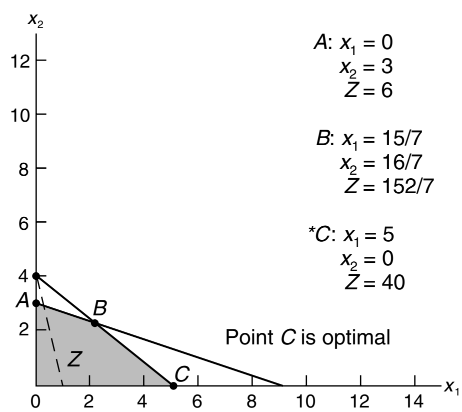
Chapter Two: Linear Programming: Model Formulation and Graphical Solution

PROBLEM SUMMARY

1. Maximization
2. Maximization
3. Minimization
4. Sensitivity analysis (2–3)
5. Minimization
6. Maximization
7. Slack analysis (2–6)
8. Sensitivity analysis (2–6)
9. Maximization, graphical solution
10. Slack analysis (2–9)
11. Maximization, graphical solution
12. Minimization, graphical solution
13. Maximization, graphical solution
14. Sensitivity analysis (2–13)
15. Sensitivity analysis (2–13)
16. Maximization, graphical solution
17. Sensitivity analysis (2–16)
18. Maximization, graphical solution
19. Standard form
20. Maximization, graphical solution
21. Standard form
22. Maximization, graphical solution
23. Constraint analysis (2–22)
24. Minimization, graphical solution
25. Sensitivity analysis (2–24)
26. Sensitivity analysis (2–24)
27. Sensitivity analysis (2–24)
28. Minimization, graphical solution
29. Minimization, graphical solution
30. Sensitivity analysis (2–29)
31. Maximization, graphical solution
32. Maximization, graphical solution
33. Minimization, graphical solution
34. Maximization, graphical solution
35. Sensitivity analysis (2–34)
36. Minimization, graphical solution
37. Maximization, graphical solution
38. Maximization, graphical solution
39. Sensitivity analysis (2–38)
40. Maximization, graphical solution
41. Sensitivity analysis (2–40)
42. Maximization, graphical solution
43. Sensitivity analysis (2–42)
44. Minimization, graphical solution
45. Sensitivity analysis (2–44)
46. Maximization, graphical solution
47. Sensitivity analysis (2–46)
48. Maximization, graphical solution
49. Sensitivity analysis (2–48)
50. Multiple optimal solutions
51. Infeasible problem
52. Unbounded problem

PROBLEM SOLUTIONS

1.



2.(a) maximize $Z = 6x_1 + 4x_2$ (profit, \$)
subject to

$$10x_1 + 10x_2 \le 100 \text{ (line 1, hr)}$$
$$7x_1 + 3x_2 \le 42 \text{ (line 2, hr)}$$
$$x_1, x_2 \ge 0$$

Wood

$$2x_1 + 6x_2 \le 36 \text{ lb}$$
$$2(6) + 6(3.2) \le 36$$
$$12 + 19.2 \le 36$$
$$31.2 \le 36$$
$$36 - 31.2 = 4.8$$

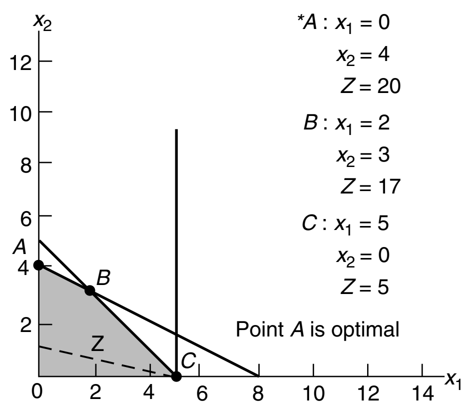
There is 4.8 lb of wood left unused.

8. The new objective function, $Z = 400x_1 + 500x_2$, is parallel to the constraint for labor, which results in multiple optimal solutions. Points B ($x_1 = 30/7$, $x_2 = 32/7$) and C ($x_1 = 6$, $x_2 = 3.2$) are the alternate optimal solutions, each with a profit of $4,000.

