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# Should corporate debt include a rating trigger?☆

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**Abstract**

Recent corporate debt offerings have included a covenant specifying a pre-determined payment to debtholders when the debt is downgraded. We examine the incentive for equityholders to increase firm risk (and the associated costs) when debt includes a "rating trigger." Equityholders of firms with a low-risk profile and operating flexibility choose debt with a trigger, while equityholders of firms with a high-risk profile and less flexibility choose regular debt. A trigger that requires an equity infusion better mitigates conflicts between equityholders and debtholders than a trigger paid by liquidating assets. A trigger that increases the coupon rate is not optimal.


*JEL classification:* G13; G32; G33

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## 1. Introduction

A "rating trigger clause" in a corporate bond indenture requires a firm to prepay part of its debt or to change the coupon rate on its debt if the firm's

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credit rating is downgraded to a specified level. Appendix A provides two examples of such a clause. Recent events have focused attention on the impact of a rating trigger on debt value and optimal capital structure. On May 15th 2002, Standard & Poors stated:[1]

> Tyco International Ltd., Vivendi Universal and twenty-two other companies have clauses in finance and trading agreements that would force them to pay back billions of dollars if their ratings fall.

Another prominent analyst is quoted in the report:

> S&P and Moody's Investors Service are under pressure to improve their scrutiny of borrowers after failing to spot the collapse of Enron last year. Enron's bankruptcy was fueled by the presence of triggers, many of which were undisclosed or buried in agreements.

A review of all listed bonds for S&P 500 companies reveals that there are 42 recent bond issues with rating triggers, offered by 17 companies (listed in Table 1). Data are obtained from the Bloomberg information services ''bond description'' page. The cash flow implications of these rating clauses are summarized on the Bloomberg information services ''bond description page'' for each bond issue. Many of the issuing firms are dominant in their industry. The purpose of this article is to analyze whether equityholders in such firms have an incentive to increase firm risk after debt is in place (asset substitution) and, as a result, cause a reduction in debt values. We ask the following questions, not previously addressed in the finance literature:

- Does a rating trigger reduce the equityholder–debtholder conflict due to asset substitution?
- For what kinds of firms does a rating trigger benefit both equityholders and debtholders?
- What type of a rating trigger clause–prepayment of debt or an increase in the coupon–best reduces the agency costs due to asset substitution and achieves a Pareto improvement in the value of the levered firm?
- When equityholders can prepay debt either by a cash infusion or by a partial sale of the firm's assets, how does the method of payment affect the agency costs due to asset substitution, the firm's value, and the choice of firm risk?

In order to analyze the impact of a rating trigger on the incentives for asset substitution, we first need a model of capital structure for the case of pure debt, and then we can investigate the impact of introducing a rating trigger in the debt contract. To achieve our objective, we use the framework developed in Leland (1994) and Leland and Toft (1996) in which the authors characterize the optimal capital structure and obtain closed-form solutions for debt values and equity values when

[1] See www.standardandpoors.com for the complete article.

Table 1
S&P500 firms with rating-based debt

This table provides a list of S&P500 firms that have issued rating-based debt. Included are metrics on company risk relative to the ten largest firms with the same SIC code. Data are collected from Bloomberg information services over the month of November 2003 and the Compustat database. Figures in parentheses are the average for companies with the same SIC code (except for EBITDA). Profit margin (gross profit divided by total revenue), debt ratios and EBITDA are the average for each quarter over the past five quarters. Rating numbers 2–8 (9–18) correspond to successive categories of A ratings (B ratings).

| Ticker | Beta | Profit margin (%) | Short - term debt to total assets (%) | Long - term debt to total assets (%) | Company credit rating (Now) | EBITDA (volatility of EBITDA) | Company credit rating (last issue date) |
|---|---|---|---|---|---|---|---|
| AOC | 0.68 | 15.78 | 2.27 | 9.46 | 9 | 428 | 7 |
| | | (24.78) | (2.25) | (13.06) | (10.67) | (56.55) | (9.75) |
| T | 0.83 | 50.85 | 7.75 | 21.33 | 11 | −2.826 | 8 |
| | | (40.65) | (9.49) | (27.24) | (8.1) | (390.11) | (8.1) |
| BCC | 1.11 | 20.23 | 4.45 | 32.42 | 13 | 120.90 | 10 |
| | | (28.69) | (4.79) | (17.68) | (14.33) | (36.22) | (10) |
| CD | 2.19 | 62.33 | 7.79 | 46.09 | 11 | 1544.80 | 8 |
| | | (41.18) | (3.43) | (24.83) | (13) | (111.20) | (11.5) |
| CMS | 1.41 | 17.68 | 5.42 | 37.78 | 14 | 74.4 | 14 |
| | | (23.36) | (8.65) | (33.7) | (9.67) | (188.86) | (9.34) |
| CMCS | 0.73 | 33.60 | 1.63 | 29.52 | 11 | 1187.50 | 12 |
| | | (39.69) | (1.20) | (44.62) | (14.11) | (201.08) | (14.375) |
| F | 1.24 | 9.88 | 21.99 | 51.96 | 11 | 2481.2 | 6 |
| | | (19.67) | (17.11) | (31.13) | (8.22) | (1307.56) | (8.25) |
| GE | 1.810 | 26.15 | 27.99 | 19.55 | 2 | 8679 | 2 |
| | | (26.63) | (10.12) | (19.60) | (8.25) | (1020.54) | (6.5) |
| HLT | 0.73 | 28.64 | 1.22 | 63.16 | 12 | 219.6 | 10 |
| | | (31.42) | (2.50) | (30.39) | (11.83) | (30.45) | (10.8) |
| IP | 0.83 | 25.64 | 3.30 | 35.71 | 11 | 789 | 9 |
| | | (22.23) | (6.13) | (41.45) | (18) | (447.23) | (11) |
| MAR | 1.13 | 8.58 | 1.72 | 22.82 | 10 | 43.80 | 9 |
| | | (31.42) | (2.50) | (30.39) | (11.83) | (30.45) | (8) |
| MHP | 0.58 | 58.07 | 55.74 | 4.79 | NA | 306.38 | NA |
| | | (51.37) | (49.31) | (13.20) | (11.33) | (104.30) | |
| ROH | 1.00 | 39.37 | 5.86 | 22.40 | 10 | 274.8 | 8 |
| | | (21.17) | (4.45) | (16.89) | (11.25) | (97.41) | (8) |
| SRE | 0.13 | 24.88 | 4.46 | 25.02 | 10 | 454 | 8 |
| | | (18.87) | (5.89) | (30.59) | (9.5) | (220.21) | (8.33) |
| SPLS | 1.02 | 26.94 | 1.93 | 12.01 | 16 | 289.97 | NA |
| | | (31.49) | (2.63) | (13.02) | (13.5) | (36.49) | |
| UTX | 1.07 | 31.45 | 2.99 | 13.72 | 8 | 1157.2 | 7 |
| | | (23.34) | (7.13) | (16.47) | (11) | (134.86) | (9.75) |
| WYE | 0.86 | 78.22 | 4.74 | 23.48 | 17 | 1124.82 | 8 |
| | | (74.92) | (8.39) | (12.02) | (3.89) | (76.41) | (8.1) |

default is determined endogenously.[2] Asset substitution, bankruptcy costs, and tax benefits have also been integrated in models such as Mello and Parsons (1992), Ross (1997), Leland (1998), Ericsson (2000), and Morellec (2001). We adapt the model of Leland (1994) and use elements of the approach in Ross (1997) and Ericsson (2000) to study the impact of a rating trigger on debt prices and agency costs. The innovation in our model is that it incorporates a rating-based trigger and at the same time allows for the possibility of tax effects, default costs, and asset substitution. A rating trigger payment is made when the value of firm assets falls below a certain level (rating trigger value). In addition, the firm defaults when the value of its assets falls below a certain prescribed level (bankruptcy triggering value). Fig. 1 illustrates two sample paths-one in bold where the firm's assets grow and remain above both the rating trigger value and the bankruptcy trigger value, and a second path (dashed) that the asset values decreasing and crossing the rating trigger value and then the bankruptcy trigger value (the firm defaults).

To capture the essential elements of asset substitution, we allow equityholders to alter the risk profile of the firm (instantaneous standard deviation of assets) after debt is sold. However, debtholders rationally anticipate the actions of equityholders. The resulting equilibrium provides interesting insights into the impact of a rating trigger on the incentives for asset substitution, as well as on optimal capital structure decisions. In particular, it is now possible to analyze how a rating trigger can align the interests of both equityholders and debtholders and increase the value of the firm. This analysis is the primary contribution of the article.

A rating trigger that forces early payment of debt by a cash injection from the equityholders reduces the per-period costs due to asset substitution. The early payment imposes an expost cost on equityholders. The equity infusion subsidizes the holders of the portion of the debt that is prepaid. In addition, by reducing leverage, a trigger lowers the wealth that equityholders can extract from debtholders. At the same time, it improves the financial condition of the firm and brings it closer to its optimal capital structure.

In general, equityholders of a firm with low-to-medium risk and with operating flexibility (characterized by the possible instantaneous standard deviations of the firm's asset value process) optimally decide to avoid high-risk strategies (and follow low-risk ones) in order to decrease the probability of reaching the trigger. Such policies are value enhancing for debtholders as well as equityholders. The existence of the refinancing trigger increases the value of the firm and increases both its debt capacity and optimal leverage. Such a firm will optimally choose debt with a rating trigger over pure coupon debt.

In contrast, a firm with a high-risk profile and similar operating flexibility has no incentive to use debt with a rating trigger. The equityholders are not able to reduce

[2]The quantitative approach to modeling a firm's assets and liabilities was pioneered by Black and Scholes (1973) and Merton (1974) and extended by Black and Cox (1976), Longstaff and Schwartz (1995), and others. This approach, commonly referred to as the "structural approach," is applied in Geske (1977), Smith and Warner (1979), Hull and White (1992), Abken (1993), Das and Tufano (1996), Leland (1994), Leland and Toft (1996), Bryis and DeVarenne (1997), Zhou (2001), Acharya and Carpenter (2002), and Brockman and Turtle (2003), among others. A second approach to modeling corporate debt is adopted by Fons (1994), Jarrow and Turnbull (1995), Jarrow et al. (1997), and Duffie and Singleton (1999), wherein the authors do not consider the relation between default and firm value in an explicit manner.

