How Does Uncertainty About The Federal Guarantee Impact The Value Of GSE Claims?

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Abstract

Recent comments by the Federal Reserve Chairman have renewed concerns about whether the government would protect bondholders in the event of default by Fannie Mae or Freddie Mac (Government Sponsored Enterprises or GSEs). We provide a model of GSEs that allows us to analyze the effect of the uncertainty in the federal guarantee on the value of GSE debt, equity and the value of the implicit guarantee. Uncertainty about the Federal Guarantee increases expected bankruptcy costs, thereby increasing the cost of funds for a GSE when debt is used to finance a part of the firm. Also, uncertainty about the guarantee reduces the profitability of GSE assets (mortgage portfolios) by increasing the costs of managing and hedging these portfolios. Counter to intuition, an increase in the likelihood that the government will not subsidize the GSE may increase the expected cost of the subsidy to the government. A cap on the value of the GSE investment portfolio is a more effective mechanism to reduce the risk exposure of the federal government.

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1. Introduction

Government-sponsored enterprises (GSEs) are financial intermediaries created by the Congress of the United States to foster liquidity in the housing loan market, and to fund loans to certain groups of borrowers such as homeowners, farmers and students. Among the biggest GSEs are the Federal National Mortgage Association (Fannie Mae) and the Federal Home Loan Mortgage Corporation (Freddie Mac). During the past decade GSEs have experienced an explosive growth in their assets. These assets (primarily mortgage backed securities) are funded mostly by selling debt. It is widely believed that these GSE debt holders will be fully reimbursed by the government were a GSE to default on its obligations. This belief that debt holders are not likely to face losses in the event of bankruptcy has lead to low GSE bond spreads and a large growth in the size of the GSE debt market. In 2005, the face value of outstanding debt of the GSEs (Fannie and Freddie) totaled more than $2 trillion. However, the assumption of a federal guarantee of GSE debt, that spurred growth in the GSE debt market, has been recently called to question. In a hearing[1] Alan Greenspan remarked:

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[1] Housing and Urban Affairs Committee hearing (July 21, 2005) chaired by Senator Richard Shelby.

> "there is a perception that debt holders are guaranteed by the full faith and the credit of the United States government, despite the fact that the debentures which they bought and literally say, as required by the law, that this instrument is *not backed* by the full faith and credit."

Subsequently, the Secretary of the Treasury (John Snow) made the following comments, when asked whether he would use his discretionary ability to bail out GSE debt holders in the event of default[2]:

> "Some commentators believe that this credit availability reinforces the perception that the Federal government backs the debt obligations of the Enterprises. This perception is false. In fact, I would exercise this line of credit only in the event that a GSE was in significant financial distress."

Seiler (2003) documents such instances when there have been public pronouncements about the likelihood of government support to GSEs, were they to default. The author finds that these pronouncements impact debt and equity prices negatively, as financial market participants reassess the risk of these assets. Such pronouncements by regulators and politicians are often motivated by concerns about the systematic risk posed by the growth in the size of these two entities as well as the potential cost that could be borne by taxpayers were these entities to default. The uncertainty about whether the government will bail out GSE debt holders raises several questions, important both for claimholders (debt and equity holders) as well as for policy makers, and are not fully addressed in the literature:

[2] See testimony of Secretary John Snow before the U.S. House Financial Services Committee Proposals for Housing GSE Reform on April 13, 2005.

- First, how and to what extent does the uncertainty about the Federal Guarantee impact GSE equity prices and debt prices?
- Second, and more importantly, does the government stance on voicing uncertainty about the guarantee help reduce the cost of this subsidy to taxpayers?

We answer the two questions posed by providing a "reduced form" model of GSEs where there is uncertainty about the government guarantee. Despite the simplicity of the model, it is useful from a practical perspective because of the limited inputs required therein. GSEs engage in two lines of business- First, they earn a fee on the mortgages they buy and then resell to investors after pooling and securitizing the claims. This constitutes their core fee based business. Second, they hold mortgage backed security portfolios to accrue the spread between their low cost of capital and the higher yield of the mortgage portfolio. The value of a GSE, in the absence of any debt financing is equal to the present value of these cash flows (in the same vein as Passmore (2005)).

GSEs however issue debt to finance the purchase of some of the assets (mortgage-backed securities). The debt financing adds value because interest payments are tax deductible, but at the same time incur the possibility of causing the firm to go bankrupt when it is unable to pay these promised interest payments. Debt issuance to finance the assets of the firm is beneficial for a number of other reasons, relative to corporations with similar business risk. First, the implicit federal guarantee allows the GSEs to sell debt securities with coupons that are marginally higher than comparable treasury bonds. GSE debt is exempt from registration with the Securities and Exchange Commission, thus reducing floatation costs. These bonds are treated as quasi-government securities by most investors because investors perceive these securities to be backed by the government. Some banks are allowed to make unlimited investments in GSE debt securities, and GSE securities are eligible as collateral for public deposits as well as for Treasury tax and loan accounts, which makes them attractive to investors. All these benefits result in a lower funding cost for a GSE.

On the other hand, a GSE may want to limit the issuance of debt relative to other corporations for a number of reasons. Because GSEs are exempt of state and local corporate income taxes, the tax advantage of leverage is lower than for regular corporations. Thus, as the bankruptcy probability increases, with more debt on the books, the tax benefits are not commensurate with the benefits that other corporations receive. GSEs are not allowed to originate mortgages, and are subject to other investment limitations. As the GSEs hold more investment securities in their portfolio, the increased credit risk, prepayment risk and interest rate risk may make the value of these earnings more volatile.