9.(a) maximize $Z = x_1 + 5x_2$ (profit, $)
subject to

$$5x_1 + 5x_2 \le 25 \text{ (flour, lb)}$$
$$2x_1 + 4x_2 \le 16 \text{ (sugar, lb)}$$
$$x_1 \le 5 \text{ (demand for cakes)}$$
$$x_1, x_2 \ge 0$$

(b)



10. In order to solve this problem, you must substitute the optimal solution into the resource constraints for flour and sugar and determine how much of each resource is left over.

Flour

$$5x_1 + 5x_2 \le 25 \text{ lb}$$
$$5(0) + 5(4) \le 25$$
$$20 \le 25$$
$$25 - 20 = 5$$

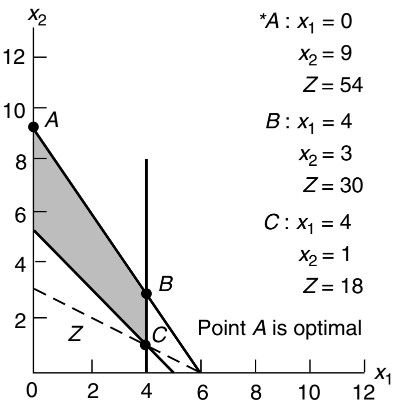
There are 5 lb of flour left unused.

Sugar

$$2x_1 + 4x_2 \le 16$$
$$2(0) + 4(4) \le 16$$
$$16 \le 16$$

There is no sugar left unused.

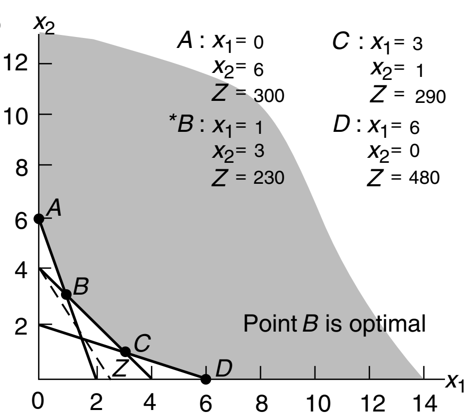
11.



12.(a) minimize $Z = 80x_1 + 50x_2$ (cost, $)
subject to

$$3x_1 + x_2 \ge 6 \text{ (antibiotic 1, units)}$$
$$x_1 + x_2 \ge 4 \text{ (antibiotic 2, units)}$$
$$2x_1 + 6x_2 \ge 12 \text{ (antibiotic 3, units)}$$
$$x_1, x_2 \ge 0$$

(b)



13.(a) maximize $Z = 300x_1 + 400x_2$ (profit, $)
subject to

$$3x_1 + 2x_2 \le 18 \text{ (gold, oz)}$$
$$2x_1 + 4x_2 \le 20 \text{ (platinum, oz)}$$
$$x_2 \le 4 \text{ (demand, bracelets)}$$
$$x_1, x_2 \ge 0$$

The extreme points to evaluate are now A, B', and C'.

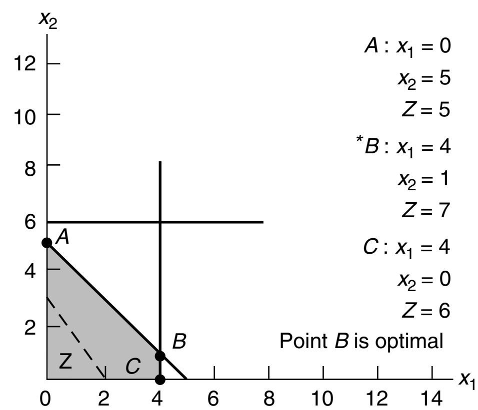
A: $x_1 = 0$
$x_2 = 30$
$Z = 1{,}200$

*B': $x_1 = 15.8$
$x_2 = 20.5$
$Z = 1{,}610$

C': $x_1 = 24$
$x_2 = 0$
$Z = 1{,}200$

Point B' is optimal

18.