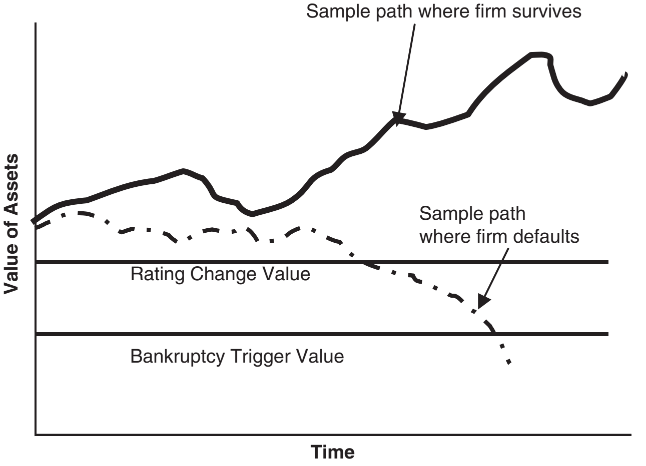



Fig. 1. Sample paths of possible asset values. One sample path (in bold) illustrates a firm whose value remains above the bankruptcy trigger level and rating change level. A second path (dashed line) illustrates the asset path of a firm that has a rating change and then defaults.

firm risk sufficiently that there is value enhancement for both equityholders and debtholders. Also, firms with asset values close to the rating trigger level–a measure of financial fragility-will choose not to issue bonds with a trigger. When the value of the firm's assets is very close to the value at which the trigger is set, debt is priced to incorporate the higher possibility of reaching the rating trigger. Therefore, the trigger will not result in a large incremental outflow from equityholders and is consequently not effective in reducing asset substitution. For asset values close to the trigger level, the potential gains from a capital structure change become small.

A rating trigger payment can be structured in different ways. In addition to prepayment of debt by cash infusion, we analyze two other variations of trigger payments: prepayment of debt by a partial liquidation of the firm's assets, and an increase in the coupon rate on the debt.

In general, a rating trigger that allows partial liquidation of the firm's assets to prepay the debt increases the equityholder-bond holder conflict due to asset substitution. This pushes the firm away from its optimal capital structure relative to a trigger paid by equity infusion. Although a portion of the debt is paid off if the trigger level is hit (and so there is no deadweight loss associated with this portion of the debt), the reduction in the firm's assets makes the portion of the debt that is not prepaid more risky. After the trigger payment is made, the debt-to-asset-value ratio of the firm increases and consequently the probability of bankruptcy increases. This in turn leads equityholders of the firm (now closer to bankruptcy) to pursue a more aggressive strategy at the expense of bondholders. By choosing a high-risk strategy, equityholders are more likely to prepay a part of the debt, but the trigger payment comes from a sale of assets that especially hurts the unpaid portion of debt.

The fact that a liquidation trigger performs differently than a refinancing trigger with a cash infusion from equityholders does not mean that it is ineffective in mitigating equityholders' incentives to follow high-risk strategies. A riskier policy has a negative impact on the tax shields because a portion of the debt is more likely to be prepaid. However, the benefit to equityholders from a riskier policy comes from the limited liability of the unprotected portion of the debt. This benefit decreases when the trigger protects a larger portion of the debt. For some proportion of prepayment of the outstanding debt, the loss in tax benefits from a riskier policy more than outweigh the agency costs that can be extracted from the unprotected portion of debt.

A coupon-based trigger neither inhibits asset substitution nor improves the financial condition of the firm. In this case, equityholders subsidize the debt by paying higher coupon payments when the trigger is reached. However, the possibility of an increase in the coupon is equivalent to an increase in debt when the firm's asset value is low, and that moves the firm farther away from the optimum leverage. The incentives for asset substitution thus increase. Further, an increase in the coupon increases the likelihood that the firm will go bankrupt. The increased bankruptcy costs more than offset the benefit to firm value from an increase in the associated tax shields. The analysis of coupon-based triggers applies equally to performance-related debt, a relatively new provision in bank debt contracts. Performance pricing makes the coupon rate charged on a bank loan a function of the borrower's financial performance as measured by debt-to-EBITDA, leverage, or interest coverage. The relation between the financial ratios and the coupon rate on debt is established at the contract's inception. Because firm value depends on these financial variables, the link between financial ratios and coupons is equivalent to the link between firm value and coupon rates that are analyzed in this article. We show that such performance-related debt contracts are inefficient.

Thus, different types of debt triggers produce very different results in moving the firm closer to the value-maximizing policy. This leads us to conclude that it is not just the existence of the debt trigger per se that matters, but the capital structure effects and the form of financing associated with a specific trigger.

A positive feature of a rating trigger clause is that it allows efficient pre-contracting, since the variables that cause a trigger payment are all observable. In addition, the decision to downgrade depends on the actions of an outside party, generally the opinion of a rating agency. Both the decision to downgrade and the associated trigger payment are not subject to expost manipulation by the parties involved. In contrast, other agency-reducing financial contracts, such as floating convertibles, analyzed in a recent article by Hillion and Vermaelen (2004), are subject to manipulation by market participants.

For the case of rating triggers that are financed by a cash infusion from equityholders, we show that the optimal trigger clause must consider the level at which the trigger is set and the amount associated with the trigger payment. For a firm with flexibility in choosing its risk level, the optimal trigger payment increases with an increase in the maximum risk that equityholders can take. In addition, this is accompanied by a corresponding decrease in the optimal trigger level (i.e., the value of firm assets at which it is optimal to pay the trigger). This is so because a higher trigger payment results in a lower amount of outstanding coupon payments, and consequently a lower optimal bankruptcy level. As a result, equityholders are able to lower the optimal rating trigger level to a point where the marginal benefit to declaring bankruptcy is the same as the benefit from the prepayment of debt.

Our article ties into a strand of literature on the effect of real flexibility on financial structure (see for example, MacKay, 2003; Morellec, 2004; Childs et al., 2004). This literature investigates whether production flexibility (process, product, or volume) affects the financial structure of a firm. Real flexibility promotes expost opportunism by broadening the menu of risky ventures available to those in control of the firm's assets. Such risk-shifting opportunities via real flexibility are also present in related work by Green and Talmor (1986) and Mello and Parsons (1992).

The article is organized as follows. Section 2 describes the model of asset price dynamics and capital structure. Closed-form values for debt and equity are derived when debt includes a rating trigger. Section 3 discusses the impact of a rating trigger when the asset value process has a fixed standard deviation. Section 4 examines the question of expost selection of asset risk and discusses the measurement of agency costs due to asset substitution. Section 5 considers the impact of a rating trigger on the costs from equityholder–debtholder conflicts and the optimality of a rating trigger under alternate conditions, and Section 6 concludes.

## 2. Model

Our objective is to evaluate the effects on the capital structure of the firm, and the incentive for asset substitution when equityholders sell debt with a rating trigger. To proceed with the analysis, we build on the framework provided by Leland (1994). First, we propose a model for the evolution of the assets of the unlevered firm. We add to it the tax benefits of debt and subtract the bankruptcy costs to give the value of the firm. The firm value net of debt value gives the equity value. Thus, we provide a formulation for firm value, debt value, and equity value. We first assume that there is no possibility of asset substitution. Our primary point of departure from Leland (1994) is in the formulation of debt value. According to practice, debt is issued at par initially, and repurchased at par when the asset value reaches a trigger level. The features of debt, in turn, affect firm value and equity value.

As in Leland (1994) and Leland and Toft (1996), we assume that the firm has productive assets whose unleveraged value $V(t)$ follows a continuous diffusion process with constant proportional volatility under the objective probability measure:

$$\frac{\mathrm{d}V(t)}{V(t)} = (\mu - \delta)\,\mathrm{d}t + \sigma\,\mathrm{d}z(t,) \tag{1}$$

where $\mu$ is the total expected rate of return on asset value $V(t)$, $\delta$ is the fraction of assets paid out each instant to equityholders, $\sigma$ is the instantaneous volatility of the firm's assets (assumed fixed for now), and d$z$ is the increment of a standard Brownian motion. The process continues without limit unless firm value falls to a default-triggering level $V_B$ (Fig. 1).

Let $\alpha$, where $0<\alpha<1$, be the fraction of asset value $V_B$ that debtholders receive in the event of bankruptcy, leaving equityholders with nothing. Assume that there exists a default-free asset that pays a constant interest rate $r$. The initial asset value is assumed to be $V(t=0)=V_0$. In addition, the value process of the firm's assets is spanned by the economy and there exists an equivalent risk-neutral measure under which the firm value process has a drift equal to the risk-free rate of interest.

Although we use a model based on asset values and balance sheet covenants, we could specify a similar model that relies on earnings (or cash flows) and income statements. The justification to model the dynamics of the claim to earnings as lognormal is presented in Goldstein et al. (2001). The value of the unlevered firm, corresponding to the term $V(t)$ in Eq. (1), is equal to the expected present value of the future stream of earnings. Now the dynamics of the value of the unlevered firm is linked to the dynamics of the earnings of the firm. The primary implication of such set up is that the mean ($\mu$) and volatility ($\sigma$) of firm value in Eq. (1) are directly related to the specification for the dynamics of the earnings of the firm. Other analysis set forth in this article carries through in such a setting.

We now formulate the functions for debt value, firm value, and equity value when debt contains a trigger payment. Recall that the equity value is the difference between firm value and debt value. The formulas reduce to the pure debt case when the rating trigger payment is set to zero.

### 2.1. Value of debt with a trigger

We assume that debt is sold only once at time zero with the following characteristics:

- Infinite maturity.
- A constant coupon flow $C$ at each instant of time.
- Debtholders receive a fraction $\Delta$ of the initial market value of debt ($0 \leqslant \Delta < 1$) from equityholders when the firm value reaches a rating trigger value $V_T$, where $V_T$ is greater than the default-triggering value $V_B$. Recall that debt is sold at par and repurchased at par. When the trigger is reached, the equity value must be greater than the repaid debt amount, i.e., $E(V_T) > \Delta D(V_0)$, where the equity and debt values are denoted by $E(V_T)$ and $D(V_0)$, respectively.[3] This is the prepayment of a portion of the outstanding debt corresponding to the description in Appendix A1.
- $V_T$ and $V_B$ are exogenous. Later, we discuss endogenous alternate possible bankruptcy trigger levels.

