



What causes mean reversion in corporate bond index spreads? The impact of survival

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Abstract

Previous studies document that the spread between the yield on commonly used corporate bond indexes (e.g., Moody's Baa index) and a comparable maturity treasury bond exhibits mean reversion. An analytical model shows that a part of the observed negative relationship between changes in the spread and the level of spreads is a natural consequence of ratings based classification of bonds included in the index and the related effects of survival. Using data on individual corporate bonds over the period January 1985 to December 1996, I corroborate the analysis and illustrate the effects of survival. The result has several implications for parametric specifications of spread dynamics in the pricing of contingent claims, for the application of spreads in tests of asset pricing models (such as the conditional version of the CAPM) and for the use of spreads in business cycle forecasts.

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1. Introduction

Commonly used indexes of corporate bond yields, such as those produced by Moody's, are constructed to track the yields on corporate bonds.¹ The difference between the yield on a given index of corporate bonds and a comparable maturity treasury bond (also referred to as the "index credit spread" or simply "credit spread") is an important input in tests of asset pricing models.² From a practical perspective, there is demand for a wide variety of financial products whose payoff is linked to the credit spread of a particular ratings class of bonds.³ The credit spread is also an important economic indicator of investor sentiment that is useful for forecasting future economic activity.⁴ Therefore an understanding of the behavior of credit spreads is of utmost importance for both academic and practical reasons. The objective of this article is to analyze the implications for spread behavior from the implicit conditioning induced by the survival of constituent bonds that are included in an index.

The finance literature has characterized the behavior of corporate bond yields, credit spreads and their relation to other asset prices.⁵ These studies reveal a tendency for spreads to revert towards a long-term mean suggestive of a long-term equilibrium. Fig. 1 is a graphical depiction of yield spreads of Moody's Baa index. The cyclical behavior and mean reversion point to the business cycle and related economic factors as an important driver of changes in index spreads. For example, Bevan and Garzarelli (2000) find a relationship between index spreads, growth in the gross domestic product, stock market volatility and other economic factors. However the overall explanatory power of economic factors at the aggregate level is modest.

Evidence at the individual bond level reveals that idiosyncratic demand and supply shocks and liquidity related effects are important factors that determine yield (and spread) changes at short horizons (e.g., Schultz, 1999; Chakravarty and Sarkar, 1999; Collin-Dufresne et al., 2001; Hotchkiss and Ronen, 2002). However liquidity related effects are unlikely to generate the mean-reverting behavior of index spreads. A look at data at the individual bond level also shows that *fluctuations in individual bond ratings* may often cause a bond to be added or excluded from an index portfolio. This observation suggests that it might be fruitful to consider the possible impli-

¹ The yield on a bond index is the average yield of constituent bonds in a particular rating class (e.g., Baa). The credit rating reflects the ability of a borrower to repay a debt.

² See for example the survey on an evaluation of asset pricing models by Ferson and Jagannathan (1996) and Harvey and Kirby (1996) and references therein.

³ Das (1998) provides a survey of the various kinds of financial products based on credit spreads.

⁴ See for example Stock and Watson (1994).

⁵ The determinants of bond yields are studied in the literature at the aggregate level (Chen et al., 1986; Keim and Stambaugh, 1986; Fama and French, 1989; Fama and French, 1993; Pedrosa and Roll, 1998; Bevan and Garzarelli, 2000), at the individual bond level (Sarig and Warga, 1989; Kwan, 1996; Duffee, 1998; Helwege and Turner, 1999; Neal et al., 2000; Elton et al., 2001; Collin-Dufresne et al., 2001) and at the portfolio level (Blume et al., 1991; Cornell and Green, 1991).

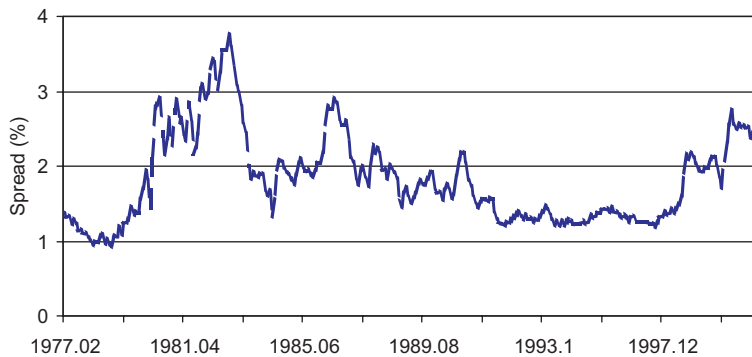


Fig. 1. Credit spreads. This figure graphs credit spreads for the period February 1977 to April 2001. The credit spread is computed as the difference between the yield on Moody's seasoned Aaa bond index (solid line) and the 20-year government bond (or Moody's seasoned Baa bond index (dashed line) and the 20-year government bond) at each point in time.

cations of this implicit ex-post conditioning, i.e., the survival of constituent bonds that are included in the index.