To analyze the tradeoffs between the advantages of debt financing (tax benefits, lower flotation costs and higher demand) and the drawbacks listed (bankruptcy costs), we provide a model that takes as inputs the cash flows from the two lines of business, the fee business and the investment portfolio business.

We obtain closed formed solutions for the value of the firm, debt and equity and relate these values to firm risk, taxes, bankruptcy costs, among other parameters[3]. Our main contribution is that we incorporate in the capital structure model, the probability that the government will either let a GSE fail or help them in the case of filing for bankruptcy as well as the effect of the uncertainty on the profitability of assets held by a GSE. This allows us to focus on the two questions posed on the impact of the uncertainty on the value of debt, equity and the cost of the subsidy.

Uncertainty about the guarantee impacts the firm via two channels. A first avenue by which the uncertainty impacts the firm value is through its impact on the expected bankruptcy costs incurred by debt holders in the event of default. When there is a larger possibility of going bankrupt, the debt holders must be compensated by a larger interest payment. This reduces the firm value and the maximum amount of debt the firm can optimally take. However the magnitude of this effect is small, given that in the current scenario, the GSE values are high enough to make the overall likelihood of bankruptcy quite low.

A second, and more important channel of the increased uncertainty, is that it reduces the earnings on its MBS portfolio. This happens because Fannie Mae and Freddie Mac make extensive use of other counterparty contracts to hedge and manage their portfolios. An increase in the uncertainty about the

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[3] The quantitative approach to modeling a firm's assets and liabilities was pioneered by Black and Scholes (1973), Merton (1974) and extended by Black and Cox (1976), and others. This

federal guarantee is likely to increase the capital requirements (margin) required by banks and financial institutions that enter into long term arrangements with the GSEs (see for example Cooper and Mello (1992)). This would consequently reduce the earnings of the mortgage portfolio and the overall value of the firm substantially in light of the fact that over 80% of the firms' earnings are derived from the investment portfolio, and hedging of the portfolio is an important aspect of its activities. *An increase in uncertainty about the guarantee that increases the costs of hedging by 10 basis points can decrease the firm value by 20%.*[4] Hence small changes in the uncertainty of the guarantee are likely to dramatically impact the firm value via a decrease in returns on the investment portfolio. This aspect of feedback of the credit risk of a firm on its earnings is discussed in Bhanot and Mello (2006) in the context of production and trading activities.

Researchers argue that the "implicit subsidy" by the federal government produces a surplus of billions of dollars directed to GSE shareholders (Lehnert, Passmore and Sherlund (2005))[5]. Our approach also helps determine the value of this implicit subsidy and the impact on the subsidy when there is some uncertainty about whether the government would step in, were the GSE to

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approach, commonly referred to as the "structural approach", is applied in Geske (1977), Smith and Warner (1979), Hull and White (1992), Leland (1994) and Leland and Toft (1996).

[4] These numbers are approximated from Fannie Mae's balance sheet for the year 2003.

[5] A clearer example how the implicit government backing works is illustrated with Fannie's problem of insolvency in the 1980s. In the beginning of such decade, the interest rates peaked and earnings on Fannie's portfolios weren't high enough to meet its liabilities. The main reason why Fannie made it through was because banks kept lending it money—based on the idea that the government stood behind Fannie. Thus, if everyone thinks that the government will not let GSEs fail, the likelihood that these companies will not be subject to market discipline will rise, further generating a moral hazard problem.

default. In related work, Lucas and McDonald (2006) specify the dynamics of the firm assets and liabilities to compute the value of the implicit subsidy. We follow their general approach even though our model is a variant of the models of capital structure presented in the corporate finance literature. In contrast to the contingent claims approach employed in this paper, Passmore (2005) directly estimates the value of the implicit guarantee using a discounted cash flow approach. This article can also be regarded as a reduced form approach to the one employed by Passmore (2005). We are able to estimate the funding advantage of GSEs, the extent to which GSEs would reduce their relative holding of GSEs in the absence of a government guarantee as well as the extent to which shareholders retain the value of the funding advantage.

The value of the implicit subsidy obtained in our setting is similar to that obtained by Passmore (2005) even though it is substantially higher than that obtained in some other studies. We determine the extent to which uncertainty about the subsidy may affect the cost of the subsidy to the government. Some policy makers argue that the government should clarify the potential misperception about the subsidy to the GSEs, and thus stem their growth rates and reduce the potential cost to the tax payer. Interestingly, an increase in the likelihood of revocation can reduce firm value dramatically that may in turn double the expected costs to tax payers. A more realistic avenue to reduce this cost to taxpayers is to cap the size of the investment portfolio relative to other options of limiting the risk posed by these entities.

The article is organized as follows. Section 2 gives background information on GSEs. Section 3 describes our model of capital structure. Closed form solutions are derived for debt, equity and firm values when there is uncertainty about the extent of the federal guarantee. Section 4 gives the input parameters, Section 5 analyzes the impact of the uncertainty about the implicit guarantee on firm value and debt and equity prices, Section 6 analyzes the implications for the value of the implicit subsidy. Section 7 discusses extensions of the model and Section 8 concludes.

2. Background

This section describes the business of GSEs[6]. GSEs such as Fannie Mae and Freddie Mac, are financial intermediaries created by the Congress of the United States to create liquidity in the housing loan market, and to fund loans to certain groups of borrowers such as homeowners, farmers and students. Fannie Mae was originally created as a wholly owned government corporation in 1938 and was converted into a GSE in 1968. Freddie Mac was created in 1970 as part of the Federal Home Loan Bank System to purchase mortgages from thrifts. Rather than hold mortgages in its portfolio, Freddie Mac pooled these mortgages and sold them after attaching a guarantee for credit risk.