The cash flow requirements for both the coupon payments and the rating trigger payment are met by selling additional equity. In Section 5.6 we consider the case where equityholders pay the rating trigger by liquidating a fraction of the firm's

[3]In order for the equityholders to decide to make a rating trigger prepayment rather than declare bankruptcy when a rating trigger is reached, it must be the case that equityholders lose more if they declare bankruptcy right away. Thus, the equity value net of the prepayment of the trigger must be larger than the payoff to equityholders from declaring bankruptcy. Declaring bankruptcy results in a deadweight cost of $(1-\alpha)$ of the firm assets. Equity values are characterized in Section 2.3. Even though it is possible to include a trigger payment when $V_T \leqslant V_B$, we do not analyze this case because it is inconsistent with the extant use of a trigger in practice.

assets. Using risk-neutral valuation, the price of debt at time 0 is written as the sum of the expected present value of three components:

(a) Coupon flows if the firm value remains above $V_B$,
(b) the bankruptcy payouts if the firm value crosses $V_B$,
(c) the prepaid principal (trigger payment) when the firm value crosses $V_T$ and the coupon flows on the prepaid principal.

The initial value of the firm is assumed to be above the rating trigger level, i.e., $V_0 > V_T > V_B$. The debt value can be written as

$$D(V_0) = \int_0^\infty e^{-rt}(1-\Delta)C[1-G(t,V_0,V_B)]\,dt + \int_0^\infty e^{-rt}\alpha V_B g(t,V_0,V_B)\,dt + \int_0^\infty e^{-rt}\Delta C[1-G(t,V_0,V_T)]\,dt + \Delta D(V_0)\int_0^\infty e^{-rt} g(t,V_0,V_T)\,dt, \tag{2}$$

where $g(t, V_1, V_2)$ denotes the density of the first passage time $t$ from a level $V_1$ to firm value $V_2$; correspondingly $G(t, V_1, V_2)$ is the cumulative distribution function of the first passage density from $V_1$ to $V_2$. The first term in the first line in Eq. (2) is the market value of debt that is not prepaid (a portion $1-\Delta$ of the total). The second line represents the portion of the outstanding debt that is prepaid when the rating trigger is reached. In other words, the coupon payments are reduced by a factor of $\Delta$ once the rating trigger $V_T$ is crossed because a part of the debt is retired.

An evaluation of Eq. (2) gives

$$D(V_0) = \frac{(1-\Delta)C/r\left(1-[V_0/V_B]^{-X}\right) + \alpha V_B[V_0/V_B]^{-X} + \Delta C/r\left(1-[V_0/V_T]^{-X}\right)}{1-\Delta[V_0/V_T]^{-X}}, \tag{3}$$

where

$$X = a + b, \quad a = \frac{r-\delta-\sigma^2/2}{\sigma^2} \text{ and } b = \frac{\sqrt{(a\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}.$$

**Proof.** See Appendix B.

Here $\Delta = 0$ and $\delta = 0$ reduces the equation to the pure debt case in Leland (1994). □

## 2.2. Bankruptcy costs and the tax benefits of debt

Debt issuance affects the total value of the firm in two ways. First, it reduces firm value because of the costs associated with possible bankruptcy. Second, it increases firm value due to the tax deductibility of the interest payments.

$$BC(V_0) = \int_0^{\infty} e^{-rt}(1-\alpha)V_B g(t, V_0, V_B)\,dt = (1-\alpha)V_B\left[\frac{V_0}{V_B}\right]^{-X}, \tag{4}$$

$$\begin{aligned} TB(V_0) &= E\left[\int_0^{\infty} \tau e^{-rt} C(1-\Delta)[1 - G(t, V_0, V_B)]\,dt \right. \\ &\quad \left. + \int_0^{\infty} \tau e^{-rs}\Delta C[1 - G(t, V_0, V_T)]\,dt\right] \\ &= \frac{\tau C}{r} - \left[\frac{\tau(1-\Delta)C}{r}\right]\left[\frac{V_0}{V_B}\right]^{-X} - \left[\frac{\tau\Delta C}{r}\right]\left[\frac{V_0}{V_T}\right]^{-X}. \end{aligned} \tag{5}$$

Here $\tau$ is the tax rate. The tax benefits $TB(V_0)$ in (5) comprise three terms representing respectively, the maximum possible benefits in the face of no default, the loss in benefits when the firm goes into default and the loss in tax benefits when the firm prepays a proportion $\Delta$ of its debt. Implicit in (5) is full deductibility of interest payments, an assumption that increases the costs associated with the trigger clause. An increase in the rating trigger payment $\Delta$ results in a higher weight on the tax benefits term tied to the rating trigger. As a result, an increase in the rating trigger payment $\Delta$ can decrease the value of the firm because tax benefits associated with the trigger payment have a higher likelihood of being terminated than the coupon payments that terminate at bankruptcy. However, an increase in the rating trigger payment reduces the outstanding debt (consequently $V_B$) and therefore the probability of going bankrupt, as discussed in footnote 4.

### 2.3. Firm value and equity value

Firm value is given as the value of the unlevered firm plus tax benefits minus bankruptcy costs:

$$\begin{aligned} F(V_0) &= V_0 + \frac{\tau(1-\Delta)C}{r}\left[1 - \left(\frac{V_0}{V_B}\right)^{-X}\right] + \frac{\tau\Delta C}{r}\left[1 - \left(\frac{V_0}{V_T}\right)^{-X}\right] \\ &\quad - (1-\alpha)V_B\left[\frac{V_0}{V_B}\right]^{-X}, \end{aligned} \tag{6}$$

Again, it is important to note that firm value is altered by the presence of a rating trigger because: (1) tax benefits do not continue to accrue on the portion of debt that is repurchased; and (2) the prepayment affects the asset value $V_B$ at which it is optimal to declare bankruptcy.

Equity value is equal to firm value (Eq. (6)) minus the debt value (Eq. (3)):

$$E(V_0) = F(V_0) - D(V_0). \tag{7}$$

Using Eqs. (3) and (6), the equity value can be written as

$$E(V_0) = V_0 + \frac{C}{r}\left[\tau - \frac{1}{\Theta}\right] + \left[\frac{V_0}{V_B}\right]^{-X}\left[-\frac{\tau(1-\Delta)C}{r} - (1-\alpha)V_B - \frac{1}{\Theta}\left(\alpha V_B - \frac{(1-\Delta)C}{r}\right)\right] + \left[\frac{V_0}{V_T}\right]^{-X}\frac{\Delta C}{r}\left(-\tau + \frac{1}{\Theta}\right), \quad (8)$$

where

$$\Theta = 1 - \Delta\left[\frac{V_0}{V_T}\right]^{-X}.$$

All else remaining constant, equity value increases with firm risk in the absence of a trigger, i.e., $\partial \mathrm{E}(V_0; \Delta = 0)/\partial\sigma^2 > 0$, as shown in Leland (1994).[4]

In summary, in Eq. (1) we propose a model for the evolution of the assets of the unlevered firm. In Eq. (6) we add to it the tax benefits of debt and the bankruptcy costs to give the value of the firm . The firm value net of debt prices from Eq. (3) gives the equity value in Eqs. (7) and (8).

## 3. Impact of a rating trigger with no risk shifting

Given the model outlined in the previous section, it is first important to understand the impact of a rating trigger when there is no possibility of asset substitution (constant asset risk). Throughout the article, we use numerical examples to elicit the primary effects of a trigger. For now, consider that the rating trigger level, $V_T$, as well as the prepaid amount of the debt, $\Delta$, are set exogenously. In Section 5.4, we compute the optimal debt trigger pair ($V_T$,$\Delta$). The bankruptcy level $V_B$ is endogenously determined.

Consider a firm whose initial assets are worth $V_0 = 150$. Assume that the risk-free rate $r = 0.07$, the payout ratio $\delta = 0.01$, the tax rate $\tau = 0.35$, the recovery rate $\alpha = 0.4$, and $\sigma = 0.2$. If the firm were to sell straight coupon debt, the coupon amount that maximizes firm value is $C = 10.43$ (Leland (1994, p. 1229, Eq. (21)). At this coupon amount, firm value is equal to 191.84, debt value is 136.24 and equity value is equal to 55.60, from Eqs. (3), (6), and (8). If instead the equityholders were to sell trigger-based debt and wanted to raise the same fixed amount of cash from debtholders (136.24), they would sell less trigger-based debt than regular debt. In our example, suppose that equityholders sell a trigger-based bond with the rating trigger

[4]Bankruptcy occurs when the asset value falls to a level $V_B$. Different assumptions will lead to alternative bankruptcy-triggering asset values. For example, Ericsson (2000) assumes that financial distress occurs when operating cash flows and debt sales are not sufficient to meet the payments to current debtholders. Leland (1994) derives an endogenous bankruptcy trigger value. If the firm is not constrained by any other covenants, bankruptcy occurs only when the firm cannot meet its required payments by selling additional equity. If the debt prepayment does not send the firm into bankruptcy, then the first derivative of Eq. (8) with respect to firm value (after the trigger has been paid) will give $V_B^* = (1-\Delta)C/r/(1-\tau)X(X+1)$.

level at $V_T = 120$ and $\Delta = 0.2$.[5] Using Eq. (3), the coupon that results in the same market value of debt as the straight debt case (i.e., 136.24) equals 9.85. At this coupon, firm value is 191.09 and equity value is 54.85. In other words, a trigger decreases the value of the firm at a cost to equityholders. When the value of the firm's assets decreases, the value of the tax shields decreases and the expected bankruptcy costs increase.

Without a trigger, the tax benefits are 46.57 and the bankruptcy costs are 4.73. A trigger reduces the tax benefits to 42.58 and decreases the bankruptcy costs to 1.48, resulting in a net decrease in firm value. However, if the trigger payment $\Delta$ is high, the reduced tax benefits offset the bankruptcy cost benefits, resulting in a net loss in firm value. The magnitude of the increase in firm value is marginal.

**Remark 1.** In the absence of operational flexibility, firm value can decrease (increase) with a rating trigger if bankruptcy costs are low (high) and tax rates are high (low).

It is worth noting that our model has a scaling property that follows from the specific stochastic process chosen and the homogeneity properties of the model. If $C$ is the value-maximizing coupon when $V = V_0$, then $\rho C$ (where $\rho$ is some constant) is the value-maximizing coupon when $V = \rho V_0$ and there is no trigger. A prepayment of a proportion $\Delta$ of the outstanding debt results in rescaling the coupon to $(1-\Delta)C$ when firm value reaches the rating trigger value $V_T$. If the rating trigger value were set to a value of $V_T = (1-\Delta)V_0$, it would correspond to an optimal rescaling of the capital structure so that firm value is once again maximized.