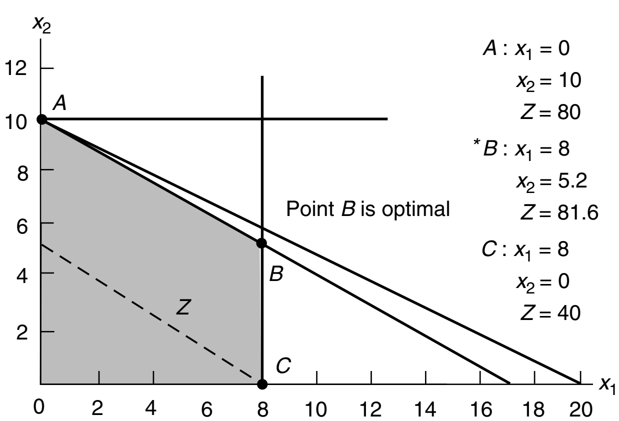


19. maximize $Z = 1.5x_1 + x_2 + 0s_1 + 0s_3$
subject to:

$$x_1 + s_2 = 4$$
$$x_2 + s_2 = 6$$
$$x_1 + x_2 + s_3 = 5$$
$$x_1, x_2 \geq 0$$

A: $s_1 = 4, s_2 = 1, s_3 = 0$
B: $s_1 = 0, s_2 = 5, s_3 = 0$
C: $s_1 = 0, s_2 = 6, s_3 = 1$

20.

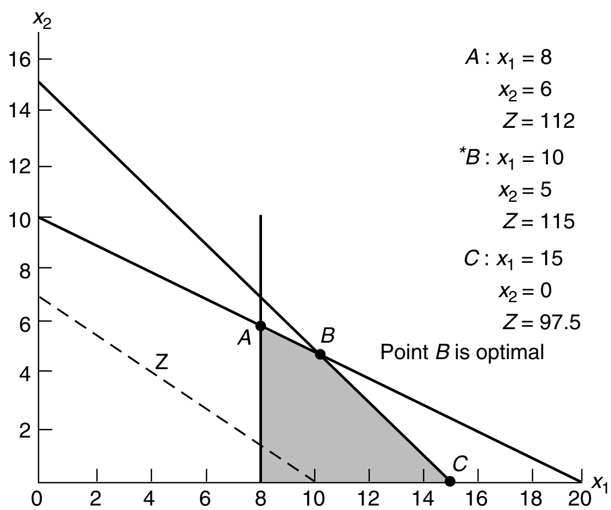


21. maximize $Z = 5x_1 + 8x_2 + 0s_1 + 0s_3 + 0s_4$
subject to:

$$3x_1 + 5x_2 + s_1 = 50$$
$$2x_1 + 4x_2 + s_2 = 40$$
$$x_1 + s_3 = 8$$
$$x_1, x_2 \geq 0$$

A: $s_1 = 0, s_2 = 0, s_3 = 8, s_4 = 0$
B: $s_1 = 0, s_2 = 3.2, s_3 = 0, s_4 = 4.8$
C: $s_1 = 26, s_2 = 24, s_3 = 0, s_4 = 10$

22.

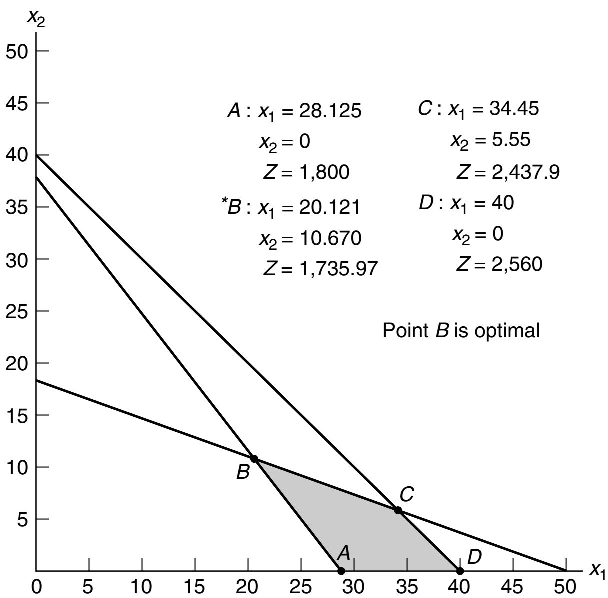


23. It changes the optimal solution to point A ($x_1 = 8$, $x_2 = 6$, $Z = 112$), and the constraint, $x_1 + x_2 \leq 15$, is no longer part of the solution space boundary.