As noted in the introduction, GSEs are hybrids of private corporations and federal entities. The GSEs are chartered by a federal statute and are exempt from state and local taxes, registration requirements. The US treasury is authorized to lend $2.25 billion to each of them. Banks are allowed to make unlimited investments in GSE debt securities, and GSE securities are eligible as collateral for public deposits as well as for Treasury tax and loan accounts. Also, GSEs are exempt from the provisions of many state investor protection laws.

The low spread on GSE debt coupled with the rapid growth of GSEs has focused attention on their impact on the systemic risk of the financial markets. Although the debt securities issued by the GSEs explicitly state that they do not carry a federal guarantee, their ties to the federal government convince investors of their ties to the federal government and the low risk of their debt. For example, Figure 1 provides a graphical depiction of the spread between GSE debt and treasury securities. The low spreads (comparable with AAA bonds), inspite of the high debt levels, are consistent with the opinions of market participants that the government will bail them out were there a likelihood of default.

Table 1 gives the outstanding debt of Fannie Mae and Freddie Mac. Their combined holding of mortgage backed securities as well as the amount of debt has grown to over or near 2 trillion dollars. Market participants as well as regulators increasingly want to determine how the size of the assets and liabilities is likely to affect the chance that the government may need to bail them out (value of the government subsidy). The uncertainty about the guarantee,

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[6] We adapt this information from other published descriptions of these entities.

addressed in this article, also leads to answers about how the implicit subsidy impacts the value of the firm and that of equity, and if the government ought to fix the potential liability of tax payers at the outset.

3. The model

Our objective is to analyze the impact of uncertainty about the federal guarantee on GSE debt and equity prices, and also its impact on the cost of the implicit subsidy. To achieve our objective we adapt the model proposed by Leland (1994) and Goldstein, Ju and Leland (2001). Our primary point of departure from these papers is in the manner in which the firm's earnings and its initial value are specified. A second difference is in the evaluation of payoffs received by debt holders were the company to default on its obligations. We start by defining the value of the firm assets when only equity is used to finance the firm (unlevered firm). Using this as a starting point, we compute the value when debt is added to the mix. We obtain the value of GSE debt and the equity prices (the residual claim on the assets of a firm).

3.1 Firm value process and uncertainty in the guarantee

First, we characterize the initial value of the firm and the dynamics of the firm value without considering bankruptcy costs and the tax benefits of debt. Fannie Mae and Freddie Mac operate two independent business lines: (1) a fee based business associated with securitizing mortgages that are sold off to other investors, and (2) a portfolio investment business that involves holding various mortgage backed securities.

Suppose the fee based business (securitization and credit guarantee business) generates a series of cash flows from the fees, denoted $\delta_1(t)$ each period. For example, in the year 2003, Fannie Mae earned approximately $2 billion in commissions and fees and incurred some administrative, tax and related costs to give net earnings of $1.4 billion per year. These earnings are stochastic and vary through time. Suppose, the earnings of this business are governed by the following process under the risk neutral measure:

$$\frac{d\delta_1}{\delta_1} = \mu_1 dt + \sigma_1 dz_1 \tag{1}$$

where μ_1 is a constant and refers to the growth rate in the business, σ_1 is that instantaneous volatility of the earnings, dt is the increment in time, dz_1 is the increment of a standard Brownian motion.

Then, the value of the business is equal to the present value of the cash flows it generates: $V_1 = E\left[\int e^{-rt}\delta_1(t)dt\right] = \frac{\delta_1}{r-\mu_1}$. When the earnings are 1.4 Billion and r=6% and with a growth rate of 1%, the value of this business works out to $28 billion. V_1 is also referred to as the unlevered value of the business because it assumes no consideration about debt financing. An application of Ito's lemma gives the dynamics of the value of the business:

$$\frac{dV_1}{V_1} = \mu_1 dt + \sigma_1 dz_1 \qquad (2)$$

In addition to this first line of business, a second line of business generates returns by holding a portfolio of securities on its books. The firm is able to generate a revenue stream equal to a proportion of the amount of security inventory on its books. In 2003, Fannie Mae earned approximately 40 billion dollars on its investment portfolio of nearly 900 billion. Then, in the absence of tax benefits of debt (because we are considering an unlevered firm) and including hedging and administrative costs, the net earnings would be around 29 billion dollars a year. The earnings of this business, per dollar of securities held (around 3.17% in our case), are governed by the following process under the risk neutral measure:

$$\frac{d\delta_2}{\delta_2} = \mu_2 dt + \sigma_2 dz_2 \qquad (3)$$

where μ_2 is a constant, σ_2 is that instantaneous volatility of the earnings and dz_2 is the increment of a standard Brownian motion. Then the corresponding value of this part of the business is equal to $V_2 = E\left[\int e^{-rt}\delta_2(t)dt\right] = \frac{F\delta_2}{r-\mu_2}$ where F is the face value of securities held. The value of this business works out to \$570 billion with rate and drift as in the first line of business. The dynamics of this line of business are correspondingly given by:

$$\frac{dV_2}{V_2} = \mu_2 dt + \sigma_2 dz_2 . \qquad (4)$$

It is important to note that the earnings of the firm from this second line of business depend on its creditworthiness. If the firm were to take on more debt or if the bankruptcy costs were to increase because the government may not support the debt holders, financial institutions that trade with GSEs would impose additional margin and other costs when they enter into long term contracts with a GSE. Hence δ_2 (earnings per dollar of securities held) of a GSE would be lower if the costs of hedging and managing the mortgage portfolio were to increase. Empirical estimates suggest that the total funding advantage of GSEs is approximately 40 basis points (e.g., Passmore (2005)). Removing the federal guarantee would increase the hedging costs to this extent so as to incorporate the increased risks borne by corporations that enter into longer term deals (e.g., swaps) with a GSE. We assume that the earnings on the mortgage portfolio are equal to δ_2 minus uncertainty about the guarantee times 40 basis points. In other words, when the government revokes the guarantee, earnings would decline by a maximum of 0.4%.