Given this base case with constant asset risk, we now proceed to analyze agency costs when equityholders can alter firm risk (the possibility of asset substitution).

## 4. Expost selection of asset volatility

In the previous section, we incorporate a rating trigger but preclude asset substitution. Past studies on capital structure assume that firm risk, or the instantaneous standard deviation of the asset value process in Eq. (1), is exogenously fixed and remains constant through time. As noted by Leland (1998), to capture the essential element of agency it must be assumed that risk choices are made ex- post, and that the risk strategy followed by the firm cannot be pre-contracted in the debt covenants.

Research by Ross (1997), Leland (1998), and Ericsson (2000) has incorporated the possibility of asset substitution. Following these models, we assume that equityholders can costlessly choose between two investment policies. These policies are characterized by the standard deviations $\sigma = \sigma_L$ (low risk) or $\sigma_H$ (high risk) of the firm value process. Even though we assume two risk levels, the results do not

[5]Given these values for $V_0$ and $V_T$, the expected time of debt prepayment is $E[\tau_{V_T}|V_0] = 1/(\mu - (\sigma^2/2)) \ln(V_0/V_T)$ , which for the parameter values used is 5.5 years. We thank the referee for pointing this out to us.

change if equityholders are allowed to choose any risk between the low and high levels. It is assumed that when the firm is initiated, equityholders operate at a certain standard deviation, $\sigma_{\text{bef}}$. The equityholders opt for an alternative policy as soon as capital structure choices are made and debt is sold. The change in investment policy is assumed a one time and irreversible change.

If exante capital structure choice and debt prices are based on the current risk level, $\sigma_{\text{bef}}$, the coupon that maximizes firm value is denoted by $C(\sigma_{\text{bef}})$, and the exante maximized firm value is given by Eq. (6) and denoted $F(V_0; \sigma_{bef}, C(\sigma_{\text{bef}}))$. However, once these capital structure choices are made, equityholders change firm risk to $\sigma_{aft} = \sigma_L$ or $\sigma_H$, in an effort to maximize equity values. As a result, there could be a gain to equityholders at the expense of debtholders. The expost firm value $F(V_0; \sigma_{\text{aft}}, C(\sigma_{\text{bef}}))$ is possibly lower than the exante firm value $F(V_0; \sigma_{\text{bef}}, C(\sigma_{\text{bef}}))$.

In a rational expectations and full information scenario, it must be the case that debtholders correctly anticipate the actions of equityholders when coupon levels and debt prices are set. Our analysis presumes that debtholders take into account the sequential actions by equityholders: the setting of debt amounts or capital structure and the subsequent alteration of firm risk after debt is sold. Agency costs refer to the loss of overall firm value because equityholders take decisions to maximize equity values rather than firm value. In order to compute agency cost impacts, we consider firm value, equity value, and debt value in two scenarios: (1) pure debt with rational expectations, and (2) debt with rating triggers and rational expectations.

## 5. The impact of triggers with shifting of asset risk

Given the setup described in the previous section, we now proceed to analyze the impact of a rating trigger on the costs created by the possibility that equityholders expost choose to shift the risk of the assets.

### 5.1. Trigger payment $\Delta = 0$

Suppose the firm has operational flexibility. Equityholders can choose firm risk $\sigma_{\text{aft}} = \sigma_L$ or $\sigma_H$ ($\sigma_H > \sigma_L$) after the initial sale of debt. For the case of pure debt ($\Delta = 0$), equityholders change firm risk expost to maximize the value of equity. Since $\partial \mathrm{E}(V_0; \Delta = 0)/\partial \sigma^2 > 0$ irrespective of the coupon, a risk choice of $\sigma_H$ is the value-maximizing strategy for equityholders. However, rational debtholders anticipate such behavior by equityholders and choose a coupon level $C(\sigma_H)$ giving a firm value $F(V_0; \sigma_H, C(\sigma_H))$.

**Remark 2.** With operational flexibility and pure debt ($\Delta = 0$), a risk level of $\sigma_H$ constitutes a rational expectations equilibrium.

Fig. 2 plots equity value and firm value as a function of firm risk (sigma) for a set of parameter values, when the initial coupon is set at a value that maximizes the value of the firm, using Eqs. (3), (6) and (8). Firm value is monotonically decreasing in sigma. For example, if the asset risk $\sigma_{\text{aft}}$ can equal 0.1 or 0.4, equityholders have

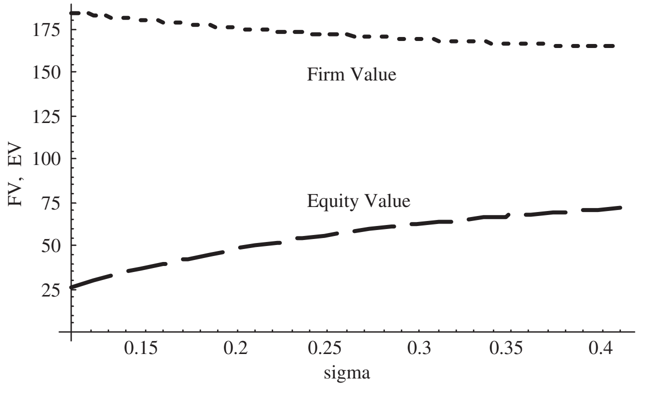
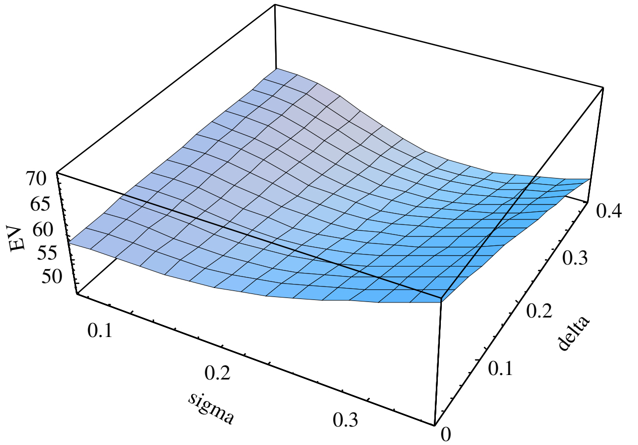



Fig. 2. The value of equity and the firm as a function of firm risk for the case of pure debt. This figure displays the equity value (long dashes) and firm value (short dashes) as a function of firm risk (sigma). We assume that the risk-free rate $r = 0.07$, the dividend payout $\delta = 0.01$, the tax rate $\tau = 0.35$, and the recovery rate $\alpha = 0.4$. The value of assets of the unlevered firm is set at $V_0 = 150$. The coupon is set at the level that maximizes firm value.

an incentive to increase risk level to 0.4 irrespective of the choice of coupon level. The rational expectations equilibrium therefore gives a firm risk of 0.4 and a firm value of 173. If firm risk could be pre-contracted to maximize firm value, shareholders would choose firm risk of 0.1 and achieve a firm value of 211. In this analysis, we assume debt of infinite maturity. The agency costs are lower if debt is of shorter maturity or if equityholders are concerned about their reputation and may need to borrow again.

### 5.2. Trigger payment $\Delta > 0$

Now consider the case where the trigger payment is positive ($\Delta > 0$). In order to define a rational expectations equilibrium, it is convenient to first analyze the expost behavior of equityholders, given a coupon level. Equityholders have an unambiguous incentive to increase firm risk to $\sigma_H$ if equity value increases with firm risk. However, in the presence of a rating trigger, the change in equity value with respect to firm risk is no longer unambiguously positive. Fig. 3 plots equity value as a function of firm risk and $\Delta$ for a set of parameter values. Note that equity value increases with firm risk (sigma) for the entire range of firm risk when $\Delta = 0$. However, when $\Delta > 0$, equity value decreases with risk for a range of risk levels. Such behavior is more apparent for larger values of $\Delta$. An examination of Eq. (8) shows that there are two opposing forces at work. An increase in firm risk increases the value of equity via a reduction in the value of debt, but increases the chance that equityholders will need to make a trigger payment. On the other hand, a decrease in asset risk can increase equity values because it reduces the chance that a trigger payment will be made and reduces the chance of going bankrupt. We see from Fig. 3 that given a rating trigger level $V_T$ and compensation $\Delta$, there is a level of firm risk ($\sigma^*$) so that equity values appreciate if risk increases or decreases beyond this level,




EV: Equity value
Sigma: Firm risk
Delta: Trigger payment

Fig. 3. The value of equity as a function of firm risk and the rating trigger payment. This figure graphs equity values (EV) as a function of firm risk (sigma) and trigger payment (delta). We assume that the risk-free rate $r = 0.07$, the dividend payout $\delta = 0.01$, the tax rate $\tau = 0.35$, the recovery rate $\alpha = 0.4$, the coupon payment $C = 9.68$, and the bankruptcy trigger $V_T = 120$. The value of assets of the unlevered firm is set at $V_0 = 150$.

i.e., $\partial \mathrm{E}(V_0)/\partial\sigma^2 > 0$ if $\sigma_{\mathrm{aft}} > \sigma^*$ and $\partial \mathrm{E}(V_0)/\partial\sigma^2 < 0$ if $\sigma_{\mathrm{aft}} < \sigma^*$. Appendix C characterizes this asset risk level $\sigma^*$. Therefore, depending on where the firm is positioned in terms of its risk choices $\sigma_L$ and $\sigma_H$, equityholders might prefer a low-risk strategy in the presence of a trigger. Equityholders prefer low risk to high risk when the expost costs of injecting new equity to subsidize the portion of the debt being repaid, in addition to the per-period agency benefit from the unpaid portion of the debt, outweigh the wealth transfers from pursuing a risky strategy at the expense of debtholders.

**Remark 3.** With operational flexibility and a rating trigger, a rational expectations equilibrium occurs at a firm risk of $\sigma_L$ or $\sigma_H$.