While earlier studies on the determinants of bond yields and associated credit spreads focus on economic factors and firm-specific factors to explain spread changes, the issue of survival and related ex-post analysis of index spread changes is yet to be addressed. In this article I derive the distributional properties of bonds that survive the conditioning based on ratings based classifications.⁶ As we would expect the spread behavior is biased by the inclusion or exclusion based on ratings changes in the underlying bonds. Researchers note the issue of survival in other contexts. Brown et al. (1995) discuss the implications of survival for equity prices. Similarly, Shiller (1989), Harvey (1995), Goetzmann and Jorion (1995) note the importance of accounting for survival in the context of determining risk and reward in the equity markets.⁷

Several factors however make this paper distinctive from extant studies. While previous studies have noted issues of survival, a precise framework that addresses the consequences of survival on credit spread behavior is yet to be put forth. In this paper, I first quantify the impact of survival on the dynamics of index spreads by characterizing the conditional distribution of spreads for surviving bonds. The analytical framework adds to the extant literature on survival by introducing two "ratings-based" barriers within which a bond must stay to be included in the index. I show via a simulation study that a large part of the observed negative correlation between spread changes and spread levels is a natural consequence of survival within these two barriers. The analytical solution is corroborated via an empirical study of corporate bond yield changes and the associated ratings changes.

⁶ Das (1998) provides tables of bond ratings changes for yearly and longer horizons. Also see default record of corporate bonds in documents at www.moodys.com.

⁷ In a recent article, Li and Xu (2002) provide an analytical framework for modeling survival.

The remainder of the paper is organized as follows. Section 2 of the article provides an overview of historical behavior of Baa index spreads, a convenient backdrop to compare my results. Section 3 outlines a theoretical model to show the consequences of survival on the behavior of index spreads. In Section 4, I simulate a bond index using 75 bonds and then use monthly data on Baa bonds from January 1985 to December 1996 to illustrate the impact of survival. Section 5 discusses some of the important implications of the results and concludes the article.

2. Background

I first give background information on the empirical properties of a commonly used bond index. This sets the stage for a discussion in subsequent sections. A credit spread on an index is computed as the difference between the average yield on bonds that comprise an index (e.g., Moody's seasoned Baa corporate bonds) and the yields on long-term constant maturity government bonds. The Baa Moody's index is colated from an equally weighted sample of yields on 75 to 100 bonds issued by large non-financial corporations. Each bond issue included in the index has an outstanding amount exceeding US \$100 million, an initial maturity of more than 20 years and a liquid secondary market. Bonds comprising an index are often "refreshed" in order to maintain constant credit quality. In other words, the yield change from one period to another does not measure the change in the same set of bonds but rather the change in the average yield on two sequential sets of bonds that share the same credit rating.

Table 1 provides summary statistics of a data set on Baa Moody's index yields. The table shows that Baa spreads provided a higher yield than government bonds (benchmark) and a correspondingly higher standard deviation. Baa index spreads

Table 1
Summary statistics

	Mean	Standard deviation	Skewness	Kurtosis	Number
Baa	10.41	2.60	0.92	−0.00	291
Govt.	8.59	2.29	0.75	−0.22	291
Spread = Baa − Govt.	1.82	0.60	0.91	0.29	291
Spread _{<i>t</i>} − Spread _{<i>t</i> − 1}	0.003	0.157	0.579	3.36	290
Spread _{<i>t</i>} − Spread _{<i>t</i> − 1} = α + β(Spread _{<i>t</i> − 1}) + error _{<i>t</i>}					
α	0.06				
β	−0.05				
(<i>T</i> -stat)	(−2.57)				
<i>R</i> -square	0.14				

This table contains summary statistics of data obtained from the Federal Reserve Board. The data comprises monthly observations of the yields (%) on a 20-year government bond and Moody's seasoned Baa bond index for the period February 1977 to April 2001. The table also reports estimates for the OLS regression on spread changes vs. spread levels: Spread_{*t*} − Spread_{*t* − 1} = α + β(Spread_{*t* − 1}) + error_{*t*} where Spread_{*t*} is the difference between the Baa index yield and the government bond.

have a mean of 1.82% and a standard deviation of 0.6%. The table also reports estimates for the OLS regression of spread changes vs. the spread level. Spread changes are negatively correlated to the level of yield spreads (italicised values of Table 1). This fact is also evident from a graphical depiction of the spreads in Fig. 1.

The next section investigates whether a part of this mean reversion is a consequence of rating based classification and the exclusion of bonds whose ratings may have changed over the past period. Theoretically the credit spread can be broken down into at least two separate components. The first component reflects the probability that default will occur and the associated capital loss. A second component is related to the risk premium that must be paid to bond holders as a compensation for the possibility of default. A mean reversion in spreads could be a result of changes in the market's assessment of aggregate default risk (some of which may be captured by economic factors) or shifts in attitudes towards risk. At an aggregate level these effects should be captured by contemporaneous or lagged values of economic variables. It is therefore puzzling why changes in a diversified index of bond yield spreads is not better explained by economic data.

3. Properties of ratings based portfolios

Virtually all bond portfolio allocation decisions are based on a segmentation of the bond markets into ratings based portfolios. Typically finance professionals suggest appropriate allocations in money market, investment grade and high yield bond funds. Allocations and related portfolio decisions, trading and statistical arbitrage rely on the notion of differential returns (spread) between different classes of bonds and the incremental risk of each of the market segments. An examination of historical data on spreads shows strong evidence in favor of mean reversion in credit spreads. What induces this mean reversion and could a part of this reversion be merely a result of this conditioning?