The total unlevered value (in the absence of debt) of the GSE is then given by the sum of the values of the two business lines. Here the value of the firm at time zero is denoted by $V(t=0)=V_0$:

$$V_0 = V_1(0) + V_2(0). \tag{5}$$

As noted, the returns to the fee business and portfolio of securities are stochastic. Assume that the correlation between the two business lines of these GSEs is $corr(dz_1, dz_2) = \rho$. Clearly the investment portfolio will bear substantial interest rate and credit risk, even if some of the risks are hedged by a GSE. Suppose, a proportion denoted y (where $0 \le y \le 1$), of the funds raised by the firm, are allocated to the holding of securities and the balance (1-y) are allocated to the fee business. Then, the proportion of each business are $V_2 = yV$ and $V_1 = (1-y)V$. Now, the overall firm value is governed by the following process under the risk neutral measure:

$$\frac{dV}{V} = \mu dt + \sigma dz \tag{6}$$

where the total value of the firm is $V = V_1 + V_2$, the drift in the overall firm value is $\mu = (1-y)\mu_1 + y\mu_2$ and $\sigma^2 = (1-y)^2\sigma_1^2 + y^2\sigma_2^3 + 2y(1-y)\rho\sigma_1\sigma_2$ is the instantaneous volatility of the firm's assets and captures all risks that may cause the value of the business to fluctuate. Thus overall asset volatility is a blend of the core business, liquidity risk, interest rate risk of the asset portfolio, among other sources of risk. Also, dt is the increment in time, dz is the increment of a standard Brownian motion[7]. The firm value evolves through time, governed by equation (6) unless the firm value reaches a default triggering value V_B (assume that this exogenous for now).

Uncertainty in the guarantee

Let $1-\alpha$, where 0< α <1, be the fraction of the firm value, V_B, that is lost to bankruptcy costs in case the government does not guarantee the liabilities. This

[7] A payout to claimholders is easily incorporated by subtracting the payout rate.

leaves debt holders with the amount αV_B, and equity holders with nothing. These losses may in part be direct legal costs, loss of human capital and other such costs. Additionally, let α^*, where $0<\alpha^*<1$, to be the amount of firm value, V_B, recovered in case of default when *government backs their debt*, while $1-\alpha^*$ is the proportion lost to bankruptcy costs incurred in this scenario. We set the value of α^* to be greater than the value of α.

In both instances GSEs file for bankruptcy, though it is expected that if government backs their debt, bondholders will recover a higher level of firm value at that time. We assume that there is an exogenous probability p that the government will not cover the losses to debt holders.

3.1 Debt value

Consider debt sold at time zero by equity holders to fund both lines of business, with the following characteristics- infinite maturity and a constant coupon flow C to debt holders each period.

Then, using risk-neutral valuation, the price of debt at time 0 is written as the sum of two components- the expected present value of:

(a) coupon flows if the firm value remains above V_B,

(b) the bankruptcy payouts if the firm value crosses V_B.

Then the debt value can be written as the sum of the two components as:

$$D(V_0)=\int_0^{\infty} e^{-rt}C[1-G(t,V_0,V_B)]dt+\left[p\int_0^{\infty} e^{-rt}\alpha V_B g(t,V_0,V_B)dt+(1-p)\int_0^{\infty} e^{-rt}\alpha^* V_B g(t,V_0,V_B)dt\right] \tag{7}$$

where $g(s,V_1,V_2)$ denotes the density of the first passage time *s* from a level V_1 to firm value V_2 (correspondingly $G(s,V_1,V_2)$ is the cumulative distribution function of the first passage density from V_1 to V_2). An evaluation of equation (2) gives:

$$D(V_0)=\frac{C}{r}\left[1-\left[\frac{V_0}{V_B}\right]^{-x}\right]+\left[p\alpha+(1-p)\alpha^*\right]V_B\left[\frac{V_0}{V_B}\right]^{-x} \tag{8}$$

where $x=\frac{1}{\sigma^2}\left[\left(\mu-\frac{\sigma^2}{2}\right)+\sqrt{\left(\mu-\frac{\sigma^2}{2}\right)^2+2r\sigma^2}\right]$, $V_0=V_1+V_2$, $V_1=\frac{\delta_1}{r-\mu_1}$, $V_2=\frac{F\delta_2}{r-\mu_2}$.

Proof. See Appendix.

Note that the coupon *C* comprises the coupons on the debt raised to finance the mortgage portfolio plus any additional debt raised to finance the core business of the firm. There are several possibilities on how to specify when the firm chooses to file for bankruptcy (the barrier V_B). Lucas and McDonald (2006) assume a level equal to 70% of the value of the liabilities as the trigger point in some examples. If returns to the asset portfolio are negative, and equity holders need to fund coupon payments to the debt holders, the endogenous bankruptcy barrier is characterized by Goldstein, Ju and Leland (2001) as $V_B=\frac{Cx}{r(x+1)}$. In our setting, using base case numbers from the balance sheet for 2003-2004, the endogenous trigger works also works out to approximately 84% of the market value of debt. The substantive implications of our results are largely unchanged for alternate trigger levels for bankruptcy. In subsequent examples we assume an endogenous barrier even though other boundary conditions could be imposed.