Let us consider a numerical example. Suppose a firm has initial assets $V_0 = 150$. Assume that the risk-free rate $r = 0.07$, the payout ratio $\delta = 0.01$, the tax rate $\tau = 0.35$, the recovery rate $\alpha = 0.4$, and the equityholders have the flexibility to choose firm risk $\sigma_{\mathrm{aft}} = 0.1$ or 0.25. If pure debt is sold, equityholders will subsequently choose firm risk equal to $\sigma_{\mathrm{aft}} = 0.25$. Debtholders rationally anticipate that equityholders will choose firm risk equal to 0.25. As a result, exante the debt is priced with the assumption of $\sigma_{\mathrm{aft}} = 0.25$ to give a coupon of 10.61, debt value of 127.86, equity value of 57.65, and a firm value of 185.51.

If the equityholders were instead to sell a trigger-based bond (assume that the rating trigger value is $V_T = 120$ and $\Delta = 0.2$), then the coupon that results in the same market value of debt (i.e., 127.86) equals 9.68. At this coupon rate, firm value is equal to 185.49, debt value is 127.86, and equity value is equal to 57.62. If equityholders subsequently decrease firm risk to 0.1, the firm value increases to 197.78 and the equity value increases to 59.42. Hence, it is optimal for equityholders to change firm risk to 0.1 rather than keep it at 0.25.

The above example illustrates that if a firm is flexible enough to decrease risk levels, it can allow for a range where firm risk decreases but both debt and equity values increase. In addition, if the total payoff to equityholders (for the same amount of cash raised though debt) is such that $E(V_0;\sigma_L,\Delta) > E(V_0;\sigma_H,\Delta)$, then it is optimal to include a rating trigger in the debt contract.

**Remark 4.** For the same face value of debt sold, a rating trigger is optimal for a firm if $\mathrm{E}(V_0;\sigma_L,\Delta) > \mathrm{E}(V_0;\sigma_H,\Delta = 0)$ and $E(V_0;\sigma_L,\Delta) > E(V_0;\sigma_H,\Delta)$.

Table 2 illustrates the benefits of a rating trigger using a numerical example. Consider a firm with operational flexibility characterized by $\sigma_L = 0.1$ and $\sigma_H = 0.25$. The upper panel of Table 1 gives the firm value, equity value, and debt value when the firm issues pure debt and the lower panel illustrates the corresponding values with rating-based debt. In each case, we compute the asset values at the equilibrium volatility level. For the case of pure debt (top panel), the equilibrium volatility is $\sigma_H = 0.25$. In contrast, we verify that the equilibrium outcome is $\sigma_L = 0.1$ in each instance in the lower panel (when $\Delta = 0.2$). In this table, we assume that the equityholders raise the same amount of cash from debt in each case. Thus, the required coupon flows are lower for the case of rating-based debt as opposed to the case of pure debt. The lower coupon associated with debt that includes a rating trigger implies lower tax benefits per period but a higher present value of tax benefits, since the trigger effectively reduces the expected costs of bankruptcy.

If the firm value is 180, for example, then the firm value-maximizing coupon is 12.73 and the debt value is 153.4 (last line of the top panel of Table 2). To raise the same amount of cash with a rating trigger ($\Delta = 0.2$), a coupon amount of 10.73 is

Table 2
Benefits of a trigger

This table gives the firm value (FV), equity value (EV), and debt value (DV) for the case of pure debt (top panel) and with a rating trigger payment of 0.2 (lower panel). We assume that the risk-free rate $r = 0.07$, the dividend payout $\delta = 0.01$, the tax rate $\tau = 0.35$, the recovery rate $\alpha = 0.4$, and the bankruptcy trigger $V_T = 120$.

| Trigger | Value of Assets | Coupon | FV | EV | DV |
|---|---|---|---|---|---|
| $\Delta = 0.0$ pure debt | | | | | |
| | 140 | 9.90 | 173.1 | 53.8 | 119.3 |
| | 160 | 11.31 | 197.9 | 61.5 | 136.4 |
| | 180 | 12.73 | 222.6 | 69.2 | 153.4 |
| $\Delta = 0.2$ | 140 | 8.35 | 179.8 | 60.5 | 119.3 |
| | 160 | 9.54 | 207.2 | 70.8 | 136.4 |
| | 180 | 10.73 | 233.5 | 80.1 | 153.4 |

required (last line of the lower panel). The value of equity for the case of rating-based debt is 80.1. In practice the tax benefits to leverage may be lower than those predicted by the model (around 20 percent). For example, Graham (2000) shows that these benefits average around ten percent. The magnitude of the benefits associated with a rating trigger should be considered with care because they represent a simulated value and cannot be dissociated from the particular model used, which tends to generate leverage ratios higher than observed in reality. It is worth noting, however, that equityholders register non-negligible gains in each case illustrated in Table 2. By reducing the deadweight costs of bankruptcy from operating the firm with less risk, the refinancing trigger increases the value of the firm, the firm's debt capacity, and the optimal leverage.

It is important to distinguish the roles of $C$ and $\Delta$ in the equityholders' incentives. $C$ is the instantaneous fee that the equityholders pay to keep managing the assets to their own benefit, capturing an agency flow per instant. While $C$ is a fixed cost per period, and therefore induces equityholders to take as much risk as possible, $\Delta$ is a deadweight cost whose likelihood of occurring depends on equityholders following risk-increasing strategies. The effectiveness of a trigger in reducing risky behavior on the part of equityholders follows from its threat to change the tradeoff equityholders face in paying the bondholders to hold on to company assets.

In the setting of Leland (1994), existing equityholders will not allow repurchase of existing debt using new equity because such repurchases result in a transfer of wealth from the current equityholders to new equityholders. In this article, existing shareholders are willing to repurchase debt if the firm asset value reaches the rating trigger. The contingent cost to equityholders is built into the computations when debt is sold at the outset and debt is purchased back at face value rather than market value, as in the case of Leland (1994). Then, there is no change in the assets of the firm or firm value and there is no transfer of wealth between existing and new equityholders at the trigger point. The reduction in debt value in our setting is exactly equal to the increase in the value of equity from the cash infusion.

In our model, equityholders are allowed to choose firm risk only once after debt is sold. If equityholders are instead allowed to switch volatility at will, they will wait longer before they switch to a low-risk strategy. This switch occurs if firm value goes down and causes the equityholders to behave more conservatively to avoid the trigger payment. With greater flexibility to switch firm risk, the agency costs are therefore higher because the equityholders can wait to switch to a low-risk strategy. In any case, equityholders behave more conservatively when the asset value nears the trigger.

The results we obtain are related to those of Purnanandam (2003), who shows that the equityholders' optimal choice of expost risk is determined by a tradeoff between the losses they bear from financial distress and the agency gains from the limited liability of the firm's equity. As he points out, accelerated debtrepayment and the requirement by lenders that equityholders maintain minimum levels of cash in the firm (the opportunity cost of a cash infusion) that we analyze in this paper represent important forms of losses from financial distress borne by equityholders. Our model, like his, predicts a positive relation between leverage and the choice of low-risk strategies for firms with flexibility and moderate levels of debt. When the firm has a large amount of debt in place, however, the equityholders still benefit from increasing firm risk. In addition, he finds a positive relation between leverage and risk management for firms with high deadweight losses, which in our model are represented by $(1-\alpha)$.

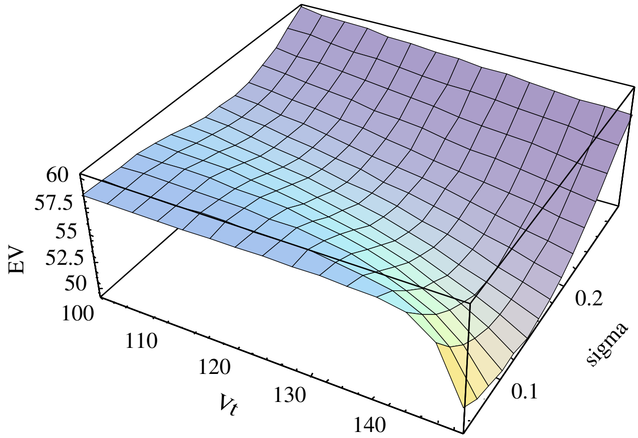
### 5.3. The effects of the trigger level on equityholders' choices.

To understand the impact of the trigger level, consider the last term in Eq. (8) for the value of equity. An increase in the trigger level $V_T$ increases the probability that the firm value will hit the rating trigger. Thus, there is a higher probability that additional equity will be required to subsidize the portion of debt being prepaid, and the tax benefits are lost on the proportion of debt that is prepaid (corresponding to the term $\Delta C/r$). These effects will reduce the value of equity. On the other hand, an increase in $V_T$ will increase the value of $1/\Theta$, the term that captures the extent to which debt prices incorporate the chance of reaching the trigger. This term can have a positive or negative impact on firm value.

Fig. 4 illustrates the effect of the trigger level ($V_T$) on equity value, given a coupon level. When the rating trigger is close to the unlevered firm value $V_0$, debt is priced to incorporate a higher possibility of reaching the rating trigger $V_T$, and a change in capital structure will not result in a large gain, since the firm's outlook has not changed much when $V_T$ is reached. Then, a trigger is not very effective in mitigating asset substitution because equityholders still benefit by increasing firm risk expost. In contrast, if the rating trigger $V_T$ is further away from the current asset value $V_0$, equityholders have a greater incentive to reduce asset risk. Fig. 4 shows that when the trigger level is 140 or higher (given that unlevered assets are at 150), equityholders have an incentive to increase asset risk to a larger extent than when the rating trigger is at 100.

The prepayment of a portion of debt results in a change in the capital structure. This prepayment can move the leverage of the firm closer to its optimal level, and therefore enhance the value of the firm. When the rating trigger is very close to the current value of assets, the magnitude of the potential change in capital structure due to the prepayment of debt is not large, and the consequent benefit from such a change is also not large. From this perspective, it is not beneficial to have a rating trigger close to the current value of assets.

Note that in practice it is possible to have more than one trigger payment that occurs at different values of firm assets. Suppose that instead of one trigger, there are two triggers at asset values of $V_{T1}$ and $V_{T2}$, where $V_{T1} > V_{T2}$. If equityholders choose a high-risk strategy, then the likelihood of paying at $V_{T1}$ goes up. From then on, the firm operates with less leverage, so the agency costs are lower and the capital structure of the firm is more balanced. With no second trigger at $V_{T2}$, equityholders will still try to extract wealth from the unpaid portion of the debt via a high-risk strategy. But, if there is a second trigger that requires equity injection to subsidize bondholders, the incentives for equityholders to pursue a high-risk strategy decrease. This is consistent with the finding of Leland (1998, p. 1,232) that if the fraction of




EV: Equity value
Sigma: Firm risk
Vt: Trigger level

Fig. 4. Equity value as a function of the rating trigger level and asset volatility. This figure shows equity values (EV) as a function of the rating trigger level ($V_T$) and asset volatility (sigma). We assume that the risk-free rate $r = 0.07$, the dividend payout $\delta = 0.01$, the tax rate $\tau = 0.35$, the recovery rate $\alpha = 0.4$, the coupon payment $C = 9.68$, and the trigger prepayment $\Delta = 0.2$. The value of assets of the unlevered firm is set at $V_0 = 150$.

retired debt increases (parameter $m$ in his paper), the point at which the equityholders switch asset risk also decreases (parameter $V_S$ in his paper).

