

# Dynamics of Credit Spreads: A Non-Parametric Analysis

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As more fixed-income market participants turn to the corporate bond market, there has been an increase in demand for a wide variety of credit-based financial products from corporations and fund managers. An understanding of the dynamics of credit spreads is thus important for practitioners and academicians alike. Modern asset pricing theory allows us to value and hedge a wide array of claims on credit spreads when we know the dynamics of the underlying state variables. Unfortunately, the theory does not provide guidance on choosing a correct specification for price changes of a financial asset.

This article uses a non-parametric approach proposed by Stanton [1997] to examine changes in credit spreads and identify the properties of possible candidate processes. The continuous-time financial theory has developed a variety of tools to value corporate bonds and associated credit derivatives, but specification of an appropriate model is for the most part an unanswered question.

In the finance literature, most models written to capture the dynamics of the spreads on the underlying state variables are continuous-time diffusions based on the specification:

$$dX_t = \mu(X_t)dt + \sigma(X_t)dZ_t \quad (1)$$

where  $\mu$  and  $\sigma$ , the drift and diffusion of the process, are functions only of the contemporaneous value of  $X_t$ , and  $Z_t$  is a standard Brownian motion.

The main differences in models are in their assumed functional forms for  $\mu$  and  $\sigma$ . The choice of the parametric drift and diffusion families is often arbitrary. One approach is to choose among possible spread processes according to how well they price specific liquid derivative securities. This is often a difficult task in practice, and when feasible tends to vary with the specific derivatives used. Moreover, the market for credit derivatives is illiquid, with wide spreads between bid and ask prices. It is therefore desirable to be able to test a specification without making use of observations on a collection of derivatives.

The non-parametric procedure proposed by Stanton [1997] allows an estimation of drift and diffusion functions from data observed at discrete intervals. It makes no parametric assumptions about the drift or the diffusion function. Using the non-parametric procedure, we examine Moody's Aaa bond index and Bbb bond index yield spreads for the sample period February 1977 through April 2001.

We find that the drift function is linear for Aaa index spread changes, but the the Bbb index spread over the benchmark bond shows evidence of substantial non-linearity. For narrow and medium spreads, there is virtually no mean reversion, but as spreads widen, the degree of mean reversion increases dramatically. Finally, we regress the non-parametric estimates of drift and standard deviation on contemporaneous spread levels to investigate the effectiveness of parametric specifications.

## I. LITERATURE OVERVIEW

Given the possibility of default, risk-averse investors must be compensated for holding corporate bonds instead of otherwise identical default-free securities. The credit spread, defined as the yield differential between seasoned corporate bond indexes and constant-maturity Treasuries, can be used to gauge the premium in aggregate that investors demand for holding securities subject to default risk.

The index represents the average of the spreads of the constituent bonds. The yield on a constituent corporate bond depends on the risk-free interest rate (or the term structure), the probability of default of a bond, and the recovery rate in the event of default.

There are two basic approaches to modeling corporate default risks. One approach, pioneered by Black and Scholes [1973] and Merton [1974] and extended by Black and Cox [1976], Longstaff and Schwartz [1995], and others, explicitly models the evolution of firm value observable by investors. This approach is commonly referred to as the “structural approach.” It has been applied in Geske [1977], Smith and Warner [1979], Cooper and Mello [1991], Hull and White [1992], Abken [1993], Leland and Toft [1997], and Zhou [1998], among others.

In structural models, the firm value is assumed to follow a diffusion process. A firm defaults on its debt if the firm value falls below the nominal value of outstanding debt. Credit spread dynamics in these models depend on the underlying diffusion specification. For example, consider the classical approach of Merton [1974]. The value of a firm  $V$  is modeled as a geometric Brownian motion, which under the martingale measure has the form:

$$dV_t = rV_t dt + \sigma V_t dZ_t \quad (2)$$

and the interest rate is assumed constant. If the firm is financed by zero-coupon debt with a face value  $D$  maturing at time  $t$ , at maturity the bondholders receive the amount:

$$\min(V_T, D) = D - \max(D - V_T, 0) \quad (3)$$

which is the payoff of a risk-free zero-coupon bond minus a put option. Therefore the credit spread itself must follow a diffusion process.