3.2 Firm value and equity value

Issuing debt can increase firm value due to tax deductibility of the interest payments, but it increases the potential bankruptcy costs. Bankruptcy costs will depend on the probability that the government will guarantee GSE debt if the firm faces financial distress. If government fully backs GSE debt then the amount lost due to bankruptcy will be low. On the other hand, if GSEs have no guarantee at all from the government, then their bankruptcy costs can reduce substantially the amount of firm value left for bondholders. Bankruptcy costs are the present value of costs conditional on default,

$$BC(V_0) = p\int_0^\infty e^{-rt}(1-\alpha)V_B g(t, V_0, V_B)dt + (1-p)\int_0^\infty e^{-rt}(1-\alpha^*)V_B g(t, V_0, V_B)dt \text{ ,}$$

which gives:

$$BC(V_0) = \left[p(1-\alpha) + (1-p)(1-\alpha^*)\right]V_B\left[\frac{V_0}{V_B}\right]^{-x} \qquad (9)$$

In equation (9), the first term is the probability weighted recovery amount and the last term is related to the probability that the firm goes bankrupt. The term x

is defined in equation (8). The tax benefits equal the tax savings conditional on not defaulting, and are correspondingly given by:

$$TB(V_0) = \frac{\tau C}{r}\left[1 - \left[\frac{V_0}{V_B}\right]^{-x}\right] \qquad (10)$$

where τ is the tax rate. The first term is the present value of the tax benefits of debt and the second term in brackets corresponds to the probability that the firm will not go bankrupt and continues to receive these tax benefits. The total value of the firm is also conditional on the probability of the government support in case of financial distress, as well as on the level of asset at which default is triggered:

$$FV(V_0) = V_0 + \frac{\tau C}{r}\left[1 - \left[\frac{V_0}{V_B}\right]^{-x}\right] - \left[p(1-\alpha) + (1-p)(1-\alpha^*)\right]V_B\left[\frac{V_0}{V_B}\right]^{-x} \qquad (11)$$

Equity value is the total value of the firm minus the value of debt:

$$E(V_0) = F(V_0) - D(V_0) \qquad \text{or}$$

$$E(V_0) = V_0 + \frac{(1-\tau)C}{r}\left[1 - \left[\frac{V_0}{V_B}\right]^{-x}\right] - V_B\left[\frac{V_0}{V_B}\right]^{-x} \qquad (12)$$

In the subsequent sections we discuss the implications of the model obtained above.

4. Model inputs

Equation (12) gives the value of equity as a function of firm value and the volatility of firm value, amongst other variables. A common problem in the implementation of the models of capital structure is that the firm value is not directly observed, and is difficult to estimate for a GSE. Also, the volatility of firm value is not observable. Crosbie and Bohn (2002) propose one solution to back out the asset volatility and the firm value from equation (12) and the relationship:

$$\sigma_E = \frac{\partial E}{\partial V}\left(\frac{V}{E}\right)\sigma . \tag{13}$$

Equation (13) follows from an application of Ito's lemma to equation (12). Note that this relationship is commonly applied by practitioners as they glean out probabilities of default from observed equity values and its volatility. In this context, Lucas and McDonald (2006) calibrate their model to observed volatility of equity values and other parameters for GSEs. The authors obtain the implied volatility of equity values of Fannie Mae from option prices, and find that these values vary between 16.7 percent and 60 percent over the year 2004. In our setting, the value of the unlevered assets can also be gleaned by using the cash flows of the firm. The implied volatility of assets then can be extracted from the equation to characterize the equity value (equation (10)).

For our model, we use our base case numbers with earnings of the first line of business equal to $\delta_1 = 1.4$, $\delta_2 = .0317$, $\mu_1 = 0.01$, $\mu_2 = 0.01$, $r = 6\%$, F=900, the value of the first line of business works out to $28 billion and the second business is worth $570 billion (from equations (2) and (4)). We fix the volatility of the first business at 5%, the implied volatility of asset values for the second line of business at 10%. The bankruptcy trigger is endogenous and C=36 billion.

5. Impact of uncertainty about the implicit federal subsidy

Our objective is to analyze the impact of uncertainty about the federal guarantee on GSE firm value, equity and debt prices. A convenient outcome of this section is that it provides an estimate of the extent to which equity holders gain by the federal subsidy.

5.1 Uncertainty in the subsidy and the value of the firm

There is an ongoing discussion about the extent to which the federal guarantee increases the value of a GSE. In this setting, the federal guarantee reduces the risk of losses to bond holders, and therefore allows the firm to earn a spread between the lower costs of debt financing, relative to the yields on mortgage backed securities. Thus the presence of a government guarantee for GSEs reduces the cost of funds for a GSE relative to other similar risk businesses run by other corporations. Therefore, the GSEs are able to accrue profits over and above what the appropriate risk return tradeoff would warrant. The reduced bankruptcy costs allows the firm to take on more debt, and avail the tax benefits of debt as well as exploit their potential funding advantage. However, an increase in debt beyond a point makes it more likely that the firm will go bankrupt. At some point the firm value stops increasing, and there is no benefit to having more debt. Also, a second effect of increasing the amount of debt is that the cost of hedging and managing the investment portfolio may increase. This impact feeds into the earnings per dollar (δ_2) of the mortgage portfolio held by the GSE.