### *5.4. Optimal triggers*

As noted in Remark 3, the equityholders choose a risk level of $\sigma_L$ or $\sigma_H$ when the firm issues debt with a rating trigger. The outcome depends in part on the rating trigger level ($V_T$) and the fraction of debt that is prepaid ($\Delta$). Therefore, in order to determine the optimal trigger, we need to analyze the joint effect of the trigger level $V_T$ and the trigger payment $\Delta$. Table 3 shows the results of simulations that compute the optimal delta and trigger level so that equity values are maximized and equityholders prefer a low-risk strategy.

We compute the optimal level of $V_T$ and $\Delta$ in the following manner. We first solve for the firm value-maximizing coupon and the corresponding debt level with no rating trigger. We then take $V_T$ and $\Delta$ as given, and compute the corresponding coupon (so that the amount of debt raised is equal to the base case without a rating trigger) and the equity value. We repeat the previous step for different combinations of $V_T$ and $\Delta$ to give the combination of $V_T$ and $\Delta$ such that equity value is maximized and equityholders prefer to include a rating trigger.

An examination of Table 3 shows that for a low-risk firm with less flexibility ($\sigma_L = 0.1$ and $\sigma_H = 0.15$), a small trigger payment $\Delta = 0.08$ is sufficient to make

Table 3
Optimal combinations of the trigger payment Δ and rating trigger level $V_T$

This table gives the optimal combinations of the trigger payment Δ and rating trigger level $V_T$ for different combinations of $\sigma_L$ and $\sigma_H$. We assume that the risk-free rate $r = 0.07$, the dividend payout $\delta = 0.01$, the tax rate $\tau = 0.35$, and the recovery rate $\alpha = 0.4$. The amount of debt raised is equal to the amount raised for the case of pure debt ($\Delta = 0$) in each case. The first entry in each cell is the trigger payment and the second entry is the trigger level.

| | $\sigma_H$ | | | |
|---|---|---|---|---|
| $\sigma_L$ | 0.15 | 0.25 | 0.35 | 0.45 |
| 0.1 | 0.08 | 0.17 | 0.29 | 0.0 |
| | 110 | 93 | 85 | |
| 0.2 | NA | 0.26 | 0.37 | 0.0 |
| | | 97 | 88 | |
| 0.3 | NA | NA | 0.0 | 0.0 |
| 0.4 | NA | NA | NA | 0.0 |

equityholders shift to a less risky strategy (see the first entry in Table 3). On the other hand, when flexibility increases while keeping constant the lower level of risk ($\sigma_L = 0.1$ and $\sigma_L = 0.25$), equityholders still benefit by reducing the risk profile of the firm, but the optimal trigger payment now increases to 0.17. A higher trigger payment is needed to counteract the increased benefit to equityholders from a high-risk strategy. However, when $\sigma_H = 0.45$, there is no possibility of offsetting the benefit of increased asset risk with a trigger payment.

From the equityholders' perspective, the rating trigger level ($V_T$) and the prepayment (Δ) define when the contingent cost is paid and the amount to be paid. Fig. 4 shows that, conditional on the coupon, equity value declines as the trigger level ($V_T$) increases. Thus, equityholders prefer a trigger far from the current value of assets. Also, Fig. 3 shows that equity value declines as Δ (delta) increases and equityholders would prefer the minimum possible prepayment (Δ). However, a smaller prepayment, Δ, and a lower trigger level, $V_T$, increase the coupon on the debt (for a fixed amount of debt), which, in turn, allows for an increased incentive for asset substitution. This explains why an increase in the minimum risk level, $\sigma_L$, increases the coupon and must be offset by an increase in Δ.

The results of the simulation help to characterize the firms for which using a debt rating trigger makes sense. For low-risk firms with low flexibility, i.e., lower values of ($\sigma_H$–$\sigma_L$), a small trigger payment is sufficient to cause equityholders to choose an equilibrium risk level of $\sigma_L$. As flexibility increases, a higher trigger payment is required. Finally, if the minimum risk level $\sigma_L$ increases, a trigger is no longer an optimal choice. The description of firms that use triggers, in Table 1, shows that many of these companies have a low-risk profile in terms of their betas and credit rating. Also, the debt ratios are lower in most cases than the industry average.

It is important to understand the difference between the optimal trigger covenant and an optimal dynamic refinancing policy freely chosen by the equityholders. The values shown in Table 3 are the optimal choices of the equityholders in terms of the risk management policy ($\sigma$), when to pay the trigger ($V_T$), and how much debt is to be retired with equity ($\Delta$). However, this choice is optimal only at the time of inception of the debt, when the firm's assets are worth $V_0$. It is not optimal for values of $V$ different from $V_0$. Therefore, when equityholders are free to choose an optimal dynamic refinancing policy, they will not choose the same $V_T$ and $\Delta$ afterwards, when $V$ differs from $V_0$.

### 5.5. How does the possibility of a cash-crunch affect the benefits of a trigger?

So far, we have assumed that a firm can raise additional equity to repay debt when the trigger level $V_T$ is reached, as long as equityholders find it optimal to do so. Suppose that equityholders are cash constrained and are not able in all instances to honor the trigger. If equityholders do not have the cash to repay a fraction $\Delta$ of the initial value of debt, the trigger pushes the firm into bankruptcy.

To analyze the effect of a cash crunch in a simple way, let $p$ be an exogenous and fixed probability that the equityholders are cash constrained when the rating trigger is reached and the firm is forced into bankruptcy. Then, the debt value (firm value) is given by 1-$p$ times the value of the debt (firm) when there is enough cash to repay the rating trigger payment, plus $p$ times the value of the debt (firm) if the firm goes bankrupt when the trigger is reached. Using Eqns. (2) and (6), this is written as (in Appendix D we provide the explicit formulae)

$$D(V_0,p) = (1-p) * D(V_0; \Delta > 0, V_T, V_B) + p * D(V_0; \Delta = 0, V_B = V_T) \tag{9}$$

$$F(V_0,p) = (1-p) * F(V_0; \Delta > 0, V_T, V_B) + p * F(V_0; \Delta = 0, V_B = V_T). \tag{10}$$

Note that equity value is the difference between firm value and debt value, i.e., $E(V_0,p) = F(V_0,p) - D(V_0,p)$. Eq. (9) assumes that the residual firm value distributed to the debtholders is less than the face value of outstanding debt.

Table 4 illustrates an example for a firm with a low-risk profile and flexibility characterized by $\sigma_L = 0.1$ and $\sigma_H = 0.25$. Column 1 gives the outcomes for the case of debt with no possibility of a cash crunch, while the subsequent two columns illustrate the outcomes when the probability of a cash crunch is 0.1 and 0.25, respectively. We tabulate the coupon level and the equity value based on the assumption that the same dollar value of debt is sold in each case.

An increase in $p$ results in a lower equity value, because a cash crunch increases the probability of incurring the deadweight costs of bankruptcy and reduces the probability of receiving a stream of tax benefits. Also, when the rating trigger level $V_T$ is moved away from the asset value $V_0$, the impact of a possible cash crunch is reduced. For example, when the rating trigger is at $V_T = 100$, the equity values are in the neighborhood of 66.8 in each case. However, when the trigger is at $V_T = 140$, the equity value is 64.7 for the firm where $p = 0.3$ in contrast to 65.8 for the firm where $p = 0.1$. If equityholders were to sell pure debt, the equity value is 57.62. Hedging is a valuable alternative in this case if a properly designed hedge moves cash from states in which the equity has enough liquidity to states in which the equity is financially constrained, see Mello and Parsons (2000).

Table 4
Equity values as a function of trigger level and trigger payment with the possibility of a cash crunch

This table gives the equity value (EV) for different combinations of the trigger payment $\Delta$ and rating trigger level $V_T$. We assume that the risk-free rate $r = 0.07$, the dividend payout $\delta = 0.01$, the tax rate $\tau = 0.35$, and the recovery rate $\alpha = 0.4$. The amount of debt raised is equal to the amount raised for the case of pure debt ($\Delta$) in each case. The probability of a cash crunch is denoted by $p$. The base case of pure debt corresponds to an equilibrium of $\sigma = 0.25$, a coupon of 9.68, debt value of 127.86, and equity value of 57.62.

| | $\Delta = 0.2, p = 0.0$ | | $\Delta = 0.2, p = 0.1$ | | $\Delta = 0.2, p = 0.3$ | |
|---|---|---|---|---|---|---|
| $V_T$ | C | EV ($\sigma$) | C | EV ($\sigma$) | C | EV ($\sigma$) |
| $\sigma_L = 0.1$ and $\sigma_H = 0.25$ | | | | | | |
| 100 | 8.95 | 66.8 | 8.95 | 66.8 | 8.96 | 66.8 |
| | | (0.1) | | (0.1) | | (0.1) |
| 120 | 8.95 | 66.8 | 8.96 | 66.7 | 8.97 | 66.6 |
| | | (0.1) | | (0.1) | | (0.1) |
| 140 | 9.74 | 55.5 | 8.98 | 65.8 | 9.06 | 64.7 |
| | | (0.25) | | (0.1) | | (0.1) |

**Remark 5.** Equityholders find it optimal to sell trigger-based debt when the probability of a cash crunch is low and the trigger level is not too close to the current asset level.

### 5.6. *Coupon-based triggers and alternative funding of triggers*

In our previous analysis, we claim that a trigger requiring debt repayment by equity infusion can be a Pareto-improving indenture. In this section, we analyze different triggers. First, we look at a trigger that increases the coupon level by a given percentage (denoted $\Delta$) when $V_T$ is crossed. Second, we investigate equityholders' choice to prepay the debt by liquidating the firm's assets instead of relying on an equity infusion.