In these models, a firm cannot default unexpectedly. If a firm cannot default unexpectedly, and if it is not currently in financial distress, its probability of defaulting in the short term is zero, and short-term credit spreads should be zero. This implication is strongly rejected by the data.

To address this problem, Zhou [1998] proposes jump diffusion to model spread changes. A bond index is a weighted average of component bond yields, however, and is therefore diversified. Hence the credit spread dynamics of an index may be appropriately represented by a diffusion process.

A second approach to modeling risky debt is adopted by Jarrow and Turnbull [1995], Jarrow, Lando, and Turnbull [1997], Madan and Unal [1998], and Duffie and Singleton [1999]. They do not consider the relation between default and firm value in an explicit manner. This approach is called the “reduced-form approach.” Contrary to the structural approach, the reduced-form approach treats default as an unpredictable event involving a sudden loss in market value.

Duffie and Singleton [1999] show that a claim on a defaultable bond may be priced as if it were default-free by replacing the usual short-term interest rate process  $r$  with the default-adjusted short rate process  $R = r + hL$ . That is, the initial market value of the defaultable claim to  $X$  is:

$$V_0 = E^Q \left[ \exp \left( - \int_0^T R_t dt \right) X \right] \quad (4)$$

where the expectation is taken under the risk-neutral measure. The authors show that the default-adjusted short rate can be modeled as the sum of two or more diffusion processes. The corresponding spread dynamics are driven by the underlying specifications.

Empirical research by Duffie and Singleton [1997], Duffie [1998], and Duffie, Pedersen, and Singleton [2000] has used variants of Equation (1) in a multidimensional setting to examine default characteristics of risky bonds. The specification of an appropriate model, however, is an open question.

## II. NON-PARAMETRIC METHODOLOGY

In valuing contingent claims it is convenient to represent the underlying state variables as a continuous-time diffusion process satisfying the time-homogeneous stochastic differential Equation (1). The usual approach is first to specify parametric forms for the drift and diffusion functions, and then to estimate the values of the parameters.

Given functions  $\mu$  and  $\sigma$ , the transition density from value  $x$  at time  $t$  to value  $y$  at some later time  $s$ ,  $p(s, y | t, x)$  must satisfy the Kolmogorov forward and backward equations (see Oksendal [1985]). In principle, for a given parameterization of  $\mu$  and  $\sigma$ , we use maximum-likelihood to estimate the model's parameters.

A potentially serious problem with any parametric model, particularly when there is no economic reason why we should prefer one functional form over another, is misspecification. Even a model that fits credit spread movements well in-sample does not necessarily price securities well. To avoid misspecification, more recent researchers have used nonparametric estimation techniques in order to avoid having to specify functional forms for  $\mu$  and  $\sigma$ .

Stanton [1997] avoids imposing any parametric restrictions on either  $\mu$  or  $\sigma$ . He uses a Taylor series approximation to show that the conditional drift and diffusion are given by (for a second-order approximation):

$$\mu(X_t) = \frac{1}{2\Delta} \{4E_t[X_{t+\Delta} - X_t] - E_t[X_{t+2\Delta} - X_t]\} + O(\Delta^2) \quad (5)$$

$$\sigma^2(X_t) = \frac{1}{2\Delta} \{4E_t[(X_{t+\Delta} - X_t)^2] - E_t[(X_{t+2\Delta} - X_t)^2]\} + O(\Delta^2) \quad (6)$$

