.
$510 + s_3 = 510$
$s_3 = 0$

therefore demand for corduroy is met.

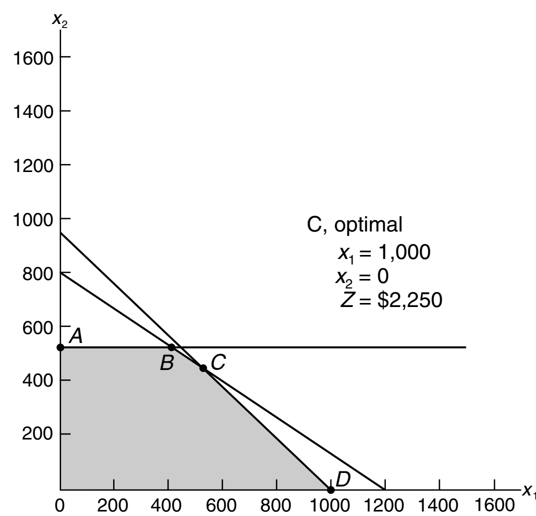
(b) In order for the optimal solution point to change from B to C the slope of the objective function must be at least as great as the slope of the constraint line, $3.0x_1 + 3.2x_2 = 3{,}000$, which is $-3/3.2$. Thus, the profit for denim would have to be,

$$-c_1/3.0 = -3/3.2$$
$$c_1 = 2.91$$

If the profit for denim is increased from \$2.25 to \$3.00 the optimal solution would change to point C where $x_1 = 1{,}000$, $x_2 = 0$, $Z = 3{,}000$.

Profit for corduroy has no upper limit that would change the optimal solution point.

(c) The constraint line for cotton would move inward as shown in the following graph where point C is optimal.



13.

***** Input Data *****

Max. $Z = 2.25x_1 + 3.1x_2$

Subject to

c_1 $5x_1 + 7.5x_2 \leq 6500$
c_2 $3x_1 + 3.2x_2 \leq 3000$
c_2 $1x_2 \leq 510$

***** Program Output *****

Final Optimal Solution At Simplex Tableau : 2

$Z = 2607.000$

Variable	Value	Reduced Cost
x_1	456.000	0.000
x_2	510.000	0.000

Constraint	Slack/Surplus	Shadow Price
c_1	395.000	0.000
c_2	0.000	0.750
c_3	0.000	0.700

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x_1	0.000	2.250	2.906	0.656	2.250
x_2	2.400	3.100	No limit	No limit	0.700

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
c_1	6015.000	6500.000	No limit	No limit	395.000
c_2	1632.000	3000.000	3237.000	237.000	1368.000
c_3	0.000	510.000	692.308	182.308	510.000

***** End of Output *****

(a) The company should select additional processing time, with a shadow price of \$0.75 per hour. Cotton has a shadow price of \$0 because there is already extra (slack) cotton available and not being used so any more would have no marginal value.

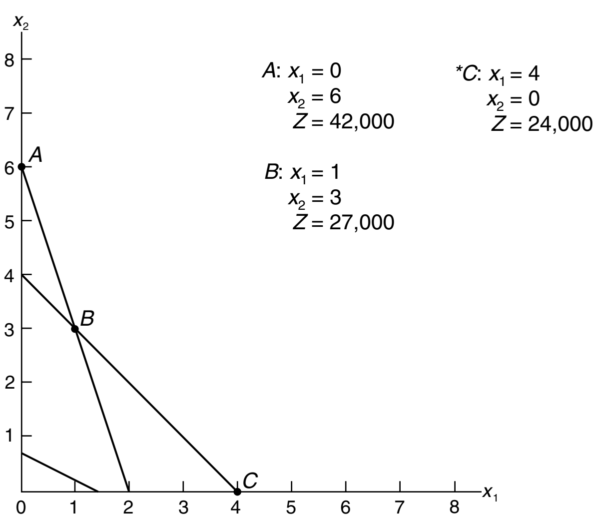
(b) $0 \leq c_1 \leq 2.906$ $\quad$ $6{,}105 \leq q_1 \leq \infty$
$2.4 \leq c_2 \leq \infty$ $\quad$ $1{,}632 \leq q_2 \leq 3{,}237$
$0 \leq q_3 \leq 692.308$

The demand for corduroy can decrease to zero or increase to 692.308 yds. without changing the current solution mix of denim and corduroy. If the demand increases beyond 692.308 yds., then denim would no longer be produced and only corduroy would be produced.