24.(a) Minimize $Z = 64x_1 + 42x_2$ (labor cost, \$)
subject to

$$16x_1 + 12x_2 \geq 450 \text{ (claims)}$$
$$x_1 + x_2 \leq 40 \text{ (workstations)}$$
$$0.5x_1 + 1.4x_2 \leq 25 \text{ (defective claims)}$$
$$x_1, x_2 \geq 0$$

(b)

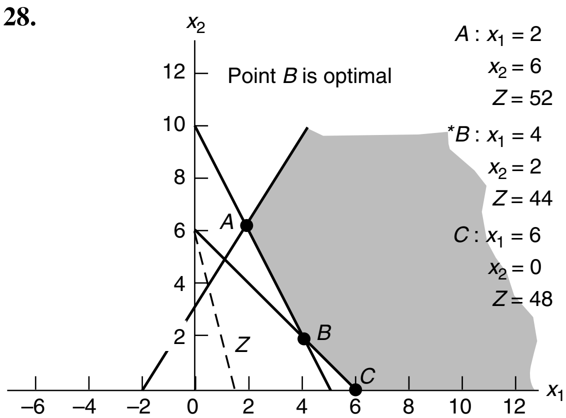


25. Changing the pay for a full-time claims processor from \$64 to \$54 will change the solution to point A in the graphical solution where $x_1 = 28.125$ and $x_2 = 0$, i.e., there will be no part-time operators. Changing the pay for a part-time operator from \$42 to \$36 has no effect on the number of full-time and part-time operators hired, although the total cost will be reduced to \$1,671.95

26. Eliminating the constraint for defective claims would result in a new solution, $x_1 = 0$ and $x_2 = 37.5$, where only part-time operators would be hired.

27. The solution becomes infeasible; there are not enough workstations to handle the increase in the volume of claims.

28.



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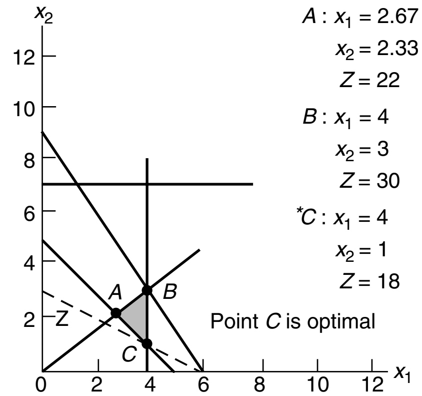
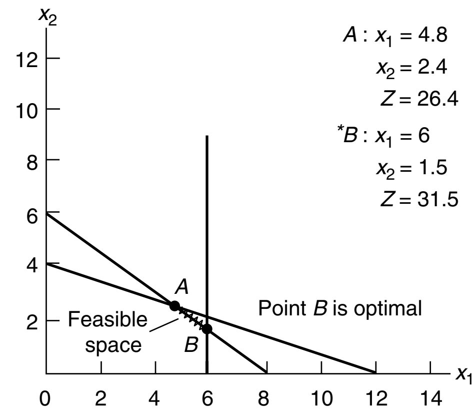


30. The problem becomes infeasible.

31.



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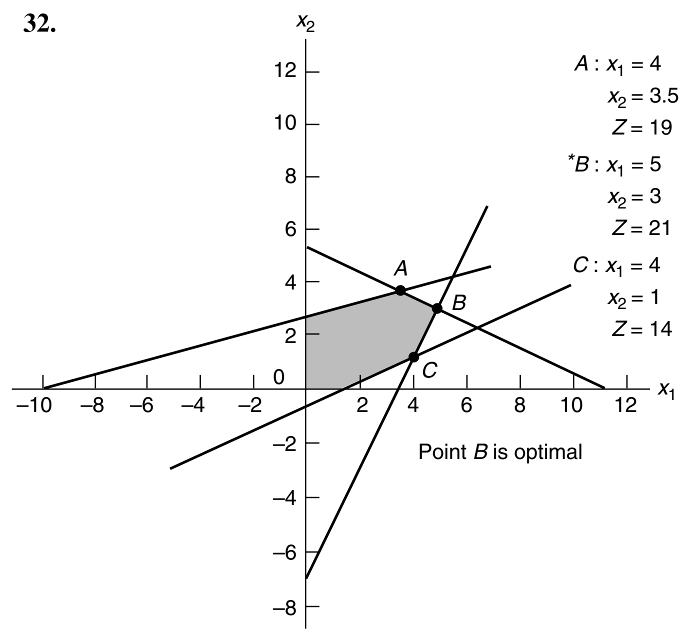
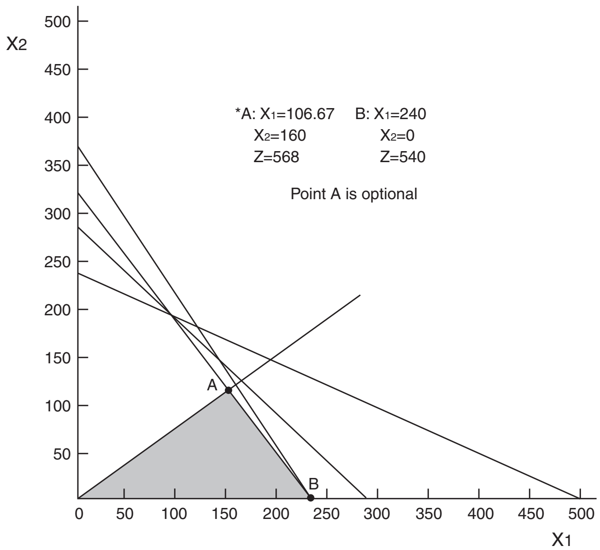