To implement the procedure, we need a means of nonparametrically estimating the conditional expectations in Equations (5) and (6). We use a kernel estimation procedure for doing this. Kernel estimation is a non-parametric method for estimating the joint density of a set of random variables. Given  $m$  dimensional vectors  $z_1, z_2, \dots, z_m$  from an unknown density  $f(z)$ , a kernel estimator of this density is

$$\hat{f}(z) = \frac{1}{Th^m} \sum_{i=1}^T K\left(\frac{z - z_i}{h}\right) \quad (7)$$

where  $h$  is the window width or smoothing parameter, and  $K(\bullet)$  is a suitable kernel function. The density at any point is estimated as the average of densities centered at the actual data points. The farther away a data point is from the estimated density, the less it contributes to the estimated density. Hence the estimated density is highest near the high concentration of data points and lowest when observations are sparse. The kernel density estimator is similar to a multidimensional histogram. Unlike a histogram, the blocks are not in general rectangular.

There is discretion in the choice of  $h$  and  $K(\bullet)$ . Results in the kernel estimation literature suggest that any reasonable kernel gives almost optimal results. Here we use a normal kernel function. The other parameter, the window width, is chosen according to the dispersion of estimates as:

$$\hat{h}_i = \hat{\sigma}_i T^{-1/(m+4)} \quad (8)$$

Another possible choice used by Ahn and Gao [1999] is:

$$\hat{h}_i = 1.5 \hat{\sigma}_i T^{-1/(m+4)} \quad (9)$$

where  $\hat{\sigma}_i$  is the standard deviation estimate of each variable  $z_i$ ,  $T$  is the number of observations, and  $m$  is the dimension of the variables. This window width has the property that for certain joint distributions it minimizes the asymptotic mean squared error of the estimated density function.

Given the density in Equation (7), we can calculate any moments we desire from the distribution. For example, the conditional expectation for the first-order approximation is estimated as:

$$E[x_{t+\Delta} - x_t | x_t = x] \approx \frac{\sum_{i=1}^T (x_{t+\Delta} - x_t) K([x - x_i]/h)}{\sum_{i=1}^T K([x - x_i]/h)} \quad (10)$$

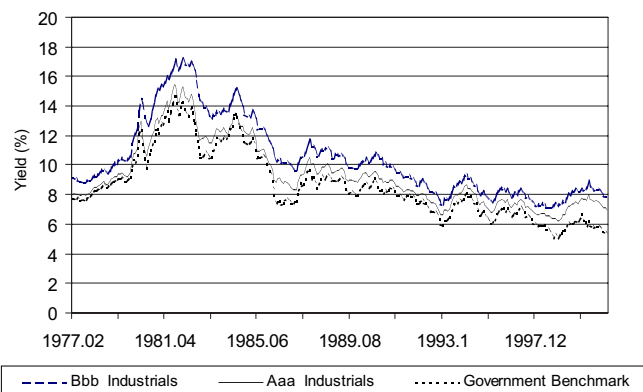
where  $K(z) = (2\pi)^{-1/2} e^{-1/2z^2}$ . This is a weighted average of observed spread changes. Similar expressions yield estimates for other conditional expectations and other moments.

### III. EMPIRICAL RESULTS

In theory, the credit spread can be broken into at least two separate components. The first component reflects the probability that default will occur and the associated default loss. A second component originates from the fact that bondholders need to be compensated for bearing extra risk. We measure the credit spread as the yield difference between the Moody's yield index for seasoned Aaa and Bbb corporate bonds and long-term constant-maturity bond yields (Exhibit 1). Exhibits 2 and 3 provide summary statistics of the data set.

## EXHIBIT 1

### Corporate Yields



Data series obtained from Federal Reserve Board for February 1977-April 2001. Yields (%) on Moody's seasoned Bbb bond index, Moody's seasoned Aaa bond index, and 30-year constant-maturity Treasury.

## EXHIBIT 2

### Summary Statistics

	Mean	Standard Deviation	Skewness	Kurtosis	Number of Observations
Aaa	9.33	2.21	0.92	-0.08	291
Bbb	10.41	2.60	0.92	-0.00	291
Benchmark	8.59	2.29	0.75	-0.22	291

Source: Federal Reserve Board.