14. x_1 = no. of days to operate mill 1
x_2 = no. of days to operate mill 2
minimize $Z = 6{,}000x_1 + 7{,}000x_2$
subject to

$$6x_1 + 2x_2 \geq 12$$
$$2x_1 + 2x_2 \geq 8$$
$$4x_1 + 10x_2 \geq 5$$
$$x_1, x_2 \geq 0$$

15.



(a) $6(4) + 2(0) - s_1 = 12$
$s_1 = 12$
$2(4) + 2(0) - s_2 = 8$
$s_2 = 0$
$4(4) + 10(0) - s_3 = 5$
$s_3 = 11$

(b) The slope of the objective function, –6000/7,000 must become flatter (i.e., less) than the slope of the constraint line, $2x_1 + 2x_2 = 8$, for the solution to change. The cost of operating Mill 1, c_1, that would change the solution point is,

$$-c_1/7{,}000 = -1$$
$$c_1 = 7{,}000$$

Since \$7,500 > \$7,000, the solution point will change to B where $x_1 = 1$, $x_2 = 3$, Z = \$28,500.

(c) If the constraint line for high-grade aluminum changes to $6x_1 + 2x_2 = 10$, it moves inward but does not change the optimal variable mix. B remains optimal but moves to a new location, $x_1 = 0.5$, $x_2 = 3.5$, Z = \$27,500.

16.

$Z = 24000$

Variable	Value
x_1	4.000
x_2	0.000

Constraint	Slack/Surplus	Shadow Price
c_1	12.000	0.000
c_2	0.000	–3000.000
c_3	11.000	0.000

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x_1	0.000	6000.000	7000.000	1000.000	6000.000
x_2	6000.000	7000.000	No limit	No limit	1000.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
c_1	No limit	12.000	24.000	12.000	No limit
c_2	4.000	8.000	No limit	No limit	4.000
c_3	No limit	5.000	16.000	11.000	No limit

(a) There is surplus high-grade and low-grade aluminum so the shadow price is \$0 for both. The shadow price for medium-grade aluminum is \$3,000 indicating that for every ton that this constraint could be reduced, cost will decrease by \$3,000.

(b) $0 \leq c_1 \leq 7{,}000$ $\quad \infty \leq q_1 \leq 24$

$6{,}000 \leq c_2 \leq \infty$ $\quad 4 \leq q_2 \leq \infty$

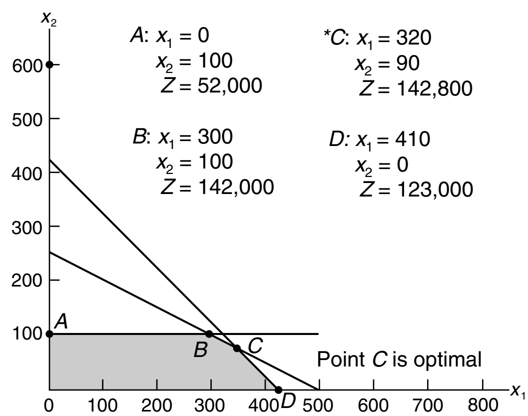
$\infty \leq q_3 \leq 16$

(c) There will be no change.

17. x_1 = no. of acres of corn
x_2 = no. of acres of tobacco
maximize $Z = 300x_1 + 520x_2$
subject to

$$x_1 + x_2 \leq 410$$
$$105x_1 + 210x_2 \leq 52{,}500$$
$$x_2 \leq 100$$
$$x_1, x_2 \geq 0$$

18.