I first characterize the credit spread of a corporate bond as a stochastic variable. An increase (decrease) in the spread to a level that is above (below) a fixed level results in the security being excluded from the index.⁸ Fig. 2 illustrates two sample paths – one for a bond that is absorbed at the upper level and another one that survives. Suppose a researcher limits his or her analysis to a spread series that survives each period. In other words, e.g., if a bond is classified as Baa at the start of a sample period, it will continue to be classified as Baa towards the end of the sample period. The researcher observes spread changes over a period of time $(0, T)$. Assume that spreads are generated by a simple absolute diffusion

⁸ The spread is a proxy for the rating category and the related probability of default. In general these upper and lower levels are allowed to increase or decrease with time though I analyze the case where the barriers are fixed. In practice ratings changes may not be contemporaneous with the spread levels. However, this does not have an effect on the results except that it may change the timing of the exclusion from the index.

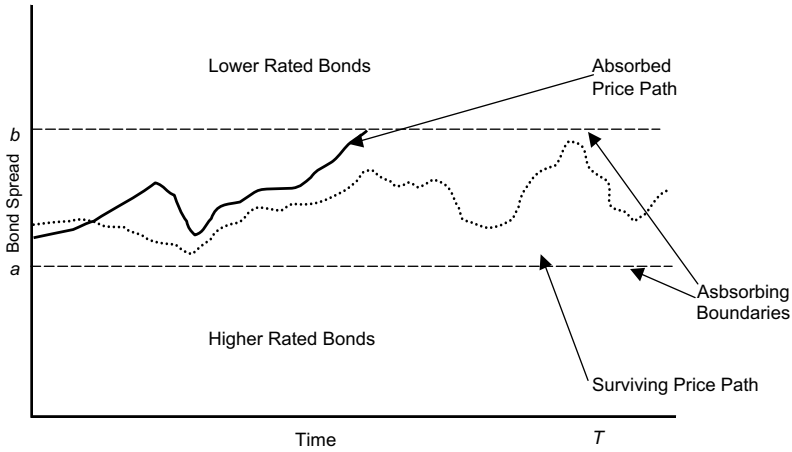


Fig. 2. Price path with two absorbing boundaries. This figure illustrates two sample paths. One sample path (in bold) illustrates a spread that starts with its level within the two boundaries but is absorbed at the upper boundary. The dashed line illustrates the spread path of a bond that does not cross the two boundaries in the given time period.

$$dx(t) = \mu dt + \sigma dz(t), \quad (1)$$

where μ and σ are parameters and z is a Brownian path. Usually we think of $x(t)$ as the log of spreads. Referring to Fig. 2, there is a reservation spread a below which a bond will be classified in a different category. Again, there is another reservation spread b above which a bond will be classified in a higher rating category. If a researcher studies spread paths that stay within the range (a, b) , what kind of return dynamics will the researcher see? Therefore the conditioning event of interest is

$$A = \{\text{Spread is greater than } a \text{ and lower than } b \text{ in the time interval } (0, T)\}.$$

More generally the set A defines the set of spread paths that survive where the ex-ante probability of survival at date T depends on the spread level at that date. The key to this analysis is to compute the conditional probability that a path belongs to the set A given that it started at a certain level within the range (a, b) at time 0. I first characterize this conditional probability and then relate the results to the time series properties of a set of sequential observations.

Let $x(T)$ be the credit spread at time T when credit spread changes are given in Eq. (1) with the initial condition $x(0) = x_0$. I wish to characterize the transition probability density $p(x(T); x(0) = x_0, A) = p(x, T)$ giving the probability density function (pdf) of $x(T)$ conditional on the initial credit spread $x(0) = x_0$ and the sample path belongs to the set A . It is well known that the transition density function for a time homogeneous diffusion satisfies the forward Kolmogorov equation:⁹

⁹ See Cox and Miller (1970, p. 213).

$$\frac{1}{2}\sigma^2 \frac{\partial^2 p}{\partial x^2} - \mu \frac{\partial p}{\partial x} = \frac{\partial p}{\partial t} \quad \text{where } a < x < b \quad (2)$$

and in my case is subject to the boundary conditions:

$$p(a; T) = 0 \quad \text{when } T > 0, \text{ and} \quad (3)$$

$$p(b; T) = 0 \quad \text{when } T > 0. \quad (4)$$

The boundary conditions (3) and (4) capture the re-classification of a bond into a higher or lower ratings category if its default risk exceeds a prescribed level. In other words, all the probability mass is concentrated on the set A . A solution to the partial differential equation with these boundary conditions can be obtained numerically. However for the simple case that I am studying, an analytic solution is available:¹⁰

$$p(x, T) = \frac{1}{\sigma\sqrt{2\pi T}} \sum_{n=-\infty}^{\infty} \left[\exp \left\{ \frac{\mu x'_n}{\sigma^2} - \frac{(x - x_0 - x'_n - \mu T)^2}{2\sigma^2 T} \right\} - \exp \left\{ \frac{\mu x''_n}{\sigma^2} - \frac{(x - x_0 - x''_n - \mu T)^2}{2\sigma^2 T} \right\} \right], \quad (5)$$

$$\text{where } x'_n = 2n(a + b) \quad \text{and} \quad x''_n = 2(a - x_0) - x'_n. \quad (6)$$

Armed with a transition density, I can now proceed to analyze the properties of index spread changes when the spread remains between the two barriers a and b .