Data series are yields (%) on Moody's seasoned Aaa bond index, Moody's seasoned Bbb bond index, and 30-year government bond. Monthly observations February 1977-April 2001.

## EXHIBIT 3

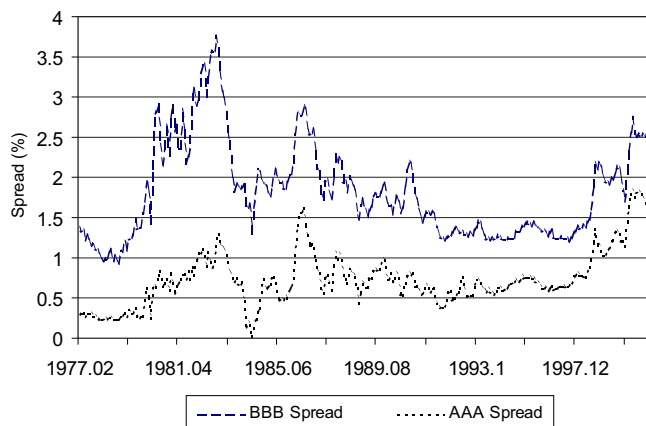
### Credit Spread Statistics

	Mean	Standard Deviation	Skewness	Kurtosis	Number of Observations
Spread Statistics					
$\Delta A = Aaa -$ Benchmark	0.74	0.35	0.98	1.17	291
$\Delta B = Bbb -$ Benchmark	1.82	0.60	0.91	0.29	291
Spread Changes					
$\Delta A_t - \Delta A_{t-1}$	0.004	0.098	0.369	1.885	290
$\Delta B_t - \Delta B_{t-1}$	0.003	0.157	0.579	3.364	290

Spreads (%) on Moody's seasoned Aaa bond index and Moody's seasoned Baa bond index over benchmark bond. The second panel reports statistics on spread changes over each month. Monthly observations February 1977-2001.

## EXHIBIT 4

### Credit Spreads



Monthly observations February 1977-April 2001. Spreads over benchmark bond.

The Moody's index is collated from an equally weighted sample of yields on 75 to 100 bonds issued by large non-financial corporations. Each bond issue included in the index has a face value exceeding US\$100 million, an initial maturity of more than 20 years, and a liquid secondary market.

A number of biases induced by the way the index is constructed have led several authors to question its reliability as a measure of corporate risk. One issue is that the index includes a large number of callable bonds. Embedded options give the issuers a right to call or repurchase a bond before its expiration. In a falling interest rate environment, this option is likely to be valuable, and correspondingly shorten the effective maturity of the index. Further, option characteristics may distort the theoretical dynamics of credit spreads derived from a hypothesized underlying model.

A second issue is that bonds constituting an index are often "refreshed" in order to maintain constant credit quality. In other words, the yield change from one period to another does not measure the change in the same set of bonds, but rather the change in the average yield on two sequential sets of bonds that share the same credit rating.

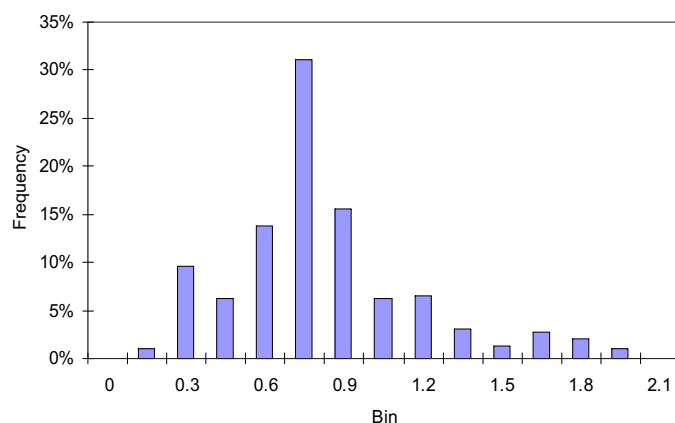
Exhibit 3 reveals that Aaa spreads have a mean of 0.74% and a standard deviation of 0.35%, while Bbb spreads have a mean of 1.82% and a standard deviation of 0.60%. Exhibits 4, 5, and 6 depict credit spreads and yield spread changes for the data set. Both Aaa spreads and Bbb spreads depict excess kurtosis that is inconsistent with a normal distribution.