Figure 3 provides a graphical analysis of the impact of the amount of debt raised (coupon payments) on the value of the firm as a function of *p*, the probability that the government will not pay bond holders at default. We set the interest rate $r = 6\%$, volatility of asset values $\sigma_1 = 0.05$ and $\sigma_2 = 0.1$. It is important to note that the cost of hedging the mortgage portfolio depends on the credit risk and the corresponding probability of no guarantee. If this uncertainty increases, δ_2 (earnings per dollar of securities held) of a GSE would be lower. We set the fractional return $\delta_2 = 0.0317 - 0.004 * p$ so that higher uncertainty results in increased hedging costs. The recovery rates are set at $\alpha = 0.5$ and $\alpha^* = 0.98$.

First note that the as the probability of no guarantee increases, the value of the firm also decreases. This occurs because there is an increased possibility of going bankrupt because of the dual costs of debt- increased probability of going bankrupt as well as reduced earnings on the mortgage portfolio. The decline in the value of the firm occurs because of increased bankruptcy costs, borne for the most part by debt holders. The more significant loss in firm value occurs because of a reduction in the value of the firm from reduced spreads earned on the investment side of the business.

Remark 1: *GSE firm value decreases with uncertainty about the guarantee because of an increase in bankruptcy costs as well as the reduced profits on its mortgage portfolios.*

5.2 Debt values and uncertainty in the subsidy

The potential funding advantage of GSEs allows the firm to raise debt financing at a lower relative spread in comparison with other firms with similar values of asset risk. This funding advantage and its impact on GSE spreads is analyzed by Ambrose and Warga (2002), and others. On the one hand the GSEs are able to raise more funds because of the funding advantage while on the other hand increase leverage may in turn increase spreads. Our objective is to understand the impact of the uncertainty about the federal guarantee on GSE debt prices and spreads.

Figure 3 illustrates the value of debt as a function of the uncertainty in the federal guarantee, when the initial value of the first line of business is set at base case numbers in Section 4, the volatility of asset values are as in the preceding example, the fractional return $\delta_2 = 0.0317 - 0.004 * p$. Note that as p increases, the value of debt decreases. Again, an increase in the uncertainty about the federal guarantee decreases the firm value, and consequently the debt value because of the increased likelihood of going bankrupt. The top line in Figure 3 does not account for increased hedging costs of the mortgage portfolio while the bottom line includes such costs. The margin earned on the investment portfolio is lower when there is a larger possibility that there will be no bail out of the debt holders.

This set up also allows us to examine the extent to which GSEs would hold mortgage portfolios were they to be financed without a government guarantee. As noted by Passmore (2005), if GSEs were purely private, they would hold far fewer MBSs on their books. The value of the portfolio to a GSE would equal its market value plus any diversification benefits that may accrue to the firm. Hence, on average, the incremental gain from holding the securities would be far less than in the current scenario.

Remark 2: *GSE debt values decrease with uncertainty about the guarantee because of an increase in bankruptcy costs as well as the reduced profits on its mortgage portfolios.*

The equation for bond prices obtained in equation (7) also allows us to evaluate the spread of bond yields over treasury bonds: $Spread = \frac{C}{D(V_0)} - r$. We can therefore analyze the extent to which the funding advantage translates into reduced spreads in comparison with similar risk entities that do not have such a government guarantee. Using our base case numbers, this spread works out to approximately 50 basis points, quite close to the estimates obtained by Passmore (2005) but higher than those estimated in Nothaft, Pearce and Stevanovic (2002) and Ambrose and Warga (2002).

Figure 4 provides a graphical depiction of equity values and debt values as a function of the uncertainty. Interestingly equity values benefit when debt values decline. This is so because a decrease in firm value makes debt more risky and there is a wealth transfer to equity holders (a common outcome often discussed in the corporate finance literature).

6. Uncertainty and the value of the implicit subsidy

The value of the implicit subsidy to tax payers has been the focus of much research in the academic literature (see for example CBO studies (2001), Hubbard (2004), Jaffee (2003), Lucas and McDonald (2006), Naranjo and Toevs (2002), Passmore (2005), Stiglitz, Orsag and Orsag (2002)). There are several problems encountered in the computation of this liability. In particular the use of complex derivatives by Fannie and Freddie Mac, limited information in their annual reports and the lack of regulatory oversight by the Securities and Exchange Commission makes it difficult to assess the value of this implicit subsidy. Our approach provides a simple way to compute the value of the subsidy as the present value of payments conditional on default, when the government chooses to pay. Even though it is a gross simplification of the structure of the GSEs, it does provide a useful starting point.

Using the set up in Section 3 the current value of the subsidy is the present value of the cost incurred by the government, conditional on default and is evaluated as:

$$S(V_0) = (1-\alpha)(1-p)V_B \left[\frac{\dfrac{\delta_1}{r-\mu_1} + \dfrac{F\delta_2}{r-\mu_2}}{V_B} \right]^{-x}$$

$$\text{where } x = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2} \right) + \sqrt{\left(\mu - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right] \qquad (14)$$

In equation (14) (1-p) is the probability that the government will reimburse bondholders and the last two terms are the present value of costs incurred. Using our base case numbers and the recovery rates are set at $\alpha = 0.5$ and $\alpha^* = 0.98$ the value of the implicit guarantee works out to 97.8 billion. This estimate is within the range of those computed by Passmore (2005) but is substantially higher than that obtained by Lucas and McDonald (2006).

Uncertainty in the guarantee impacts the payout probability via the first term. Also, as p increases, the value of the subsidy may decrease δ_2 and increase the value of the subsidy because the value of the firm declines, and it makes bankruptcy more likely. For example with a 10 basis point increase in the probability of no support by the government, and using $\delta_2 = 0.0307$, the value of the subsidy increases by 6 billion.