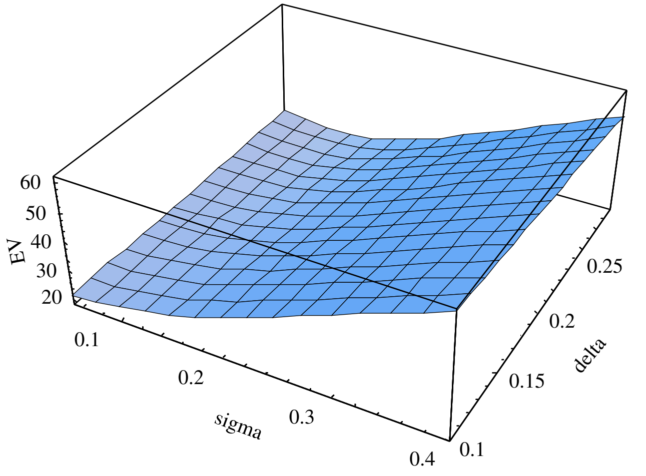
To answer the first point we reformulate the value of the different claims with the new coupon structure (Appendix E provides the explicit formulae). Note that debt values now incorporate two opposing effects: a higher coupon increases the value of the debt but also raises the bankruptcy trigger value $V_B$. Thus, debt values increase initially when $\Delta$ increases but decline as $\Delta$ moves beyond a certain point. In addition, raising the coupon gives a tax advantage relative to the trigger with debt repayment, but will force the firm into bankruptcy more often. In general, the latter effect dominates firm value decreases because of a coupon increase. The intuition for this result is simple when viewed from an optimal capital structure perspective. When the value of the assets declines, it is optimal for the firm to reduce its debt to avoid higher expected bankruptcy costs. Repaying the debt moves the firm closer to the value-maximizing financial policy. By contrast, an increase in the coupon amount when the firm's assets are low is tantamount to increasing the amount of debt, which moves the firm away from the value-maximizing financial policy.

A recent innovation in bank contracts, termed "performance related debt contracts," requires that the coupon charged on bank debt be explicitly linked to financial ratios. Because firm value is a function of underlying financial state variables (e.g., EBIT), an increase in coupon payments when financial performance declines is equivalent to an increase in coupon payments when firm value decreases. Hence, step-up coupon payments on such performance pricing contracts (and the reverse, lower coupon payments to reward credit rating upgrades) moves the firm away from its optimal capital structure, and are not efficient.

To highlight the incentive effects, we plot in Fig. 5 the residual equity values as a function of firm volatility ($\sigma$) and the increase in the coupon ($\Delta$), for the same parameter values as Fig. 3. Unlike the case of a trigger prepayment (refer to Fig. 3), Fig. 5 shows that for most of the domain equityholders increase firm risk.

**Remark 6.** In general, an increase in the coupon level decreases firm value and does not inhibit (and may even stimulate) asset substitution. In general, performance-related contracts are not efficient.

Turning to the equityholders' choice of financing to prepay the debt, we reformulate Eqs. (3)–(8) with the assumption that equityholders liquidate a fraction of the firm's assets to pay for the trigger (Appendix F). Now, the asset value falls from $V_T$ to a value of ($V_T$ minus the trigger payment) when the rating trigger payment is made. Because the liquidation affects the firm value process (jumps




EV: Equity value
Sigma: Firm risk
Delta: Trigger payment

Fig. 5. The value of equity as a function of firm risk when the coupon rate increases by "delta" percent at the trigger. This figure graphs equity value (EV) as a function of asset volatility (sigma) and coupon increase (delta). We assume that the risk-free rate $r = 0.07$, the dividend payout $\delta = 0.01$, the tax rate $\tau = 0.35$, the recovery rate $\alpha = 0.4$, the coupon payment $C = 9.68$, and the bankruptcy trigger $V_T = 120$. The value of assets of the unlevered firm is set at $V_0 = 150$.

down), the debt value cannot be isolated even though it can be solved numerically using the equations in Appendix F.

Consider first the case of trigger payments by a cash infusion for a firm with operational flexibility characterized by standard deviations $\sigma_L = 0.1$ and $\sigma_H = 0.25$. A coupon payment of $C = 9.68$ is required each period when the rating trigger level is set at $V_T = 120$, $\Delta = 0.2$, the risk-free rate $r = 0.07$, the tax rate $r = 0.35$, and the recovery rate $\alpha = 0.4$. In this case, the debt value is 127.86 and the equity value is 59.42. Here, a risk level of 0.1 is optimal for both equityholders and debtholders.

If equityholders were instead to sell rating-based debt with a liquidation trigger with the same face value of debt, a rating trigger would no longer be optimal because equityholders benefit by increasing firm risk to 0.25. With a liquidation trigger, if debt is priced with the assumption that firm risk is equal to 0.1, debt value equals 149.03 and equity value equals 52.82. If equityholders subsequently decrease firm risk to 0.25, the equity values increases to 53.92, at a cost to debt, which decreases to 127.86.

These numbers appear to indicate that a rating trigger paid by liquidating the firm's assets induces asset substitution. The fall in the firm's assets associated with the trigger makes the remaining debt riskier (the debt-to-value ratio of the firm increases and the probability of bankruptcy also increases). This motivates equityholders to follow a more aggressive strategy at the expense of debtholders. Even if a portion of the debt is fully prepaid, the cash comes from the sale of assets, which especially hurts the unpaid portion of the debt. This conclusion is summarized in the top part of Table 5.

However, a liquidation trigger can also make equityholders choose a low-risk strategy. If $\Delta = 0.5$, equityholders prefer low risk, i.e., $\sigma = 0.1$. The reason is that a high-risk policy makes the debt more likely to be prepaid, and the larger the $\Delta$, the more of the debt is prepaid, thus reducing the tax benefits to the equity. At some value of $\Delta$, the loss in tax shields from pursuing a riskier policy more than outweighs the wealth extracted from the unprotected portion $(1-\Delta)$ of the debt.

The primary implication is that the agency costs from expost risk shifting are more severe in the asset liquidation case than in the case of an equity injection. By increasing asset volatility, equityholders are able to affect debt prices in the first case by a larger extent However, even an asset liquidation-based trigger, which does not involve explicit subsidization of the protected portion of the debt by the equityholders, can lead equityholders to optimally choose a low risk strategy.

**Remark 7.** In general, a liquidation trigger has higher agency costs of debt from asset substitution compared to an equity infusion trigger. The higher the portion $\Delta$ of the debt prepaid in the trigger, the higher is the incentive for equityholders to follow a low-risk policy.

## 6. Summary and conclusions

A "rating trigger clause" in a corporate bond indenture requires a firm to prepay its debt or to increase the coupon rate on the debt if the firm's credit rating falls to a

Table 5
Liquidation Trigger
This table gives the values of the equity, debt, firm, bankruptcy costs, and tax shields, when a portion $\Delta = 0.2$ or 0.5 of the debt is prepaid by liquidating assets. Each of these values is determined at both inception and when the assets reach the trigger level $V_T$. We assume that the risk-free rate $r = 0.07$, the dividend payout $\delta = 0.01$, the tax rate $\tau = 0.35$, the recovery rate, $\alpha = 0.4$, $V_T = 120$, and the initial asset value $V_0 = 150$. Equityholders optimally manage the firm and can choose either $\sigma = 0.1$ or $\sigma = 0.25$.

| | When assets are at $V_0 = 150$ and $V_T = 120$ | | When assets reach the trigger level $V_T = 120$ | |
|---|---|---|---|---|
| | | | Before trigger is paid | After trigger is paid |
| $\Delta = 0.2$ | | | | |
| $\sigma$ | 0.1 | 0.25 | 0.25 | 0.25 |
| Equity | 52.28 | 53.92 | 27.52 | 24.45 |
| Debt | 149.03 | 127.86 | 112.51 | 90.01 |
| Firm | 201.39 | 181.78 | 140.03 | 114.45 |
| Bank. costs | 0.10 | 5.92 | 9.30 | 9.30 |
| Tax benefits | 51.46 | 37.69 | 29.33 | 29.33 |
| Coupon | 10.45 | 10.45 | 10.45 | 10.45 |
| $\Delta = 0.5$ | | | | |
| $\sigma$ | 0.1 | 0.25 | 0.1 | 0.1 |
| Equity | 53.77 | 47.97 | 2.82 | 8.96 |
| Debt | 145.28 | 127.86 | 140.14 | 70.08 |
| Firm | 199.05 | 175.83 | 142.97 | 79.04 |
| Bank. costs | 0.08 | 3.79 | 1.26 | 1.26 |
| Tax benefits | 49.14 | 29.63 | 24.23 | 24.23 |
| Coupon | 10.18 | 10.18 | 10.18 | 10.18 |

specified level. Recent corporate debt offerings by some large firms in the S&P 500 index include such a trigger clause. The purpose of this article is to analyze the implications of a rating trigger when equityholders can alter the risk profile of the firm after debt is sold.

Low-to-medium risk firms with operating flexibility find that a rating trigger enhances value for both debtholders and equityholders. In contrast, firms with a high-risk profile and with similar operating flexibility, as well as firms with asset values close to rating trigger levels, have no incentive to use debt with a rating trigger. The paper thus provides a convenient way to analyze and explain why a rating trigger might be appropriate for a given firm.

Rating triggers can be structured in different ways. We investigate the impact of the trigger design on the incentives of equityholders to pursue first best policies, as well as from the optimal capital structure of the firm. A trigger that requires that debt be paid by an infusion of equity is often effective. A trigger that allows partial liquidation of the firm's assets to prepay the debt is generally less effective, but also induces equityholders to follow low-risk strategies when the fraction of the debt prepaid is significant. A coupon-based trigger does not inhibit asset substitution and moves the firm away from the optimal capital structure. That different debt trigger types produce different results in moving the firm closer to the value-maximizing policy means that it is not just the existence of the debt trigger per se that matters, but also the capital structure effects and the form of financing associated with the trigger. The paper draws important implications for the design of performance-related debt contracts.

## Appendix A. Two examples of a rating trigger clause

### *A.1. First example*

In the case of any "Triggering Event" occurring on or prior to maturity, each holder of the notes will have the right, at the holder's option, subject to the terms and conditions of the Indenture, to require us (the company) to purchase all or any part (provided that the principal amount is $1,000 or an integral multiple thereof) in not less than 30 nor more than 60 business days after the occurrence of such a triggering event at a cash price equal to the principal amount thereof plus accrued interest.