Our objective in this exercise is to characterize the dynamics of the index.

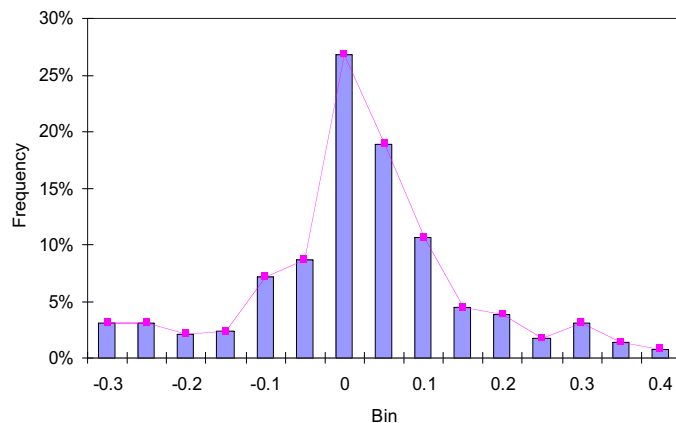
## EXHIBIT 5

### Histograms of Aaa Spreads and Spread Changes

#### A. Spread Distributions



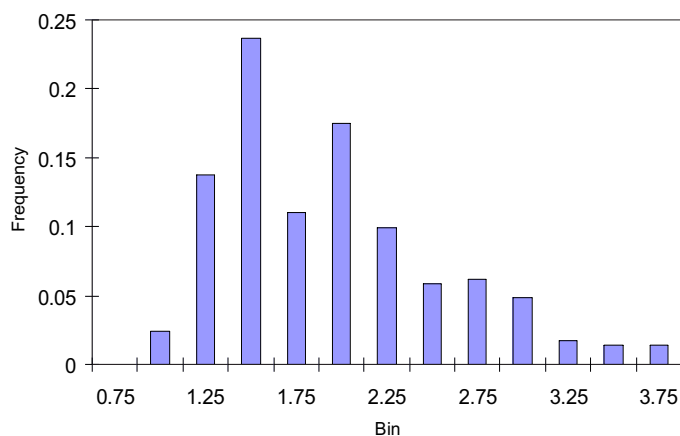
#### B. Spread Changes



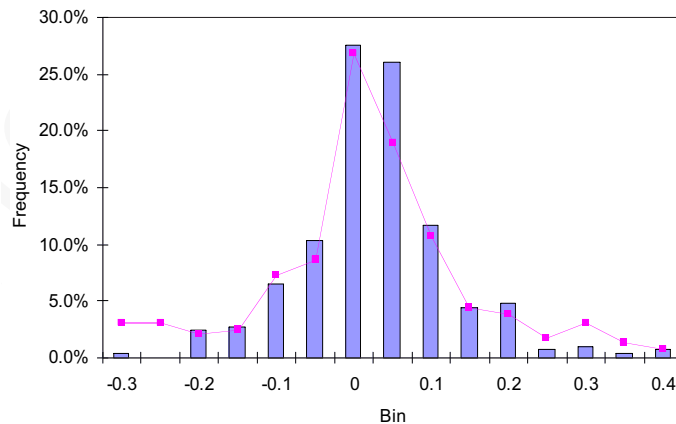
## EXHIBIT 6

### Histograms of Bbb Spreads and Spread Changes

#### A. Spread Distributions

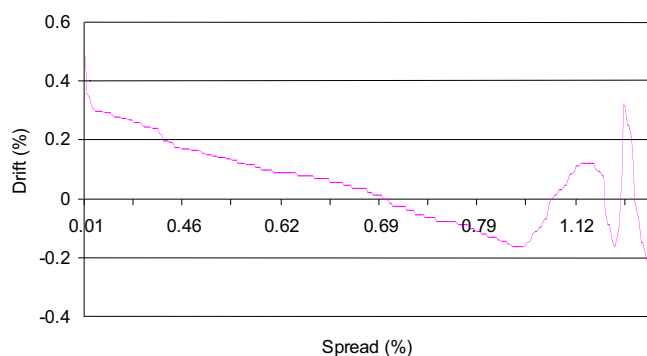


#### B. Spread Changes



## EXHIBIT 7

### Non-Parametric Estimate of Drift for Aaa Spreads

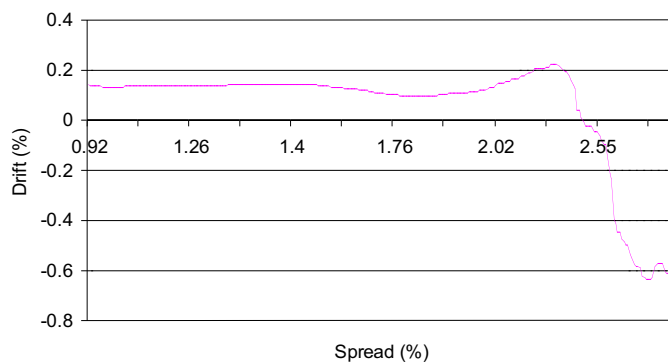


Exhibits 7, 8, 9, and 10 provide the primary results of the article. The results are obtained by an analysis of time series data, so the parameter estimates are obtained under a true measure rather than a risk-neutral measure. The conditional drift and variance are obtained by applying Equations (5) and (6) to the data set for both Aaa index yield spreads and Bbb index yield spreads.