Remark 3: *The cost of the Federal subsidy to taxpayers may increase with uncertainty about the guarantee. The extent of the increase depends on the feedback of the uncertainty on the costs of hedging its asset portfolio.*

Fannie Mae and Freddie Mac assume a large proportion of the credit risk and prepayment risk of the United States housing market that is currently valued at over 9 trillion dollars. In the presence of a funding advantage, GSEs will optimally raise debt financing in order to maximize the value of the firm. At present regulation limits the risk taking by a restriction on investments to conventional and conforming mortgages where the size of the loan is limited. This limit excludes Fannie Mae and Freddie Mac from only a small fraction of the market. Also, there is a capital regulation equal to 2.5% of the balance sheet assets and .45% of off-balance sheet assets. This capital requirement is small in proportion to the amount of debt on the books and is unlikely to significantly impact the bankruptcy barrier or buffer the losses given default. As pointed out, if the government were to take away this guarantee, the expected cost to the government increases. A cap on the size of the portfolio will naturally increase the size of equity relative to debt through time and consequently allow the government to reduce the extent of the subsidy in an orderly manner.

7. Related issues

In our setting, we model a firm where the extent of leverage is fixed. However, in our setting the proportion of the security holdings are adjusted downwards as the business deteriorates. In practice as a firm reduces its holdings of mortgage backed securities, it may also reduce the amount of leverage, were the value of the mortgage portfolio to decline. The impact of such an action is to reduce the overall leverage, but at the same time also reduce the value of the firm. Such an un-levering of the firm reduces firm value and thus increases the probability of going bankrupt on the one hand but reduces the bankruptcy barrier and the point at which bankruptcy is reached on the other hand. In our setting the volatility of the earnings may decline when the firm sells a part of its mortgage portfolio and offload its relatively more risky business.

We assume that the volatility of the firm value and the face value of mortgage holdings are constant even though in reality the volatility of earnings of the firm and the holdings may depend on the value of assets. Such a generalized model can be obtained using the set up, but may not add much to the analysis. One option is to increase the overall value of the input to the volatility of the firm as to account for such a possibility and make the face value of the holdings a function of the firm value.

8. Conclusions

We provide a model of capital structure that incorporates the ability of a GSE to generate a revenue stream by selling mortgage backed securities, as well as by holding these securities on its books via debt financing. The model can be regarded as a reduced form cash flow approach that allows us to analyze the impact of uncertainty about the implicit federal guarantee on GSE debt capacity, bond spreads and equity values.

Regulators are concerned with providing a market based mechanism to control the growth of the GSE portfolios. We show that a government pronouncement that increases the uncertainty about the federal guarantee to a small extent can cause GSEs values to decline by large amounts. The increase in the likelihood that the government will not subsidize the GSEs may increase the expected costs of the subsidy to the government. Thus we argue that a cap on the value of GSE investment portfolios is a more effective mechanism to reduce the growth rate of these entities. Our model is easily applied for discussion about the impact of the implicit subsidy on the values of various financial claims of a GSE.

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Appendix

From Goldstein, Ju and Leland (2001):

$$\int_0^\infty e^{-rt} C\left[1 - F\left(t, V_0, V_T\right)\right] dt = \frac{C}{r}\left(1 - \left[\frac{V_0}{V_T}\right]^{-x}\right) \text{ where}$$

$$x = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right].$$

Also, $\int_0^\infty e^{-rt} \alpha V_B g\left(t, V_0, V_B\right) dt$ is evaluated as: $\left[\alpha V_B\right]\left[\frac{V_0}{V_B}\right]^{-x}$.

Substituting these results in equation (8) gives the desired result.

Table 1
Outstanding Debt and MBS holdings for Fannie Mae and Freddie Mac
Data is obtained from the Bloomberg Database and the Department of Housing and Urban Development's Office of Federal Housing Enterprise Oversight, the Federal Housing Finance Board and Fannie Mae's and Freddie Mac's financial statements.

Date	Fannie Mae		Freddie Mac	
	MBS	Debt	MBS	Debt
1985	55	94	100	13
1986	96	94	169	15
1987	136	97	213	20
1988	170	105	226	27
1989	217	116	273	26
1990	288	123	316	31
1991	355	134	359	30
1992	424	166	408	30
1993	471	201	439	50
1994	486	257	461	93
1995	513	299	459	120
1996	548	331	473	157
1997	579	370	476	173
1998	637	460	478	287
1999	679	548	538	361
2000	707	643	576	427
2001	859	763	653	578
2002	1,029	851	749	666
2003	1,300	962	773	740
2004	1,403	945	852	732

Figure 1
GSE spreads

This figure illustrates the spread between 10yr Fannie Mae notes and 10yr Treasury notes (in percentage points). Data is obtained from the Bloomberg Information Services.