Fig. 3 is a graphical depiction of the transition density for sample parameter values. Note that the conditional pdf has a value of zero outside the barrier values of $a = 1\%$ and $b = 3\%$ because once the credit spread increases or decreases enough to touch the barrier, the classification of the bond changes. The entire area under the graph represents the probability that a bond survives the barriers (conditional on its initial value):

$$\pi(x_0, T) = \Pr(A|x_0, T). \quad (7)$$

Fig. 3(a) shows that the pdf is centered on its initial value of 1.8%. Hence an analysis of the instantaneous drift of the sample path at this point is likely to reveal a nearly zero drift. On the other hand if the initial spread is near one of the barriers, a or b , this scenario changes. Fig. 3 provides two other plots of the pdf when the initial spread is near the lower boundary (Fig. 3(b)) and a second where the initial spread is near the upper boundary (Fig. 3(c)). When the initial value is near the lower boundary, the conditional pdf is skewed and the conditional mean is higher than the initial value. The opposite is observed when the initial value is near the upper boundary. Thus a security that starts at the upper or lower boundary is likely to show a reversion towards the mean if it survives the barriers. This conclusion is intuitively sound because if a bond did not have a rating change over a given time period, it is natural that the yield would be constrained to a certain band.

¹⁰ This problem is analyzed in Cox and Miller (1970) on p. 220.

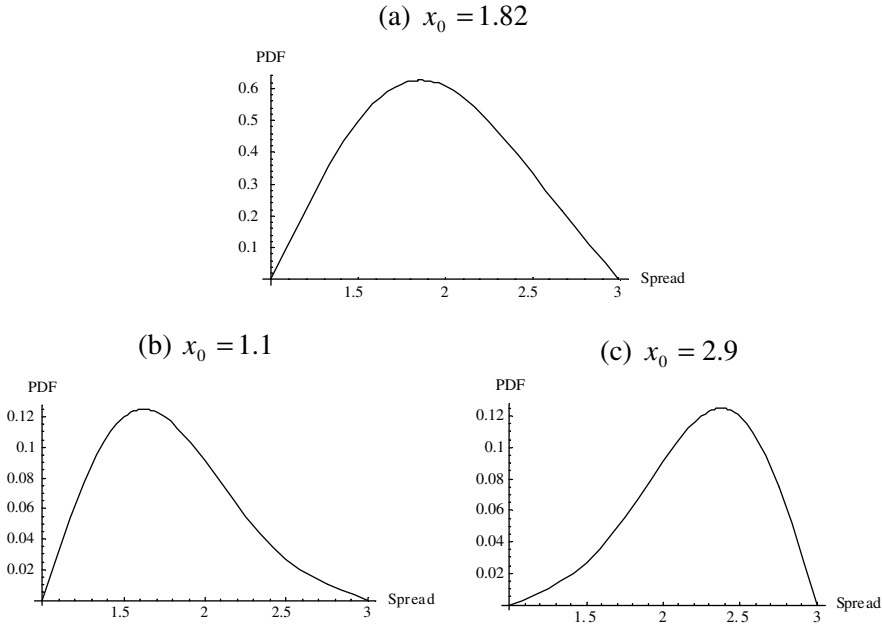


Fig. 3. Probability density function conditional on survival. This figure shows the probability density (conditional on survival) for a credit spread that follows a Brownian motion with absorbing barriers at $a = 1\%$ and $b = 3\%$ levels. The conditional density is given by:

$$p(x, t) = \frac{1}{\sigma\sqrt{2\pi t}} \sum_{n=-\infty}^{\infty} \left[\exp \left\{ \frac{\mu x'_n}{\sigma^2} - \frac{(x - x_0 - x'_n - \mu t)^2}{2\sigma^2 t} \right\} - \exp \left\{ \frac{\mu x''_n}{\sigma^2} - \frac{(x - x_0 - x''_n - \mu t)^2}{2\sigma^2 t} \right\} \right]$$

where $x'_n = 2n(a + b)$, $x''_n = 2(a - x_0) - x'_n$, $\sigma = 0.62$, $\mu = 1.82$, $t = 1$ and $x_0 = 1.82$.

Eq. (7)) changes. The conditional drift function for a surviving security can be computed as a sum of the true drift and a factor that accounts for the fact that the security has survived. An *ex-post* analysis of a sequence of bond spreads and spread changes is likely to be negatively related to spread levels and exhibit mean reversion even though none were present in the original data! Though the results are derived for a relatively simple case, the basic logic remains impervious to more complicated processes for yield spreads. Later, I investigate and quantify the extent of changes in mean reversion under a more complicated scenario that proxies the actual behavior of corporate bonds and index spreads.

4. Constructing an index – a simulation and empirical study

Analysis in the previous section suggests that one reason why index spreads are negatively related to the spread level is because the returns of constituent bonds *must* revert to the mean after they approach any of the barriers or they will be excluded

from the bond index. Several questions arise in this context. Is the mean reversion because of these inclusions and exclusions *economically significant*? Will I observe similar behavior in a scenario when default and yield curve changes are modeled in a more realistic fashion? To address these issues I run two simulation experiments – one where I study the behavior of a single bond and a second experiment where 75 bonds are modeled separately and then combined to yield a hypothetical index. I also conduct an empirical study wherein I examine cohorts of Baa bonds and their inclusion and exclusion from a hypothetical index. Both the simulation and the empirical study reveal that survival accentuates the extent of mean reversion in credit spreads.