Exhibits 7 and 8 graph the conditional drift for Aaa spreads and Bbb spreads. Aaa spreads show a mean reversion toward the unconditional mean of 0.74% for most of the range. The drift function seems to be linearly related to the spread. The drift function drops monotonically, but behaves erratically toward high levels of spreads. Exhibit 8 by contrast shows that Bbb spreads are non-linear in the drift function. Around its mean, the Bbb

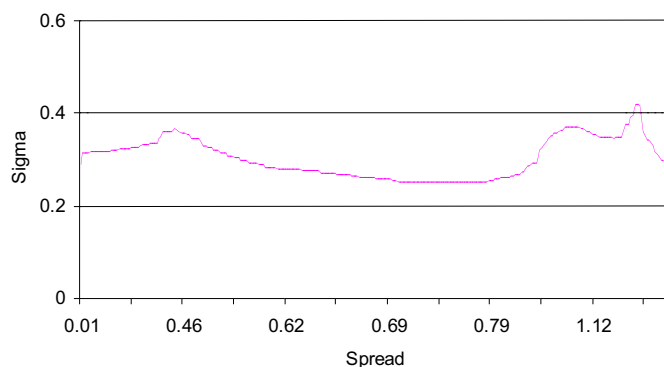
## EXHIBIT 8

### Non-Parametric Estimate of Drift for Bbb Spreads



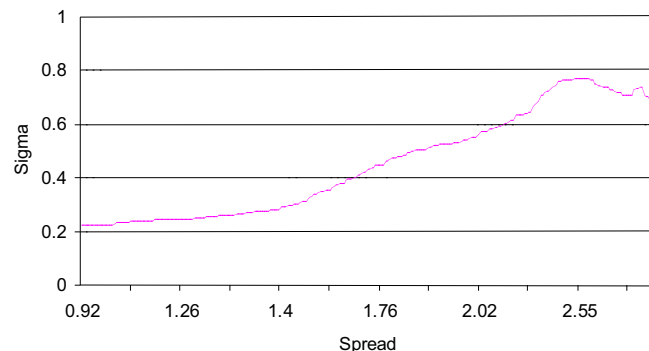
## EXHIBIT 9

### Non-Parametric Estimate of Standard Deviations for Aaa Spreads



## EXHIBIT 10

### Non-Parametric Estimate of Standard Deviations for Aaa Spreads



drift is essentially zero and behaves like a random walk, but it then mean-reverts strongly when far away from the mean.