(a) $x_1 = 320, x_2 = 90$

$320 + 90 + s_1 = 410$

$s_1 = 0$ acres uncultivated

$90 + s_3 = 100$

$s_3 = 10$ acres of tobacco allotment unused

(b) At point D only corn is planted. In order for point D to be optimal the slope of the objective function will have to be at least as great (i.e., steep) as the slope of the constraint line, $x_1 + x_2 = 410$, which is –1. Thus, the profit for corn is computed as,

$$-c/520 = -1$$
$$c_1 = 520$$

The profit for corn must be greater than \$520 for the Bradleys to plant only corn.

(c) If the constraint line changes from $x_1 + x_2 = 410$ to $x_1 + x_2 = 510$, it will move outward to a location which changes the solution to the point where $105x_1 + 210x_2 = 52{,}500$ intersects with the axis. This new point is $x_1 = 500$, $x_2 = 0$, $Z = \$150{,}000$.

(d) If the constraint line changes from $x_1 + x_2 = 410$ to $x_1 + x_2 = 360$, it moves inward to a location which changes the solution point to the intersection of $x_1 + x_2 = 360$ and $105x_1 + 210x_2 = 52{,}500$. At this point $x_1 = 260$, $x_2 = 100$ and $Z = \$130{,}000$.

19.

***** Input Data *****

Max. $Z = 300x_1 + 520x_2$

Subject to

$c_1 \quad 1x_1 + 1x_2 \leq 410$
$c_2 \quad 105x_1 + 210x_2 \leq 52500$
$c_2 \quad 1x_2 \leq 100$

***** Program Output *****

$Z = 142800.000$

Variable	Value
x_1	320.000
x_2	90.000

Constraint	Slack/Surplus	Shadow Price
c_1	0.000	80.000
c_2	0.000	2.095
c_3	10.000	0.000

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x_1	260.000	300.000	520.000	220.000	40.000
x_2	300.000	520.000	600.000	80.000	220.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
c_1	400.000	410.000	500.000	90.000	10.000
c_2	43050.000	52500.000	53550.000	1050.000	9450.000
c_3	90.000	100.000	No limit	No limit	10.000

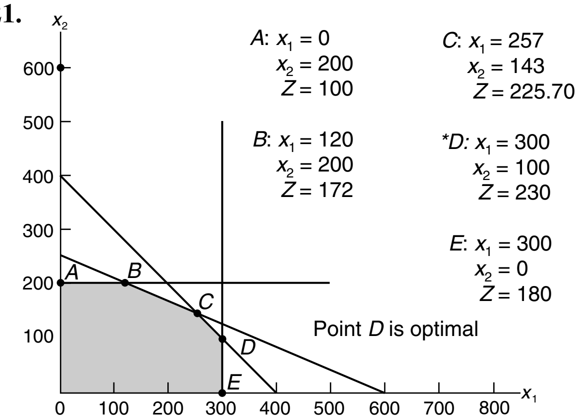
(a) No, the shadow price for land is \$80 per acre indicating that profit will increase by no more than \$80 for each additional acre obtained. The maximum price the Bradley's should pay is \$80 and the most they should obtain is at the upper limit of the sensitivity range for land. This limit is 500 acres, or 90 additional acres. Beyond 90 acres the shadow price would change.

(b) The shadow price for the budget is \$2.095. Thus, for every \$1 dollar borrowed they could expect a profit increase of \$2.095. If they borrowed \$1,000 it would not change the amount of corn and tobacco they plant since the sensitivity range has a maximum allowable increase of \$1,050.

20. x_1 = no. of sausage biscuits
x_2 = no. of ham biscuits
maximize $Z = .60x_1 + .50x_2$
subject to

$$\begin{aligned} .10x_1 &\leq 30 \\ .15x_2 &\leq 30 \\ .04x_1 + .04x_2 &\leq 16 \\ .01x_1 + .024x_2 &\leq 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

21.