47. A new constraint is added to the model in

$$\frac{x_1}{x_2} \geq 1.5$$

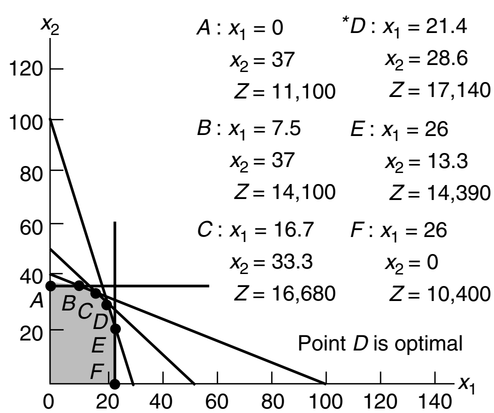
The solution is, $x_1 = 160$, $x_2 = 106.67$, $Z = \$568$



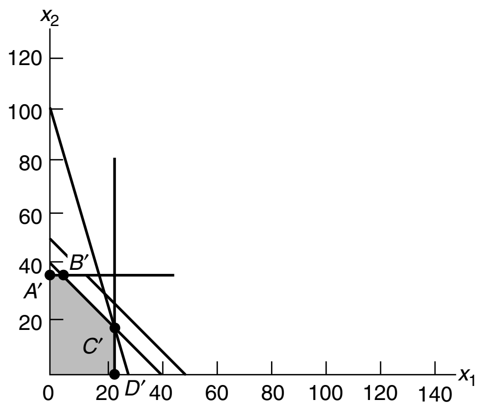
48.(a) maximize $Z = 400x_1 + 300x_2$ (profit, \$)
subject to

$$x_1 + x_2 \leq 50 \text{ (available land, acres)}$$
$$10x_1 + 3x_2 \leq 300 \text{ (labor, hr)}$$
$$8x_1 + 20x_2 \leq 800 \text{ (fertilizer, tons)}$$
$$x_1 \leq 26 \text{ (shipping space, acres)}$$
$$x_2 \leq 37 \text{ (shipping space, acres)}$$
$$x_1, x_2 \geq 0$$

(b)



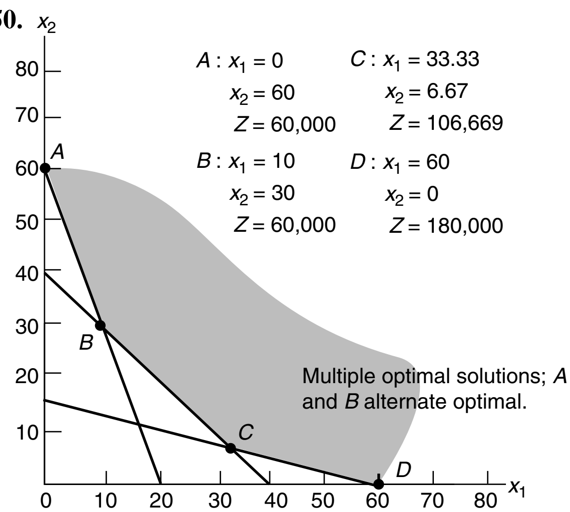
49. The feasible solution space changes if the fertilizer constraint changes to $20x_1 + 20x_2 \leq$ 800 tons. The new solution space is $A'B'C'D'$. Two of the constraints now have no effect.