4.1. Simulation study

The yield on a corporate bond depends on the risk-free interest rate (or the term structure), the probability of default of a bond, the market price of risk and the recovery rate in the event of default. To simulate the behavior of a corporate bond I need two primary ingredients (the market price of risk is assumed to be a function of the current spread level): (a) An assumption on the behavior of the term structure and (b) an assumption on the manner in which default occurs and the recovery rate.

4.1.1. Assumptions

My yield curve model follows a popular specification studied in the literature.¹¹ Denote the instantaneous default-free interest rate r_t . I assume that r_t equals the sum of a constant and two factors $s_{1,t}$ and $s_{2,t}$, that follow independent square root stochastic processes:

$$r_t = \alpha_r + s_{1,t} + s_{2,t}, \quad (8)$$

where the dynamics of $s_{1,t}$ and $s_{2,t}$ under the *true measure* are given by

$$ds_{i,t} = k_i(\theta_i - s_{i,t})dt + \sigma_i\sqrt{s_{i,t}}dZ_{i,t}, \quad i = 1, 2, \quad (9)$$

with parameter values noted in Table 2 and $Z_{1,t}$ and $Z_{2,t}$ are independent Brownian motions. With this specification and parameter values noted in Duffee (1999), closed form solutions are available for bond prices (see Duffee, 1999, for details).

The default intensity of the process for firm j at time t under the equivalent martingale measure is denoted by $h_{j,t}$. The key assumption is that the default intensity process is the sum of a translated square root process plus two components tied to the default-free interest rate factors.¹²

¹¹ Several studies are available on models of the term structure. See for example Chapter 7 of Duffee (1996).

¹² There are two basic approaches to modeling corporate default risks. One approach, pioneered by Black and Scholes (1973) and Merton (1973) and extended by Black and Cox (1976), Longstaff and Schwartz (1995) and others, explicitly models the evolution of firm value observable by investors. This approach is commonly referred to as the “structural approach”. A second approach to modeling risky debt is adopted by Fons (1994), Duffee and Singleton (1999), Jarrow and Turnbull (1995), Jarrow et al. (1997), Madan and Unal (1994) wherein the authors do not consider the relation between default and firm value in an explicit manner. This approach is called the reduced form approach.

Table 2

Simulation results for Baa spreads obtained using sample default process

	Mean	Standard deviation	Skewness	Kurtosis	Time periods
<i>Spread statistics</i>					
Spread = Baa – Govt.					
Rebalanced	1.97	0.34	0.88	0.57	240
Not rebalanced	2.17	0.48	0.86	0.29	240
<i>Spread changes</i>					
Spread _t – Spread _{t-1}					
Rebalanced	0.00	0.26	-11.3	156	240
Not rebalanced	0.00	0.27	-10.1	139	240
ρ_1					
Rebalanced	0.01				240
Not rebalanced	0.00				240
Cor(Spread _{t-1} , Spread _t – Spread _{t-1})					
Rebalanced	-0.17*				240
Not rebalanced	-0.11*				240

This table contains simulation results for credit spreads obtained by running 10,000 sample paths with 240 data points along each sample path. For each time point along a sample path, the instantaneous interest rate is generated by: $r_t = \alpha_r + s_{1,t} + s_{2,t}$ where the dynamics of $s_{1,t}$ and $s_{2,t}$ under the true measure are given by an Euler approximation of $ds_{i,t} = k_i(\theta_i - s_{i,t})dt + \sigma_i\sqrt{s_{i,t}}dZ_{i,t}$, $i = 1, 2$ with parameter values noted in Duffee (1999). Bond yields are subsequently obtained using a multi-factor version of the CIR model. I then generate the instantaneous default risk: $h_{j,t} = \alpha_j + h_{j,t}^* + \beta_{1,j}(s_{1,t} - \bar{s}_{1,t}) + \beta_{2,j}(s_{2,t} - \bar{s}_{2,t})$ where $h_{j,t}^*$ follows a square root process given by an Euler approximation of $dh_{j,t}^* = k_j(\theta_j - h_{j,t}^*)dt + \sigma_j\sqrt{h_{j,t}^*}dZ_{j,t}$ with parameter values noted in footnote 13. Defaultable bond yields are obtained using recovery adjusted par yields. This table reports summary measures of credit spreads and credit spread changes generated in the simulations. I assume a recovery rate of 0.44 in the event of default.

$$h_{j,t} = \alpha_j + h_{j,t}^* + \beta_{1,j}(s_{1,t} - \bar{s}_{1,t}) + \beta_{2,j}(s_{2,t} - \bar{s}_{2,t}), \quad (10)$$

where the dynamics of $h_{j,t}^*$ under the *true measure* are given by

$$dh_{j,t}^* = k_j(\theta_j - h_{j,t}^*)dt + \sigma_j\sqrt{h_{j,t}^*}dZ_{j,t} \quad (11)$$

with parameter values noted in Duffee (1999) and $Z_{j,t}$ is a Brownian motion. Again details on the bond valuation formulae are available in Duffee (1999).

4.1.2. Simulation results

The first simulation experiment elicits the properties of spread changes of a single bond. My objective is to determine whether the presence of barriers has a significant impact on the properties of credit spread changes for a realistic specification for default processes and the risk-free yield curve. I run 10,000 sample

paths for the with 240 time points in each sample path using the following procedure:¹³

For Simulation Number = 1 to 10,000 do steps 1, 2 and 3

1. For time = 1 to time = 240 repeat (a), (b) and (c) to generate the default-free yield curve:

- a. Compute the spot rate as the rate in the previous period rate plus the factor shocks:

$$r_t = r_{t-1} + \Delta s_{1,t} + \Delta s_{2,t} \text{ where}$$

$$\Delta s_{i,t} = k_i(\theta_i - s_{i,t})\Delta t + \sigma_i \sqrt{s_{i,t}} \varepsilon_{i,t} \sqrt{\Delta t}, \quad \Delta t = \frac{1}{12}, \quad \varepsilon_{i,t} = N(0, 1)$$

with parameter values in appendix.