(a) $x_1 = 300$, $x_2 = 100$, $Z = \$230$

$.10(300) + s_1 = 30$
$s_1 = 0$ left over sausage
$.15(100) + s_2 = 30$
$s_2 = 15$ lbs. left over ham
$.01(300) + .024(100) + s_4 = 6$
$s_4 = 0.6$ hr.

(b) The slope of the objective function, –6/5, must become flatter (i.e., less) than the slope of the constraint line, $.04x_1 + .04x_2 = 16$, for the solution to change. The profit for ham, c_2, that would change the solution point is,

$$-0.6/c_2 = -1$$
$$c_2 = .60$$

Thus, an increase in profit for ham of 0.60 will create a second optimal solution point at C where $x_1 = 257$, $x_2 = 143$ and $Z = \$225.70$. (Point D would also continue to be optimal, i.e., multiple optimal solutions.)

(c) A change in the constraint line from, $.04x_1 + .04x_2 = 16$ to $.04x_1 + .04x_2 = 18$ would move the line outward, eliminating both points C and D. The new solution point occurs at the intersection of $0.01x_1 + .024x_2 = 6$ and $.10x = 30$. This point is $x_1 = 300$, $x_2 = 125$, and $Z = \$242.50$.

22.

***** Input Data *****

Max. $Z = .6x_1 + .5x_2$

Subject to

c_1 $.1x_1 \leq 30$
c_2 $.15x_2 \leq 30$
c_3 $.04x_1 + .04x_2 \leq 16$
c_4 $.01x_1 + .024x_2 \leq 6$

***** Program Output *****

$Z = 230.000$

Variable	Value
x_1	300.000
x_2	100.000

Constraint	Slack/Surplus	Shadow Price
c_1	0.000	1.000
c_2	15.000	0.000
c_3	0.000	12.500
c_4	0.600	0.000

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x_1	0.500	0.600	No limit	No limit	0.100
x_2	0.000	0.500	0.600	0.100	0.500

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
c_1	25.714	30.000	40.000	10.000	4.286
c_2	15.000	30.000	No limit	No limit	15.000
c_3	12.000	16.000	17.000	1.000	4.000
c_4	5.400	6.000	No limit	No limit	0.600

(a) The shadow price for sausage is $1. For every additional pound of sausage that can be obtained profit will increase by $1. The shadow price for flour is $12.50. For each additional pound of flour that can be obtained, profit will increase by this amount. There are extra ham and labor hours available, so their shadow prices are zero, indicating additional amounts of those resources would add nothing to profit.

(b) The constraint for flour, indicated by the high shadow price.

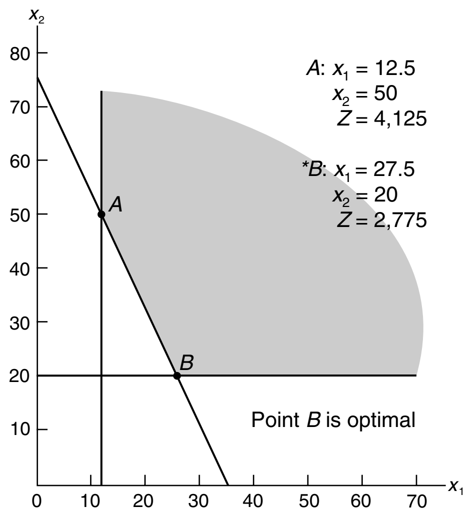
(c) $.50 \le c_1 \le \infty$
$25.714 \le q_1 \le 40$

The sensitivity range for profit indicates that the optimal mix of sausage and ham biscuits will remain optimal as long as profit does not fall below $0.50. The sensitivity range for sausage indicates the optimal solution mix will be maintained as long as the available sausage is between 25.714 and 40 lbs.