## EXHIBIT 11

### Regression Estimates of Conditional Drift and Volatility Spreads

	a	b	R <sup>2</sup>	F-Stat	Number of Observations
Aaa Spreads					
Drift					
Estimate	-0.01	-0.17	0.45	272	290
T-Stat	-2.28	-16.50			290
Sigma					
Estimate	0.30	0.002	0.03	0.26	290
T-Stat	95.12	0.51			290
Bbb Spreads					
Drift					
Estimate	0.30	-0.46	0.41	236	290
T-Stat	16.01	-15.37			290
Sigma					
Estimate	0.13	0.58	0.92	3400	290
T-Stat	20.51	57.73			290

A non-linear drift function is observed in examinations of the short-term interest rate (e.g., Ait-Sahalia [1997] and Ahn and Gao [1999]). These studies suggest that the primary reason for rejection of the affine class of diffusion processes as valid representations of dynamics of interest rates is misspecification of the drift function. A corresponding analysis for index spreads shows that the same observation is valid for the Bbb index spreads.

Exhibits 9 and 10 graph the conditional standard deviations for Aaa spreads and Bbb spreads. Exhibit 9 shows that Aaa spreads behave like a random walk with a conditional standard deviations in the neighborhood of 0.35%. Therefore, given the drift and diffusion functions, the classic Vasicek [1979] specification for evolution of Aaa spreads is likely to work well for most of the domain. Exhibit 10 reveals that the conditional standard deviations of Bbb spreads increase with the spread level. The slope of the increase suggests a great sensitivity to interest rate levels.

An examination of the relationship of spreads to the conditional drifts and standard deviation via a regression sheds some light on some of the parametric specifications that are likely to work well in practice. Exhibit 11 shows regression estimates of the equations:

$$\text{Non-Parametric Drift} = a + b[\text{Log}(\text{Spread})] + \text{Error}$$

$$\text{Non-Parametric Volatility} = a + b[\text{Log}(\text{Spread})] + \text{Error} \quad (11)$$

for both Aaa and Bbb index spreads.



The first panel of Exhibit 11 reveals that the non-parametric volatility ( $\sigma$ ) of Aaa spreads is independent of the spread level, while the volatility of Bbb spreads has a sensitivity of 0.58 to the spread level. Note that the sensitivity to spread level is more than 0.58 for high levels of Bbb spreads and lower for low levels of spreads. In the classic square root model of Cox, Ingersoll, and Ross [1985], the exponent is 0.5 while it is 1.5 for the non-linear model of Ahn and Gao [1999].

The drift function of Bbb spreads has a sensitivity of -0.46 to the spread level. This result is driven largely by the increased mean reversion at the high levels of Bbb spreads. Therefore, for pricing applications, Aaa spread dynamics are represented by the affine class of functions (e.g., the Vasicek model [1979]). For Bbb spreads, the affine class of models may not adequately capture the non-linearity of the drift function. A maximum-likelihood investigation (results not reported here) of various parametric specifications reveals that a non-linear drift specification substantially improves the explanatory power of the model as measured by a chi-square statistic for the goodness of fit (see Ait-Sahalia [1999]).

#### IV. SUMMARY

We examine the conditional drift and variance of spreads using a non-parametric approach proposed by Stanton [1997]. The data include Moody's Aaa and Bbb index yields for the period February 1977 through April 2001. We show that the drift and diffusion is linear in Aaa index spreads, while Bbb spreads show a strong non-linearity in the drift function. Around its mean, the Bbb drift is essentially zero and behaves like a random walk; it then mean-reverts strongly when far away from the mean. Therefore, for pricing applications, the affine class of parametric diffusion specifications is likely to work well for Aaa spreads, but a non-linear specification is preferable for Bbb spreads.