- b. Compute zero coupon bond yields for maturity $T = 1, \dots, 20$.
- c. Compute the par coupon yield for a 20-year par coupon bond.
- d. Compute factor means \bar{s}_1 and \bar{s}_2 .

2. For time = 1 to time = 240 repeat (a), (b), (c) and (d) to generate the defaultable yield curve:

- a. Compute the default intensity as

$$h_{j,t} = \alpha_j + h_{j,t}^* + \beta_{1,j}(s_{1,t} - \bar{s}_{1,t}) + \beta_{2,j}(s_{2,t} - \bar{s}_{2,t}),$$

where

$$h_{j,t}^* = h_{j,t-1}^* + k_j(\theta_j - h_{j,t}^*)\Delta t + \sigma_j \sqrt{h_{j,t}^*} \varepsilon_{j,t} \sqrt{\Delta t}, \quad \Delta t = \frac{1}{12}, \quad \varepsilon_{j,t} = N(0, 1)$$

with parameter values in appendix.

- a. Compute zero coupon bond yields for maturity $T = 1, \dots, 20$.
 - b. Compute the par coupon yield for a 20-year par coupon bond.
 - c. Compute the spread over the 20-year default free bond and weight it by the recovery rate.
3. Compute sample statistics.

Table 2 has sample statistics of data generated via this simulation study. First note the difference between the parameter values for credit spread changes for both the case without barriers and the one with barriers. As expected, the overall mean (rows 1 and 2) is lower for a rebalanced portfolio. This is so because the distribution for spread changes is bounded below at zero in both cases but has no constraint on the other side for the portfolio that is not rebalanced. In particular note the correlation between spread level and spread changes is negative in both

¹³ Parameter values are set at those reported by Duffee (1999) in Table 2 (top panel) and in Table 4 (for Baa bonds).

cases (numbers in bold). The presence of barriers increases the negative correlation value from -0.11 to -0.17 . In other words, the presence of barriers increases the negative correlation by approximately 50%. In other words, if we analyze only those price paths that survive, these price paths must show mean reversion in the ex-post analysis!

The second simulation experiment determines whether the presence of barriers has a significant impact on the properties of credit spread changes *for an index* of 75 bonds. Each of the 75 bonds have different parameter values distributed evenly within a variation of one standard deviation (based on Duffee, 1999) around the mean parameter values reported in the appendix. I again run 10,000 sample paths for each of the 75 bonds with 240 time points in each sample path. Bonds are excluded from the index if the yield spread increases or decreases sufficiently to cross out of the range (1%, 2.5%). I also report statistics on an index where there is no rebalancing of the index portfolio.

Table 3 has sample statistics of data generated by these simulations. Again note the difference between the parameter estimates for credit spread changes for both the case without barriers and the one with barriers. The presence of barriers increases the negative correlation between spread changes and spread levels from -0.12 to -0.18 in this case. Therefore, the simulations support the thesis of observed mean reversion on account of survival. I now proceed to an analysis of bond data to explore the same issue.

4.2. An empirical study

4.2.1. Data and methodology

I obtain proprietary data on Baa corporate bond prices for the period January 1985 to January 1996.¹⁴ The data set gives month end prices for a large array of Baa bonds with an initial maturity of greater than 10 years. Table 4 gives summary statistics of the number of bonds in the data set at the beginning of each year. There are an average of 63 bonds in the data set at the beginning of each year. I also collect data on dates on which the ratings of these bonds was changed (if at all) from the initial Baa rating. On average eight bonds had a rating change during the subsequent year and this number varies considerably across the sample period. Corresponding 10-year government bond yields are obtained from the Federal Reserve Bank home page.

My objective is to determine the impact of rating changes on credit spread behavior. I form 11 portfolios, one each at the beginning of years 1985 through 1996. My objective is to analyze the performance of each cohort (a) without taking into account rating changes and (b) by including only those bonds that have not experienced a rating change. For each cohort constructed at time

¹⁴ I am thankful to USAA mutual funds and Carlos Molina for the data. Note that the number of bonds is slightly lower than the number in other popularly used databases such as Datastream and Lehman brothers but it covers most of the large issues.