23. x_1 = no. of telephone interviewers
x_2 = no. of personal interviewers
minimize $Z = 50x_1 + 70x_2$
subject to

$$80x_1 + 40x_2 \ge 3{,}000$$
$$80x_1 \ge 1{,}000$$
$$40x_2 \ge 800$$
$$x_1, x_2 \ge 0$$

24.



(a) The optimal point is at B where $x_1 = 27.5$ and $x_2 = 20$. The slope of the objective function –50/70, must become greater (i.e., steeper) than the slope of the constraint line, $80x_1 + 40x_2 = 3{,}000$, for the solution point to change from B to A. The cost of a telephone interviewer that would change the solution point is,

$$-c_1/70 = -2$$
$$c_1 = 140$$

This is the upper limit of the sensitivity range for c_1. The lower limit is 0 since as the slope of the objective function becomes flatter, the solution point will not change from B until the objective function is parallel with the constraint line. Thus,

$$0 \le c_1 \le 140$$

Since the constraint line is vertical, it can increase as far as point B and decrease all the way to the x_2 axis before the solution mix will change. At point B,

$$80(27.5) = q_1$$
$$q_1 = 2{,}200$$

At the axis,

$$80(0) = q_1$$
$$q_1 = 0$$

Summarizing,

$$0 \le q_1 \le 2{,}200$$

(b) At the optimal point, B, $x_1 = 27.5$ and $x_2 = 20$.

$80(27.5) - s_2 = 1{,}000$
$s_2 = 1{,}200$ extra telephone interviews
$40(20) - s_3 = 800$
$s_3 = 0$

(c) A change in the constraint line from $40x_2 = 800$ to $40x_2 = 1{,}200$, moves the line up, but it does not change the optimal mix. The new solution values are $x_1 = 22.5$, $x_2 = 30$, $Z = \$3{,}225$.

25.

***** Input Data *****

Min. $Z = 50x_1 + 70x_2$

Subject to

c_1 $\quad 80x_1 + 40x_2 \ge 3000$
c_2 $\quad 80x_1 \ge 1000$
c_3 $\quad 40x_2 \ge 800$

***** Program Output *****

$Z = 2775.000$

Variable	Value
x_1	27.500
x_2	20.000

Constraint	Slack/Surplus	Shadow Price
c_1	0.000	–0.625
c_2	1200.000	0.000
c_3	0.000	–1.125

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x_1	0.000	50.000	140.000	90.000	50.000
x_2	25.000	70.000	No limit	No limit	45.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
c_1	1800.000	3000.000	No limit	No limit	1200.000
c_2	No limit	1000.000	2200.000	1200.000	No limit
c_3	0.000	800.000	2000.000	1200.000	800.000

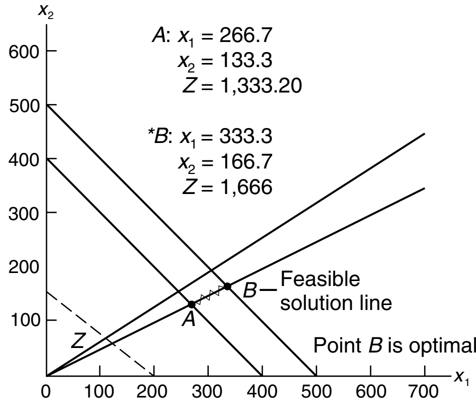
(a) Reduce the personal interview requirement; it will reduce cost by \$0.625 per interview, while a telephone interview will not reduce cost; i.e., it has a shadow price equal to \$0.

(b) $25 \leq c_2 \leq \infty$
$1{,}800 \leq q_1 \geq \infty$

26. x_1 = no. of gallons of rye
x_2 = no. of gallons of bourbon
maximize $Z = 3x_1 + 4x_2$
subject to

$$\begin{aligned} x_1 + x_2 &\geq 400 \\ x_1 &\geq .4(x_1 + x_2) \\ x_2 &\leq 250 \\ x_1 &= 2x_2 \\ x_1 + x_2 &\leq 500 \\ x_1, x_2 &\geq 0 \end{aligned}$$

27.