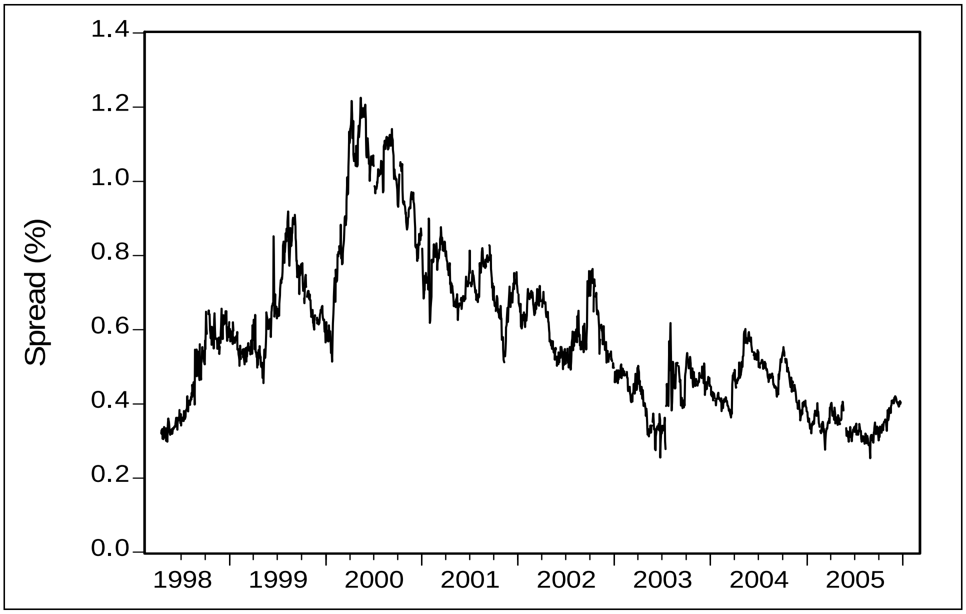

Figure 2
Firm value with uncertainty about the subsidy

This figure illustrates the value of debt (Firm value) as a function of uncertainty about the guarantee (*p*). The earnings of the first line of business is $\delta_1 = 1.4$, $\delta_2 = .0317$, the interest rate $r = 6\%$, volatility of asset values $\sigma_1 = 0.05$, $\sigma_2 = 0.1$, and the recovery rates are set at $\alpha = 0.5$ and $\alpha^* = 0.98$.

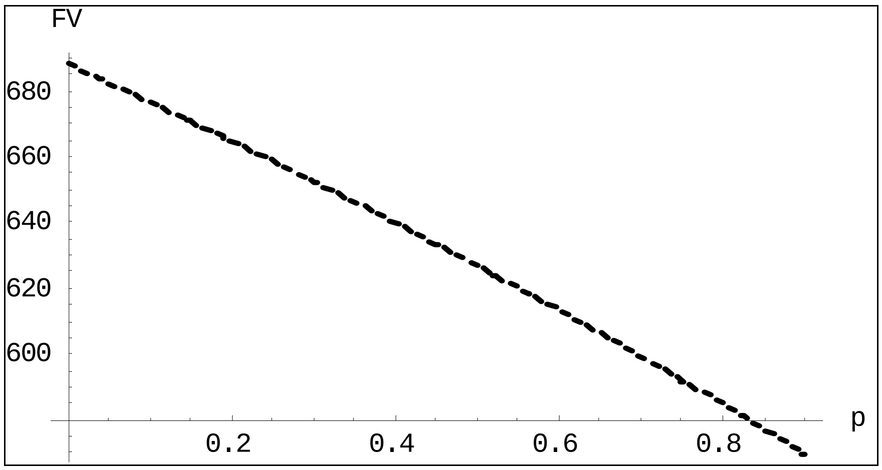

Figure 3
Debt value and uncertainty about the subsidy

This figure illustrates the value of debt (Debt value) as a function of uncertainty about the guarantee (*p*). The earnings of the first line of business is $\delta_1 = 1.4$, $\delta_2 = .0317$, the interest rate $r = 6\%$, volatility of asset values $\sigma_1 = 0.05$, $\sigma_2 = 0.1$, and the recovery rates are set at $\alpha = 0.5$ and $\alpha^* = 0.98$.

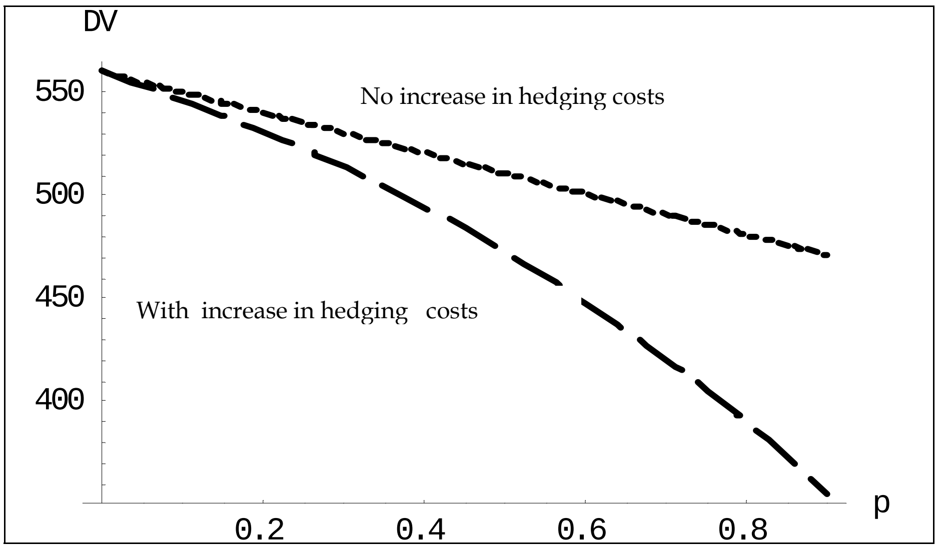

Figure 4

Equity value and Debt Values and uncertainty about the subsidy

This figure illustrates the value of debt (DV) and Equity (EV) as a function of uncertainty about the guarantee (*p*). The earnings of the first line of business is $\delta_1 = 1.4$, $\delta_2 = .0317$, the interest rate $r = 6\%$, volatility of asset values $\sigma_1 = 0.05$, $\sigma_2 = 0.1$, and the recovery rates are set at $\alpha = 0.5$ and $\alpha^* = 0.98$.