Table 3
Simulation results for credit spread index

	Mean	Standard deviation	Skewness	Kurtosis	Time periods
<i>Spread statistics</i>					
Spread = Baa – Govt.					
Rebalanced	1.97	0.29	0.87	0.42	240
Not rebalanced	2.07	0.34	0.93	0.56	240
<i>Spread changes</i>					
Spread _t – Spread _{t-1}					
Rebalanced	0.00	0.11	-0.34	0.84	240
Not rebalanced	0.00	0.12	-0.39	1.84	240
ρ_1					
Rebalanced	0.00				240
Not rebalanced	0.00				240
Cor(Spread _{t-1} , Spread _t – Spread _{t-1})					
Rebalanced	-0.18*				240
Not rebalanced	-0.12*				240

This table contains simulation results for credit spreads obtained by running 10,000 sample paths with 240 data points along each sample path for each bond (for a total of 75 bonds). For each time point along a sample path, the instantaneous interest rate is generated by: $r_t = \alpha_r + s_{1,t} + s_{2,t}$ where the dynamics of $s_{1,t}$ and $s_{2,t}$ under the true measure are give by an Euler approximation of $ds_{i,t} = k_i(\theta_i - s_{i,t})dt + \sigma_i\sqrt{s_{i,t}}dZ_{i,t}$, $i = 1, 2$. Bond yields are subsequently obtained using a multi-factor version of the CIR model. I generate the instantaneous default risk for 75 bonds: $h_{j,t} = \alpha_j + h_{j,t}^* + \beta_{1,j}(s_{1,t} - \bar{s}_{1,t}) + \beta_{2,j}(s_{2,t} - \bar{s}_{2,t})$ where $h_{j,t}^*$ follows a square root process given by an Euler approximation of $dh_{j,t}^* = k_j(\theta_j - h_{j,t}^*)dt + \sigma_j\sqrt{h_{j,t}^*}dZ_{j,t}$. The parameter values are evenly distributed within one standard error of reported mean values in footnote 13. Index spreads are the averages for the 75 bonds. Defaultable bond yields are computed with recovery adjusted par yields. This table reports summary measures of the index credit spreads and credit spread changes generated in the simulations. I assume a recovery rate of 0.44.

* Significant at 95%.

$T = 1985, 1986, \dots, 1996$, I compute the average yield and yield spread on all the bonds as follows:

$$\text{AllBonds}_{T,t} = \frac{1}{n_T} \sum_1^{n_T} y_{i,t}, \quad t = 1, 2, \dots, 12, \quad (12)$$

where n_T is the number of bonds in the date T cohort, t is the month in year T and $y_{i,t}$ is the yield to maturity on the bond in month t . This gives the average yield on all the bonds in the cohort. Thus the 11 cohorts give me a set of 11 times 12 or 132 data points for yields on a rebalanced portfolio. Similarly I construct a second set of yields by including only those bonds that do not experience a rating change in the subsequent year as follows:

$$\text{SurvivingBonds}_{T,t} = \frac{1}{s_{T,t}} \sum_1^{s_{T,t}} y_{i,t}, \quad t = 1, 2, \dots, 12, \quad (13)$$

Table 4
Parameter estimates
Panel A^a

Year	Total number of bonds	Number survived
1985	66	57
1986	62	52
1987	56	52
1988	66	55
1989	61	51
1990	48	44
1991	36	33
1992	47	43
1993	64	57
1994	78	73
1995	81	73
1996	85	74
Average yield (std. dev.)	9.83 (1.35)	9.70 (1.29)
Average spread over 10-year treasury (std. dev.)	2.05 (0.51)	1.93 (0.37)

Panel B^b

	α	β	<i>T</i> -stat of β	<i>R</i> -square
All bonds	0.34	−0.16	−3.65*	0.08
Surviving bonds	0.49	−0.25	−4.59*	0.14

^a This table provides summary statistics of the data used in the empirical study. The data comprises monthly observations of Baa rated bonds for the period January 1985 to December 1996. The table reports the number of unique bonds in each cohort, the average yield to maturity and average spread of the bonds over a comparable 10-year treasury bond.

^b I form 11 portfolios at the beginning of each year from 1985 through 1996. I compute the monthly average yield and yield spread on each cohort for that year (a) for “All bonds” without taking into account rating changes and (b) for “Surviving bonds” by including only those bonds that have not experienced a rating change. This table reports estimates for the OLS regression on spread changes vs. spread levels for each portfolio:

$$\text{Spread}_t - \text{Spread}_{t-1} = \alpha + \beta(\text{Spread}_{t-1}) + \text{error}_t.$$

The data comprises monthly observations of Baa rated bonds for the period January 1985 to December 1996 with an initial maturity of more than 10 years.

* Significant at 99%.

where $s_{T,t}$ is the number of surviving bonds in the date T cohort, t is the month in year T and $y_{i,t}$ is the yield to maturity on the bond in month t . This gives the average yield on all the bonds in the cohort that do not experience a rating change. For example Table 4 shows that on average the yield on all bonds is 9.83% while the yield on surviving bonds is 9.70%. The corresponding yield spread is computed as the difference between a corporate bond yield and the government bond yield at the end of each month. Another point to note is that the standard deviation of the yield and the

credit spread on all bonds is higher than the corresponding standard deviations for the cohort of surviving bonds.

4.2.2. Results

Fig. 4 reports the average yield spreads on surviving bonds computed using the methodology described earlier (bold line). A second line depicts the credit spread on surviving bonds minus all bonds. Notice that when credit spreads are high (e.g., in 1985–1986 period), the second line is negative. In other words excluding some bonds because of rating changes reduced the overall spread on the surviving bonds. We can observe the same kind of behavior during the 1989–1991 time period. On the other hand in the 1991–1995 period, there are bond exclusions because of upgrades and some the overall portfolio yield is lower than that of the surviving period. Thus, the evidence corroborates the fact that when yield spreads are high, some bonds are excluded thus constraining yields below a certain level. On the other hand when yields are low the spreads are constrained above a certain level by excluding bonds that may have experienced upgrades.