(a) Optimal solution at B: $x_1 = 333.3$ and $x_2 = 166.7$

$(333.3) + (166.7) - s_1 = 400$
$s_1 = 100$ extra gallons of blended whiskey produced

$.6(333.33) - .4(166.7) - s_2 = 0$
$s_2 = 133.3$ extra gallons of rye in the blend

$(166.7) + s_3 = 250$
$s_3 = 83.3$ fewer gallons of bourbon than the maximum

$(333.3) + (166.7) + s_4 = 500$
$s_4 = 100$ gallons of blend production capacity left over

(b) Because the "solution space" is not really an area, but a line instead, the objective function coefficients can change to any positive value and the solution point will remain the same, i.e., point B. Observing the graph of this model, no matter how flatter or steeper the objective function becomes, point B will remain optimal.

28.

***** Input Data *****

Max. $Z = 3x_1 + 4x_2$

Subject to

c_1 $\quad 1x_1 + 1x_2 \geq 400$
c_2 $\quad .6x_1 - .4x_2 \geq 0$
c_3 $\quad 1x_2 \leq 250$
c_4 $\quad 1x_1 - 2x_2 = 0$
c_5 $\quad 1x_1 + 1x_2 \leq 500$

***** Program Output *****

$Z = 1666.667$

Variable	Value
x_1	333.333
x_2	166.667

Constraint	Slack/Surplus	Shadow Price
c_1	100.000	0.000
c_2	133.333	0.000
c_3	83.333	0.000
c_5	0.000	3.333

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x_1	–2.000	3.000	No limit	No limit	5.000
x_2	–6.000	4.000	No limit	No limit	10.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
c_1	No limit	400.000	500.000	100.000	No limit
c_2	No limit	0.000	133.333	133.333	No limit
c_3	166.667	250.000	No limit	No limit	83.333
c_4	–250.000	0.000	500.000	500.000	250.000
c_5	400.000	500.000	750.000	250.000	100.000

(a) $-2.0 \leq c_1 \leq \infty$
$-6.0 \leq c_2 \leq \infty$

Because there is only one effective solution point the objective function can take on any negative (downward) slope and the solution point will not change. Only “negative” coefficients that result in a positive slope will move the solution to point A, however, this would be unrealistic.

(b) The shadow price for production capacity is \$3.33. Thus, for each gallon increase in capacity profit will increase by \$3.33.

(c) This new specification changes the constraint, $x_1 - 2x_2 = 0$, to $x_1 - 3x_2 = 0$. This change to a constraint coefficient cannot be evaluated with normal sensitivity analysis. Instead the model must be solved again on the computer, which results in the following solution output.

***** Input Data *****

Max. $Z = 3x_1 + 4x_2$

Subject to

c_1	$1x_1 + 1x_2 \geq 400$
c_2	$.6x_1 - .4x_2 \geq 0$
c_3	$1x_2 \leq 250$
c_4	$1x_1 - 3x_2 = 0$
c_5	$1x_1 + 1x_2 \leq 500$

***** Program Output *****

$Z = 1625.000$

Variable	Value
x_1	375.000
x_2	125.000

Constraint	Slack/Surplus	Shadow Price
c_1	100.000	0.000
c_2	175.000	0.000
c_3	125.000	0.000
c_5	0.000	3.250

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x_1	–1.333	3.000	No limit	No limit	4.333
x_2	–9.000	4.000	No limit	No limit	13.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
c_1	No limit	400.000	500.000	100.000	No limit
c_2	No limit	0.000	175.000	175.000	No limit
c_3	125.000	250.000	No limit	No limit	125.000
c_4	–500.000	0.000	500.000	500.000	500.000
c_5	400.000	500.000	1000.000	500.000	100.000