"A Triggering Event" means the occurrence of a rating decline to (i) with respect to S&P, any of the following categories: BB, B, CCC, CC, C and D (or equivalent successor categories), (ii) with respect to Moody's, any of the following categories: Ba, B, Caa, Ca, C and D (or equivalent successor categories), and (iii) the equivalent of any such category of S&P or Moody's used by another rating agency.

### *A.2. Second example*

At any time on or after March 27, 2001, to and including March 15, 2006, the interest rate payable on each series of Exchange Notes will be subject to adjustment from time to time if either Moody's Investors Service, Inc. ("Moody's") or Standard & Poor's Ratings Services, a division of McGraw-Hill, Inc. ("S&P") downgrades the rating ascribed to the Exchange Notes as set forth below.

(a) If the rating from Moody's is decreased to a rating set out below, the interest rate will increase from the rate set forth on the cover page of this prospectus for each series of Exchange Notes by the percentage set opposite that rating:

| RATING | PERCENTAGE |
|---|---|
| Baa1 | .25% |
| Baa2 | .50% |
| Baa3 | .75% |
| Ba1 | 1.00% |

(b) If the rating from S&P is decreased to a rating set out below, the interest rate will increase from the rate set forth on the cover page of this prospectus for each series of Exchange Notes by the percentage set opposite that rating:

| RATING | PERCENTAGE |
|---|---|
| BBB + ......................................................... | .25% |
| BBB......................................................... | .50% |
| BBB−......................................................... | .75% |
| BB + ......................................................... | 1.00% |

Each adjustment required by any decrease in rating above, whether occasioned by the action of Moody's or S&P, shall be made independent of any and all other adjustments. If Moody's or S&P subsequently increases its ratings of the Exchange Notes to any of the thresholds set forth above, the interest rate on each series of Exchange Notes will be readjusted downwards to the percentage set forth opposite such ratings threshold above, provided that in no event shall (a) the interest rate for any series of Exchange Notes be reduced to below the interest rate set forth for such series on the cover page of this prospectus, and (b) the total increase in the interest rate on any series of Exchange Notes exceeds 2.00%.

## Appendix B. Debt values with a rating clause

From Eq. (2), debt values are given by

$$D(V_0) = \int_0^\infty e^{-rt}(1-\Delta)C[1-G(t,V_0,V_B)]\,dt + \int_0^\infty e^{-rt}\alpha V_B g(t,V_0,V_B)\,dt + \int_0^\infty e^{-rt}\Delta C[1-G(t,V_0,V_T)]\,dt + \Delta D(V_0)\int_0^\infty e^{-rt} g(t,V_0,V_T)dt. \tag{B.1}$$

Rearranging the equation gives

$$D(V_0) = \frac{\int_0^\infty e^{-rt}(1-\Delta)C[1-G(t,V_0,V_B)]\,dt + \int_0^\infty e^{-rt}\alpha V_B g(t,V_0,V_B)\,dt + \Delta C\int_0^\infty e^{-rt}[1-G(t,V_0,V_T)]\,dt}{1-\Delta\int_0^\infty e^{-rt} g(t,V_0,V_T)\,dt}. \tag{B.2}$$

From Leland and Toft (1996), $\int_0^\infty e^{-rt}(1-\Delta)C[1-G(t,V_0,V_B)]\,dt + \int_0^\infty e^{-rt}\alpha V_B g(t,V_0,V_B)\,dt$ is evaluated as

$$\frac{(1-\Delta)C}{r} + \left[\alpha V_B - \frac{(1-\Delta)C}{r}\right]\left[\frac{V_0}{V_B}\right]^{-X} \tag{B.3}$$

where $X = a + b$, $a = \frac{r-\delta-\sigma^2/2}{\sigma^2}$ and $b = \frac{\sqrt{(a\sigma^2)^2+2r\sigma^2}}{\sigma^2}$.

Similarly using the results in (B.3), the trigger payment is given by

$$\int_0^\infty e^{-rt}\Delta C[1-G(t,V_0,V_T)]\,dt = \frac{\Delta C}{r}\left(1-\left[\frac{V_0}{V_T}\right]^{-X}\right), \tag{B.4}$$

and the term in the numerator in Eq. (B.2) equals

$$\int_0^\infty e^{-rt} g(t, V_0, V_T)\,dt = \left[\frac{V_0}{V_T}\right]^{-X}. \tag{B.5}$$

Substituting Eq. (B.3), (B.4) and (B.5) in (B.2) gives the bond price.

## Appendix C. Equity Prices and firm risk (Remarks 3 and 4)

Using Eq. (7), $\partial E/\partial\sigma^2 = \partial F/\partial\sigma^2 - \partial D/\partial\sigma^2$. Rewrite debt value (Eq. (3)) as the sum of coupon payments and bankruptcy payouts: $D = CP + BP/\Theta$. Then, the slope of equity values with respect asset risk (keeping all other parameters fixed) is given by $\partial E/\partial\sigma^2 = \partial F/\partial\sigma^2 - (1/\Theta)\partial CP/\partial\sigma^2 - (1/\Theta)\partial BP/\partial\sigma^2 - (CP + BP)\partial(1/\Theta)/\partial\sigma^2$. Now $\partial F/\partial\sigma^2 < 0$, $\partial CP/\partial\sigma^2 < 0$, $\partial BP/\partial\sigma^2 > 0$, and $\partial(1/\Theta)/\partial\sigma^2 > 0$. The minimum is obtained at $\sigma = \sigma*$ so that $\partial F/\partial\sigma^2 - (1/\Theta)\partial BP/\partial\sigma^2 - (CP + BP)\partial(1/\Theta)/\partial\sigma^2 = 1/\Theta\partial CP/\partial\sigma^2$. This risk level can be computed numerically.

## Appendix D. Debt and firm values with the possibility of a cash crunch

Using Eqs. (1)–(5) we can rewrite the debt value and other components of firm value as

$$\begin{aligned} D(V_0,p) =&(1-p)\frac{(1-\Delta)C/r\left(1-[V_0/V_B]^{-X}\right) + \alpha V_B[V_0/V_B]^{-X} + \Delta C/r\left(1-[V_0/V_T]^{-X}\right)}{1-\Delta[V_0/V_T]^{-X}} \\ &+ p\left[C/r\left(1-[V_0/V_T]^{-X}\right) + \alpha V_B[V_0/V_T]^{-X}\right] \end{aligned} \tag{D.1}$$

$$BC(V_0,p) = (1-p)(1-\alpha)V_B[V_0/V_B]^{-X} + p(1-\alpha)V_T[V_0/V_T]^{-X} \tag{D.2}$$

$$\begin{aligned} TB(V_0,p) =& (1-p)\left(\frac{\tau C}{r} - \left[\frac{\tau(1-\Delta)C}{r}\right]\left[\frac{V_0}{V_B}\right]^{-X} - \left[\frac{\tau\Delta C}{r}\right]\left[\frac{V_0}{V_T}\right]^{-X}\right) \\ &+ p\left(\frac{\tau C}{r} - \left[\frac{\tau C}{r}\right]\left[\frac{V_0}{V_T}\right]^{-X}\right) \end{aligned} \tag{D.3}$$

Firm value is given as the sum of the unlevered firm value plus tax benefits minus bankruptcy costs: $FV(V_0,p) = V_0 + TB(V_0,p) - BC(V_0,p)$. Equity value is equal to firm value minus the debt value: $E(V_0,p) = F(V_0,p) - D(V_0,p)$.

## Appendix E. Debt and firm values with an increase in the coupon by Δ percent

Using Eqs. (1)–(5) we can rewrite the debt value and other components of firm value as (when the coupon is increased by Δ on reaching the rating trigger firm value):

$$D^*(V_0) = \frac{C}{r}\left(1 - \left[\frac{V_0}{V_T}\right]^{-X}\right) + \frac{(1+\Delta)C}{r}\left(1 - \left[\frac{V_T}{V_B}\right]^{-X}\right)\left[\frac{V_0}{V_T}\right]^{-X} + \alpha V_B \left[\frac{V_0}{V_B}\right]^{-X}, \tag{E.1}$$

$$BC^*(V_0) = (1-\alpha)V_B\left[\frac{V_0}{V_B}\right]^{-X}, \tag{E.2}$$

$$TB^*(V_0) = \frac{\tau C}{r}\left(1 - \left[\frac{V_0}{V_T}\right]^{-X}\right) + \frac{\tau(1+\Delta)C}{r}\left(1 - \left[\frac{V_T}{V_B}\right]^{-X}\right)\left[\frac{V_0}{V_T}\right]^{-X} \tag{E.3}$$

Firm value is given as the sum of the unlevered firm value plus tax benefits minus bankruptcy costs: $F^*(V_0) = V_0 + TB^*(V_0) - BC^*(V_0)$. Equity value is equal to the firm value minus the debt value: $E^*(V_0) = F^*(V_0) - D^*(V_0)$.

## Appendix F. Debt value and firm value with a liquidation trigger

Using Eqs. (1)–(5) we can rewrite the debt value and other components of firm value as:

$$D'(V_0) = \frac{(1-\Delta)C/r\left(1 - [V_0/V_B]^{-X}[(V_T - \Delta D(V_0))/V_T]^{-X}\right) + \alpha V_B[V_0/V_B]^{-X}[(V_T - \Delta D(V_0))/V_T]^{-X} + \Delta C/r\left(1 - [V_0/V_T]^{-X}\right)}{1 - \Delta[V_0/V_T]^{-X}} \tag{F.1}$$

$$BC'(V_0) = (1-\alpha)V_B\left[\frac{V_0}{V_B}\right]^{-X}\left[\frac{V_T - \Delta D(V_0)}{V_T}\right]^{-X}, \tag{F.2}$$

$$TB'(V_0) = \frac{\tau C}{r} - \left[\frac{\tau(1-\Delta)C}{r}\right]\left[\frac{V_0}{V_B}\right]^{-X}\left[\frac{V_T - \Delta D(V_0)}{V_T}\right]^{-X} - \left[\frac{\tau\Delta C}{r}\right]\left[\frac{V_0}{V_T}\right]^{-X}. \tag{F.3}$$

Firm value is given as the sum of the unlevered firm value plus tax benefits minus bankruptcy costs: $FV'(V_0) = V_0 + TB'(V_0) - BC'(V_0)$. Equity values are equal to the firm value minus the debt values: $E'(V_0) = F'(V_0) - D'(V_0)$.

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