The new optimal solution is point C':

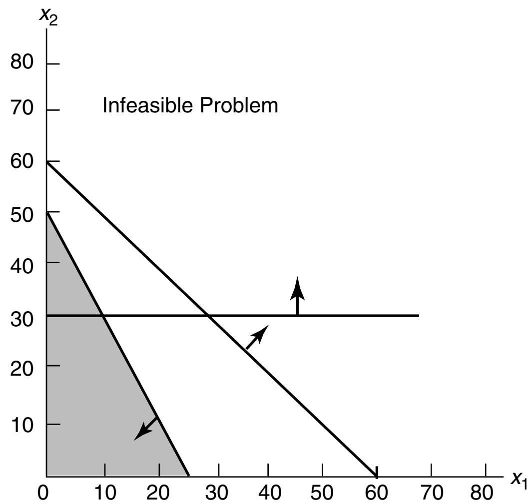
A':	$x_1 = 0$ $x_2 = 37$ $Z = 11{,}100$	$*C'$:	$x_1 = 26$ $x_2 = 14$ $Z = 14{,}600$
B':	$x_1 = 3$ $x_2 = 37$ $Z = 12{,}300$	D':	$x_1 = 26$ $x_2 = 0$ $Z = 10{,}400$

50.



Multiple optimal solutions; A and B alternate optimal

51.



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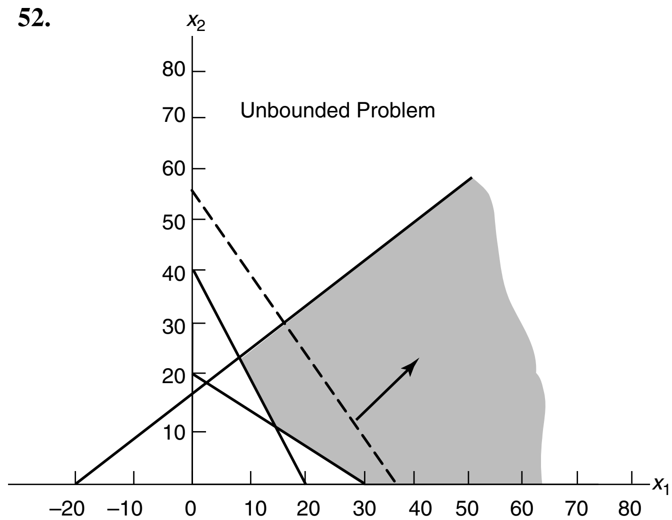


CASE SOLUTION: METROPOLITAN POLICE PATROL

The linear programming model for this case problem is

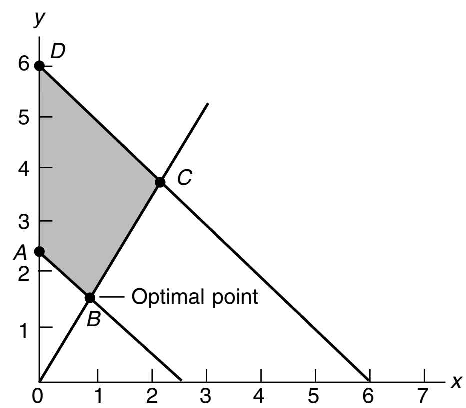
minimize $Z = x/60 + y/45$

subject to

$$2x + 2y \geq 5$$
$$2x + 2y \leq 12$$
$$y \geq 1.5x$$
$$x, y \geq 0$$

The objective function coefficients are determined by dividing the distance traveled, ie., $x/3$, by the travel speed, ie., 20 mph. Thus, the x coefficient is $x/3 \div 20$, or $x/60$. In the first two constraints, $2x + 2y$ represents the formula for the perimeter of a rectangle.

The graphical solution is displayed as follows.



The optimal solution is $x = 1$, $y = 1.5$, and $Z = 0.05$. This means that a patrol sector is 1.5 miles by 1 mile and the response time is 0.05 hr, or 3 min.

CASE SOLUTION: "THE POSSIBILITY" RESTAURANT

The linear programming model formulation is

Maximize = $Z = \$12x_1 + 16x_2$

subject to

$$x_1 + x_2 \leq 60$$
$$.25x_1 + .50x_2 \leq 20$$
$$x_1/x_2 \geq 3/2 \text{ or } 2x_1 - 3x_2 \geq 0$$
$$x_2/(x_1 + x_2) \leq .10 \text{ or } .90x_2 - .10x_1 \geq 0$$
$$x_1, x_2 \geq 0$$

The graphical solution is shown as follows.