#### ENDNOTE

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#### REFERENCES

- Abken, P. "Valuation of Default Risky Interest Rate Swaps." *Advances in Futures and Options Research*, 6 (1993), pp. 931-116.
- Ahn, D., and B. Gao. "A Parametric Nonlinear Model of Term Structure Dynamics." *The Review of Financial Studies*, 12 (1999), pp. 721-762.
- Ait-Sahalia, Y. "Testing Continuous Time Models of the Spot Interest Rate." *Review of Financial Studies*, 9 (1996), pp. 385-426.
- . "Transition Densities for Interest Rate and Other Non-linear Diffusions." *Journal of Finance*, 54 (1999), pp. 1361-1396.
- Black, F., and J.C. Cox, "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions." *Journal of Finance*, 31 (1976), pp. 351-367.
- Black, F., and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy*, 81 (1973), pp. 637-659.
- Cooper, I.A., and A.S. Mello. "The Default Risk of Swaps." *Journal of Finance*, 47 (June 1992), pp. 597-620.
- Cox, J.C., J.E. Ingersoll, Jr., and S.A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53 (March 1985), pp. 385-407.
- Duffee, G.R. "Treasury Yields and Corporate Bond Yield Spreads: An Empirical Analysis." *Journal of Finance*, 53 (December 1998), pp. 2225-2242.
- Duffie, D., L.J. Pederson, and K.J. Singleton. "Modeling Sovereign Yield Spreads: A Case Study of Russian Debt." Working paper, Stanford University, 2000.
- Duffie, D., and K.J. Singleton. "An Econometric Model of the Term Structure of Interest Rate Swap Yields." *Journal of Finance*, 52 (September 1997), pp. 1287-1323.
- . "Modeling Term Structure of Defaultable Bonds." *Review of Financial Studies*, 12 (1999), pp. 687-720.
- Geske, R. "The Valuation of Corporate Liabilities as Compound Options." *Journal of Financial and Quantitative Analysis*, 12 (1977), pp. 541-552.
- Hull, J., and A. White. "The Impact of Default Risk on the Prices of Options and Other Derivative Securities." *Journal of Banking and Finance*, 19 (May 1995), pp. 299-322.
- Jarrow, R.A., D. Lando, and S.M. Turnbull. "A Markov Model for the Term Structure of Credit Risk Spreads." *Review of Financial Studies*, 10 (1997), pp. 481-523.

Jarrow, R.A., and S.M. Turnbull. "Pricing Derivatives on Financial Securities Subject to Credit Risk." *Journal of Finance*, 50 (March 1995), pp. 53-85.

Leland, H.E., and K.B. Toft. "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure and Credit Spreads." *Journal of Finance*, 51 (July 1996), pp. 987-1019.

Longstaff, F., and E. Schwartz. "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt." *Journal of Finance*, 50 (September 1995), pp. 789-819.

Madan, D., and H. Unal. "Pricing the Risks of Default." *Review of Derivatives Research*, 2 (1998), pp. 121-160.

Merton, R.C. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance*, 29 (May 1974), pp. 449-470.

Oksendal, B. *Stochastic Differential Equations*. Springer Verlag, 1992.

Smith, C.W., and J.B. Warner. "On Financial Contracting: An Analysis of Bond Covenants." *Journal of Financial Economics*, 7 (1979), pp. 117-161.

Stanton, R. "A Nonparametric Model of Term Structure Dynamics and the Market Price of Interest Rate Risk." *Journal of Finance*, 52 (1997), pp. 1973-2002.

Vasicek, O. "An Equilibrium Characterization of the Term Structure." *Journal of Financial Economics*, 5 (1979), pp. 177-188.

Zhou, C.S. "A Jump-Diffusion Approach to Modeling Credit Risk and Valuing Defaultable Securities." Working paper, Federal Reserve Board, 1998.