Another way to ascertain the impact of the rating change based exclusions is to run a regression between spread changes and spread levels for all bonds and for the surviving bonds. Table 4 shows that the beta estimate (coefficient on spread level) is -0.16 for all bonds but increases to -0.25 when only surviving bonds are analyzed. The null hypothesis for equality of these beta estimates is rejected at the

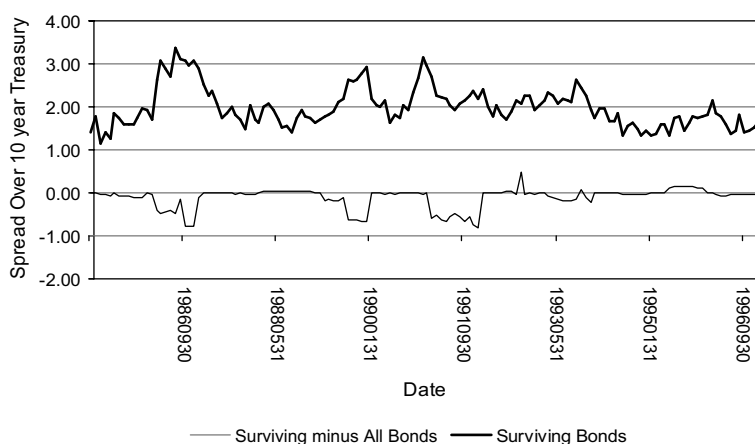


Fig. 4. Credit spreads of surviving bonds. The data comprises monthly observations of Baa rated bonds for the period January 1985 to December 1996 with an initial maturity of more than 10 years. I form 11 portfolios at the beginning of each year from 1985 through 1996 and compute the monthly average yield and yield spread on each cohort for that year (a) for “All Bonds” without taking into account rating changes and (b) for “Surviving Bonds” by including only those bonds that have not experienced a rating change. The bold line depicts average spreads on bonds that retain their Baa rating (surviving bonds) and the light line depicts average spreads on surviving bonds minus all bonds. The credit spread is computed as the difference between the yield on a corporate bond and the 10-year government bond at each point in time.

95% confidence level. Thus survival causes a statistically significant increase in mean reversion of the surviving bonds. As the business cycle progresses, it is likely that surviving bond spreads will narrow and some will be upgraded thus restricting the bond spreads to a certain range.

In summary, the simulations and the empirical evidence shows that spread behavior of an index is influenced by survival because ratings changes often cause bonds to be excluded from the index.

5. Implications and concluding comments

Analysis in the previous section suggests that mean reversion in index spreads is accentuated by survival. In particular this result provides an explanation for the observed mean reversion at both high and low levels of spreads. In the finance literature, most models written to capture the dynamics of the spreads or the underlying state variables are continuous time diffusions. The main difference in the extant models lies in their assumed functional forms for the drift and volatility. Our analysis in Section 4 shows that the average spread change (drift function) for the index spread should show a high mean reversion at large spread levels and low spread levels. For low and medium spreads, there is little mean reversion but as the spreads continue to climb, the degree of mean reversion again should increase. Such a non-linear drift function is observed in examinations of the short-term interest rate (e.g., Ait-Sahalia, 1999; Ahn and Gao, 1999). My study provides a definite rationale for the presence of non-linearity in the drift function.¹⁵ Note that the underlying specification is likely to considerably alter the conditional probability density of spread levels. Hence claim prices whose payoffs depend on spread levels are likely to change.

Because the spread is considered a measure of investor risk aversion, the conditional asset pricing models use an index spread to gauge changes in investor appetite for risk. This study shows that changes in the spread may not be linearly related to changes in risk premia but there may be a non-linear component during troughs and upturns of the business cycle. If the asset pricing model implies that the expected return can be written as a linear relationship between return and the spread, the coefficient estimate for spread changes will likely reflect in part the impact of such non-linear behavior. In related work, financial economists have long understood that financial market variables like stock prices and interest rates contain considerable information about the *current and future* state of the economy.¹⁶ In terms of the current state of the economy, an ex-post analysis of changes in spreads and their rela-

¹⁵ In order to lend credence to my assertion, I conduct a maximum-likelihood investigation of three competing parametric specifications for Baa index credit spreads – the first two (Vasicek and CIR) are commonly used to describe spread changes while the third (non-linear) accounts for the postulated behavior of credit spreads. The non-linear specification provides a better in-sample fit.

¹⁶ See, for instance Bernanke (1990), Diebold and Rudebusch (1989), Stock and Watson (1990) and Stock and Watson (1994).

tionship to the stock market or economic variables can be driven in part by the spurious shifts in spreads as a result of ratings based classifications. As a result more work is needed to separate out the components that drive spread changes. This will enhance our understanding of the issues elaborated above.

In conclusion, research on the behavior of credit spreads reveals that the spread between a corporate bond index yield and comparable treasury bonds is negatively related to the spread level. Efforts to explain the negative relationship via macro-economic factors and other firm-specific factors have had limited success. I show that a large part of the observed mean reversion is a natural consequence of ratings based classifications of bonds and the related effects of survival. An important finding of the article is that the observed behavior of index spreads at low and high spread levels can be explained by survival. Research by Moody's investor services on ratings changes and credit quality migration shows that credit downgrades and upgrades effect a significant portion of the bonds in each rating category. During economic boom times or during recessions the proportion of ratings changes may be high. It is precisely in these situations where survival affects the dynamics of spreads to the largest extent.

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